Family Bargaining Powers, Education and Fertility
Decisions, and Policy

Akira Yakita

1. Introduction

Women are widely recognized as more strongly preferring to rear children than men are. Lundberg et al. (1997) and Attanasio and Lechene (2002), as a result of their “experiments,” report positive effects of women’s income share on family expenditures for children’s clothing and food. Such a perception has also been reflected in theoretical studies as a higher utility weight on the welfare of their children (e.g., Doepke and Tertilt 2009; Yakita 2018). By contrast, whether the preference for the number of children is stronger for women or not is not clear. The OECD Family Database reports that the average personal ideal numbers of children for women and men aged 15-64 in OECD 22 countries in Europe and some other countries were 2.29 for women and 2.17 for men in 2011. However, after surveying empirical works, Klaus and Strulik (2014) report that women tend to prefer to have fewer children than men in developing, especially, African countries but no difference in Asian countries.¹

Different preferences between women and men might necessitate negotiation about how many children to have and how to rear them at home after marriage. The possibility of such bargaining induces strategic behaviors of women and men related to self-education investment even before marriage and gives rise to inefficiency in family decision-making. This paper first presents an analysis of outcomes of bargaining games related to investment in the premarital personal education and child-rearing time of

¹) The OECD Family Database also reports that the mean personal numbers of children of women and men of aged 15–64 were, respectively, 2.52 and 2.24 in France, 2.32 and 2.14 in the UK, 2.22 and 2.08 in Germany, and 2.01 and 2.00 in Italy in 2011. The URL of the database is http://www.oecd.org/social/family/database.htm. See Figure 1.
spouses, assuming that the difference between women and men is only the difference in the degree of tolerance for child rearing at home. We do not specify either of spouses is more tolerant of child-rearing at home \textit{a priori}. Then the policy implications are derived.

Mazzocco (2007) and Rasul (2008) test intra-family commitment and conclude that non-commitment models of family decision-making are appropriate for policy analyses. Konrad and Lommerud (2000) demonstrate by comparison with results derived from full non-cooperation that overinvestment in education tends to occur and that family public goods such as child-rearing at home might be underprovided in a two-stage bargaining game between women and men. Kemnitz and Thum (2015) endogenize the bargaining power of spouses to demonstrate that family bargaining causes a systematic downward bias on fertility decisions. Our contribution is to present an examination of policy implications related to family bargaining in this analytical line of the literature.

This paper presents consideration of the following two-stage cooperative game, based on a collective-model approach. Women and men choose education levels personally before marriage in stage 1. In stage 2, they marry. Then, taking their wage rates as given,

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Mean personal ideal number of children: mid 2000s}
\end{figure}

\begin{itemize}
\item Note: Women in countries on the left of the 45 degree line prefer to have more children, while men in countries on the right of the 45 degree line prefer to have more children.
\item Source: OECD Family Database (2017)
\end{itemize}
they determine the number of children in the family by contributing to child rearing at home. Without intra-family commitment, the contributions to child-rearing time are determined through family bargaining. The first, and most important, feature of this paper is consideration of children as family public goods. The number of children depends on the child-rearing time spent by parents, as assumed in Galor and Weil (1996).\textsuperscript{2}\hspace{1em} The second feature is heterogeneity between women and men of psychic burden from contributing to family public goods provision, i.e., rearing children at home, although, from the evidence described above, we cannot infer a priori who is more tolerable.\textsuperscript{3}\hspace{1em} Finally, the distribution of bargaining power between spouses in stage 2 is assumed to depend on their relative wage rates, which are determined personally in stage 1.\textsuperscript{4}\hspace{1em}

For analytical purposes, we first define Pareto-efficient solution as the one that would be obtained if women and men make family decisions unitarily, i.e., maximizing the family welfare subject to the resource constraint of the family. Then, the outcome of the family bargaining is assessed from the efficiency. The main results are the following. First, family bargaining tends to engender overinvestment in the education of a more child-rearing-tolerant spouse, underinvestment in the education of the other, and underprovision of family public goods, i.e., a lower fertility rate, relative to the efficient solution. Second, to alleviate or eliminate inefficiency caused by family bargaining, government must resort to education policy (i.e., taxes on education of the more tolerant spouse and subsidies for the other’s education) rather than a family/child policy. For government to alleviate inefficiency caused by strategic behaviors of persons, differentiated taxes on education investment between women and men are required rather than child and family policies. The differentiated tax policy has not been argued in the literature of demography. Actually, despite the ideal numbers of children, many developed countries are well known to have fertility rates only blow the replacement rate of 2.07. If these observations are consequences of individuals’ strategic behaviors, then our results presented in this paper have important policy implications.

The remainder of this paper is structured as the following. Section 2 introduces a model. Assuming the bargaining powers of persons to be constant throughout stages, a

\textsuperscript{2}\hspace{1em} For a seminal paper by Becker and Barro (1988) both parents’ rearing time and purchased goods are assumed to be used for rearing children. However, most subsequent reports of the literature include the assumption that child-rearing costs consist only of foregone income of parents or sometimes mothers. Exceptionally, Apps and Rees (2004), Hirazawa and Yakita (2009), and Kemnitz and Thum (2015) assume both factors in rearing children.


\textsuperscript{4}\hspace{1em} In the present model of two-stage bargaining, as Pollak (2011) argues, wage rates rather than earnings are determinants of bargaining power, whereas Lundberg et al. (2016) infer that the gender earnings ratio is a more appropriate measure of the gender wage gap than the wage ratio.
Pareto-efficient solution is obtained in Section 3. Section 4 presents an analysis of the consequences of family bargaining, with comparison to the efficient solution. Policy implications are discussed in Section 5. Section 6 concludes the paper.

2. Model

A family is assumed to consist of a woman and a man: a couple. Our main concern is the provision of family public goods, i.e., child rearing, in stage 2. Therefore, we assume that persons of the same gender are all identical and every adult marries. Equal sizes of male and female populations are assumed to avoid the issue of matching individuals. The marriage matching process is also assumed to be exogenous. We call a spouse person $F$ and the other person $M$. The payoff of person $i$ in a family is assumed as

$$U^i = c_i + G - \gamma_i a(g_i) - b(w_i),$$

where $c_i$ represents the goods consumption of person $i$, $w_i$ stands for the wage rate, $g_i$ signifies the time-contribution to child rearing, and $G$ denotes the number of children of the family ($i = F, M$). Function $a(g_i)$ is the psychic cost function of the child-rearing contribution. Function $b(w_i)$ stands for the effort function representing educational effort to obtain wage rate $w_i$. We assume that these two functions satisfy the following inequalities: $a'(g_i) > 0$, $a''(g_i) > 0$, $b'(w_i) > 0$, and $b''(w_i) > 0$. Parameter $y$ denotes the personal time endowment. Consumption of person $i$ is given by the product of the labor supply and the wage rate, i.e., $c_i = (y - g_i)w_i$, where the labor supply of person $i$ is the time endowment less the contribution time to child rearing $y - g_i$. It is noteworthy that although the psychic cost is introduced by Konrad and Lommerud (2000), we assign different utility weights on the psychic costs for women and for men, which are the source of conflicts of interest between sexes. Parameter $\gamma_i$ represents the degree of psychic pain of person $i$ from contributing to child rearing. A higher (lower) $\gamma_i$ reflects that the person is heavily (lightly) burdened by contributing to child rearing. For example, Buddelmeyer et al. (2018) report that births of children cause a substantial rise in mothers' time stress while they tend to cause financial stress in fathers' financial stress greater than mothers.

---

5) Although children are also family members, they make no economic decisions. Therefore, to examine family decisions especially, this paper puts them out of the analytical scope.

6) A symmetric equilibrium among families is considered in this paper.

7) This specification is similar to that assumed in Konrad and Lommerud (2000). The quasi-linear payoff function enables us to obtain explicit solutions. Actually, quasi-linearity implies that there are no income effects on contributions to child rearing. A more general payoff function might make it difficult to solve the solution explicitly.

8) Evidence of the return to education of girls and boys is mixed for education levels (Psacharopoulos and Patrinos 2004). Therefore, we assume the same effort function for women and men.
and that the departure of a child reduces parents’ time stress.\(^9\) Whether the stress of births is greater to mothers or fathers might depend on how these stresses are weighted.\(^10\) A greater net benefit from child rearing means a lower utility cost of contributing to child rearing. In the following, we define person \(F\) as a spouse who is more tolerant for child rearing at home although \(F\) can be female or male:

\[
\gamma_F < \gamma_M. \quad (2)
\]

The production function of children is assumed following Galor and Weil (1996) to be given as a linear function of child-rearing time provided by parents as

\[
G = \phi(g_F + g_M). \quad (3)
\]

where \(\phi\) is a positive constant. For analytical simplicity, we assume that childbirth (and bearing) per se does not cause any burden to mothers. We assume that no differences exist between child-rearing services provided by women and by men. They are perfectly substitutable. Therefore, children can be reared equally well either by women or by men.\(^11\)

Budgets of women and men are not pooled in family bargaining. Therefore, spouses face their respective budget constraints.

3. Pareto-efficient solution under given family-welfare weights

In this section, we first derive a Pareto-efficient solution in which the welfare weights of spouses remain constant. The welfare function of a family is

\[
(1 - \bar{\theta})U_F + \bar{\theta}U_M, \quad (4)
\]

where \(\bar{\theta}\) denotes the relative welfare weight of person \(M\). In the Pareto-efficient solution with full intra-family commitment to plans in stage 2. Contributions of each person to family public goods \(g_i\), investment in one’s own education, and consumption of women and men are the choice variables. Education effort is a single-valued function of the wage rate. Therefore, education investment can be represented by the wage rate \(w_i\) \((i = F, M)\).

Pareto-efficient solution can be obtained by maximizing the welfare function subject to

\(^9\) They use panel data of Australia and Germany. Time stress is identified as the Lagrange multiplier on each spouse’s time constraint and financial stress is as the Lagrange multiplier on goods constraint of each spouse.

\(^10\) Although parenting strain is indexed in the psychology literature (Nakajima et al., 1999), mostly the concern is mothers’ stress.

\(^11\) Sometimes child rearing is distinguished from child bearing (Lundberg et al., 2016). The latter can be done only by mothers. We assume away the latter for the analyses described in this paper.
the family’s resource constraint, \((y - g_F)w_F + (y - g_M)w_M - c_F - c_M = 0\). The first-order conditions for the family welfare maximization are the following:

\[
\phi - (1 - \bar{\theta})w_F - (1 - \bar{\theta})\gamma_F a'(g_F) = 0, \tag{5}
\]

\[
\phi - \bar{\theta}w_M - \bar{\theta}\gamma_M a'(g_M) = 0, \tag{6}
\]

\[
y - g_F - b'(w_F) = 0, \tag{7}
\]

\[
y - g_M - b'(w_M) = 0 \quad \text{and} \tag{8}
\]

\[
1 - 2\bar{\theta} = 0. \tag{9}
\]

Because of quasi-linearity of the payoff function, there might not exist a solution satisfying the above first-order conditions. To guarantee an interior solution we must assume that \(\bar{\theta} = 1/2\). The welfare weights of payoff of person \(F\) and \(M\) are identical and equal to \(1/2\). The first terms on the right-hand side of (5) and (6) respectively represent the marginal benefits of child-rearing time for person \(F\) and for person \(M\), whereas the second terms represent the marginal costs of the contribution time. The third terms are the marginal psychic costs. Terms \(y - g_i\) on the right-hand sides of (7) and (8) respectively denote the marginal utility of consumption brought about by an increase in the wage rate for person \(i\) \((i = F, M)\). The last term on the right-hand side of (7) and (8) is the marginal effort cost. From these conditions, we obtain

\[
\phi = \frac{1}{4} \left[ b'^{-1}(y - g_M) + \gamma_M a'(g_M) \right] + \frac{1}{4} \left[ b'^{-1}(y - g_F) + \gamma_F a'(g_F) \right]. \tag{10}
\]

The left-hand side of (10) is the marginal benefit of an additional child rearing time to the family, whereas the right-hand side shows the marginal cost. The marginal cost is represented by the weighted average of the marginal costs for spouses, which in turn consists of the opportunity time cost of child rearing at home and the psychic cost for spouses. This condition corresponds to the Samuelson rule for public goods provision.

Assuming that the second-order (sufficient) conditions are satisfied, we must have

\[
1 - \gamma_i a''(g_i)b''(w_i) < 0 \quad (i = F, M) \tag{11}
\]

(see Appendix A).

From conditions (7) and (8), one can readily obtain

\[
g_M \lessapprox g_F \Leftrightarrow b'(w_M) \lessapprox b'(w_F) \Leftrightarrow w_M \lessapprox w_F. \tag{12}
\]

From conditions (5) and (6), it follows that

12) The utilitarian social objective function has often been assumed in the taxation literature such as Apps and Rees (1988) and Cremer et al. (2016).
From (13) we have

\[
(i) \quad \frac{w_M}{w_F} \geq 1 \geq \gamma_F \frac{\alpha'(g_F)}{\alpha'(g_M)} \quad \text{or} \quad (ii) \quad \gamma_M \frac{\alpha'(g_M)}{\alpha'(g_F)} \geq 1 \geq \frac{w_M}{w_F}.
\]

The only heterogeneity of preferences is the different utility weights of the psychic cost of child rearing: \( \gamma_F < \gamma_M \). Therefore, if one assumes the existence of an efficient solution, then it can be shown that \( g_F > g_M \) and \( w_F < w_M \) must hold from (12), i.e., case (i) of (14). Person \( F \) spends more time on child rearing at home in stage 2 and undertake less education investment in stage 1 than person \( M \) does. Therefore, we have the following proposition:

**Proposition 1:** Assume that the Pareto-efficient solution exists and is interior. Then, in the efficient solution with a lower person \( F \)'s utility weight on the psychic cost of child rearing at home, the education investment of person \( F \) is less than that of person \( M \), although the child-rearing time of person \( F \) is greater than that of person \( M \).

This result can be interpreted intuitively as described below. The psychic cost of child rearing at home for person \( F \) is lower than that for person \( M \). Therefore, the efficiency requires a greater contribution of person \( F \) to rearing children with lower psychic costs. Person \( F \) obtains greater payoffs from a greater number of children even with a lower education investment and lower wage income (see (12)). In contrast, person \( M \) will obtain greater payoffs from investing in education than person \( F \) does (i.e., \( w_F < w_M \)), although the contribution to child rearing at home is smaller. Person \( F \) might avoid expending educational effort, whereas person \( M \) can avoid the burdens of child rearing at home. Person \( F \) might specialize in child rearing at home; person \( M \) shoulders greater burdens of education investment. Increases in the payoffs of spouses raise the family welfare. Full cooperation with intra-family commitment involves the intra-family redistribution of utility in the sense that a higher utility of family welfare deriving from a greater number of children and a higher utility from a higher wage income are shared between the spouses.\(^{13} \)

A spouse spends more time to child rearing while the other earns more. Nevertheless, full-cooperation or unitary-family models are no longer supported empirically (e.g., Lundberg et al., 1997; Attanasio and Lechene, 2002).

Before considering a family game with endogenous bargaining powers, we present the efficient solution \((g_F, g_M, w_F, w_M)\) visually. We can show that combinations of \( g_i \) and \( w_i \)

---

\(^{13}\) The condition is reminiscent of the necessary condition for marriage reported by Becker (1973): the total income produced by the marriage is greater than or equal to the sum of the maximum outputs of single people. In the context of this paper, the family welfare of a couple is not less than the sum of the utilities of single women and men.
satisfying (7) and (8) must be on line \( w_i = W_i(g_i) \) in the \((g, w)\) space, thereby satisfying

\[
\frac{dw_i}{dg_i} = W'(g_i) = \frac{-1}{b'(w_i)} < 0. \tag{15}
\]

The loci \( w_i = W_i(g_i) \) are identical for both persons. Combinations of \( g_i \) and \( w_i \) satisfying (5) and (6), respectively can be regarded as those on lines \( w_i = L_i(g_i) \) \((i = F, M)\). We also have

\[
\frac{dw_i}{dg_i} = L_i'(g_i) = -\gamma_i \alpha'(g_i) < 0. \tag{16}
\]

From (11), (15), and (16), we can show that \( W'(g) > L_F'(g) > L_M'(g) \). Therefore, assuming the existence of the efficient solution, from the second-order condition (11), the two lines \( w_i = L_i(g_i) \) satisfying (15) must intercept line \( w_i = W_i(g_i) \) while satisfying (15) from the northwest to the southeast \((i = F, M)\). This situation is represented in Figure 2, in which the loci are depicted as straight lines for exposition.

4. Family bargaining without intra-family commitment

This section presents analyses of family bargaining outcomes. In the absence of intra-family commitment and intra-family transfers, women and men might renegotiate the allocation of child-rearing time, revealing the bargaining powers posited in stage 2. We analyze the optimal strategy of persons assuming that the distribution of bargaining powers between spouses depends on the relative wage rates, which are already determined by education investment in stage 1. For analytical purposes, we define the bargaining power of person \( M \) in stage 2 as \( \theta(w_M/w_F) \). Function \( \theta(w_M/w_F) \) satisfies \( \theta(w_M^P/w_F^P) = 1/2 \), \( \theta(0) \geq 0 \) and \( \theta(\infty) = \hat{\theta} = 1 \) where \( w_M^P/w_F^P \) presents the wage ratio at Pareto.
efficiency discussed in Section 3. Spouses choose their respective contributions to child rearing at home for given wage rates. Their potential (full-time) earnings ratio is the wage ratio in fertility choice.\footnote{14} We consider that changes in the bargaining power distribution within a family derive deviations from the welfare weights described in the previous section.

It is still plausible that $g_F > g_M$ under assumption (2). In this case, we still have case (i) in (14), i.e., $w_F < w_M$. The range of the bargaining power function must be continuous and $\hat{\theta} \leq \theta(w_M/w_F) \leq \hat{\theta}$ where $\hat{\theta} \equiv \theta(1) > 0$.\footnote{15} We also assume that the bargaining power function satisfies $\theta'(w_M/w_F) \geq 0 \geq \theta''(w_M/w_F)$ for $1 \leq w_M/w_F$.\footnote{16}

Now, we can ascertain a person’s education investment decision by maximizing the family welfare at stage 1 and by anticipating its effects on bargaining power at stage 2. We can solve for the family problem by backward induction. First, the optimal contributions of spouses to family public goods are obtainable by choosing $g_i$ to maximize payoffs (4) for given wage rates $w_i$ ($i = F, M$). The first-order conditions are

\begin{align}
\phi - (1 - \theta)w_F - (1 - \theta)\gamma_F a'(g_F) &= 0 \quad \text{and} \\
\phi - \theta w_M - \theta \gamma_M a'(g_M) &= 0,
\end{align}

where $\theta$ denotes the bargaining power of person $M$. Assuming the existence of the solution, from these conditions, we have $g_i = g[w_i, \theta(w_M/w_F)]$ ($i = F, M$).\footnote{17}

Next, we examine the respective optimum education investments of person $F$ and $M$ at stage 1. It is noteworthy that the effect of education decisions of person $F$ ($M$) on the bargaining power is not controllable to person $M$ ($F$). Therefore, because of the endogeneity of bargaining power, the outcome of collective family decision making might deviate from the efficient one. Anticipating its effect on bargaining power at stage 2, persons choose their education levels strategically, i.e., without caring about the partner’s behavior. The optimal strategies are obtainable as

\begin{align}
- \theta'(\frac{w_M}{w_F}) (U_M - U_F) + (1 - \theta)[y - g_F - b'(w_F)] &= 0 \quad \text{and}
\end{align}

14) Therefore, spouses who fail to accumulate human capital will be disadvantaged by future bargaining. A higher wage rate translates into greater bargaining power. By contrast, Kemnitz and Thum (2015) assume that bargaining power is determined by the actual earnings ratio because the number of children is already determined in choosing how to apportion the income between market childcare and consumption of spouses. Related to this point, see also Basu (2006).

15) We do not rule out the case of $\hat{\theta} \leq 1/2$ in this paper.

16) We cannot rule out the possibility of $w_M/w_F < 1$ in a general case. For analytical convenience, we assume away this possibility for the analyses presented in this paper.

17) In the present setting in which individuals of each sex are identical, we need not consider the issue of marriage.
\[ \theta \left( \frac{1}{w_F} \right) (UM - UF) + \theta (y - g_M - b'(w_M)) = 0, \]  

(20)

where we use the envelop theorem. Although optimal conditions with respect to education investment are independent of the weights when the weights are constant, persons will choose education investment considering its effects on the bargaining power. By rewriting these conditions, we obtain the following.

\[ b'(w_F) = -\left( \frac{\theta'}{1 - \theta} \right) \left( \frac{w_M}{w_F} \right) (UM - UF) + y - g_F, \]

(19)

\[ b'(w_M) = \left( \frac{\theta'}{\theta} \right) \left( \frac{1}{w_F} \right) (UM - UF) + y - g_M, \]

(20)

where it can be shown that in the absence of intra-family transfers, \( UM < UF \) using \( \gamma_M > \gamma_F \) (see Appendix B).

Our purpose in this paper is to present an analysis of the effects of strategic behaviors of persons related to education investment and fertility. Equations (17)–(20) do not determine the bargaining power \( \theta \) if it does not depend on the wage ratio. For expositional purposes, we assume that the gender wage rate is identical with, or sufficiently close to, that at Pareto efficiency if the welfare weight does not depend on the wage rate, i.e., \( w_M / w_F = w_M^P / w_F^P \) if \( \theta' = 0 \), in this section.\(^{18}\) Under the assumption, the respective right-hand sides of (19') and (20') are the same as those of (7) and (8) because \( \theta(w_M^P / w_F^P) = \bar{\theta} = 1/2 \) when \( \theta' = 0 \). However, when bargaining power is dependent on the relative wage rate, i.e., when \( \theta' \neq 0 \), the right-hand side of (19') differs from that of (7) by the absolute value of the first terms: The combination of \( w \) and \( g \) satisfying (19') is located on the left of those satisfying (7) by the right-hand side of (19') in the \((g, w)\) space. This line is shown as \( (F) \) in Figure 2. Similarly, because the right-hand side of (20') differs from that of (8) by the absolute values of the first term, the line for person \( F \) moves rightward by the absolute value of the first term on the right-hand side of (20'). This line is depicted as \( (M) \). Line \( w = W(g) \) when \( \theta' = 0 \) is represented by a dotted line in Figure 3.

To assess the effects of family bargaining on the bargaining power, we assume that the bargaining power determined under family bargaining coincides with that without bargaining in Figure 3, i.e., \( \theta' = 0 \). The change in the bargaining power generated by renegotiation might be interpreted as a deviation from \( \theta' = 0 \). Both cases of \( \theta' = 0 \) and \( \theta' \neq 0 \) are presented in Figure 3, in which lines \( w_i = L_i(g_i) \) are the same as those in Figure 2 (\( i = F, M \)).\(^{19}\) Comparison of the two cases shows that the contribution of person \( F \) to child rearing

\(^{18}\) When \( \theta' = 0 \), equations (17)–(20) determine four variables, \( w_F, w_M, g_F, \) and \( g_M \) for a given \( \theta \). If \( \theta = \theta(w_M^P / w_F^P) \), these variables are equal to those in Pareto efficiency.

\(^{19}\) Precisely, when \( \theta' \neq 0 \), both lines \( w_i = L_i(g_i) \) shift in response to changes in \( \theta \): \( w_F = L_F(g_F) \) moves upward and \( w_M = L_M(g_M) \) shifts downward. We assume that these shifts are not significantly great, neglecting them in Figure 3. If these shifts are not negligible, the results in Proposition 2 should be modified. However, the total fertility rates in developed countries are below 2, such a case might not
at home is less when $\theta' \neq 0$ than when $\theta' = 0$, although the contribution of person $M$ is greater when the bargaining power depends on the wage ratio (i.e., when $\theta' \neq 0$). The education investment of person $F$ is higher when $\theta' \neq 0$ than when $\theta' = 0$, although education investment of person $M$ is lower when $\theta' \neq 0$. We have $\theta = \theta' (w_M/w_F)$ when $\theta' = 0$. Therefore, the absolute value of the first term on the right-hand side of (20) is greater than that of (19). Therefore, we still have $W_i(g) > L_F(g) > L_M(g)$, even when $\theta' \neq 0$. The contribution of person $F$ to child rearing at home becomes smaller, although the contribution of person $M$ is greater. Correspondingly, the education investment of person $F$ is greater, although the education investment of person $M$ is smaller. From inequalities $W_i(g) > L_F(g) > L_M(g)$, it can also be shown that the reduction of child-rearing contribution of person $F$ is sufficient to offset the increase in the contribution of person $M$. Therefore, the total child-rearing time of the family is less when $\theta' \neq 0$ than when $\theta' = 0$. The number of children is smaller when the bargaining power depends on the gender wage ratio, i.e., lower fertility.

Therefore, we have the following proposition:

Proposition 2: If the bargaining power depends on the gender wage ratio and the effect of marginal changes in bargaining power is significant, then the number of children is lower than the Pareto-efficient solution and the education level of person $F$ is higher and the education level of person $M$ is lower than in the Pareto-efficient solution.

The intuition underlying this result can be interpreted as follows. Because $U^M < U^F$, be plausible although we cannot rule out the case theoretically. Feyrer et al. (2008) illustrate that greater bargaining power by women in richer economies induces fathers to spend more time with their children, which in turn engenders greater fertility.
person $F$ strategically raises the welfare weight by increasing the education investment and reducing child-rearing time. The reduction in the number of children (i.e., public goods) induces person $M$ to spend more time on child rearing and to contribute more to family public goods provision. Consequently, person $M$ reduces education investment and consumption. If these behaviors engender deviations from the Pareto efficiency, the level of family welfare would be lowered.

Our results are apparently consistent with the result described by Kemnitz and Thum (2015), who do not consider education investment. Family bargaining might engender a downward bias to fertility. Our results also show that, if women are more tolerant to child rearing, family bargaining might cause an upward bias in the education investment of women and a downward bias in the education investment of men. The upward bias in women’s education is consistent with the result described by Iyigun and Walsh (2007). However, these studies assume that only mothers care for their children at home, the assumption is that it is a source of conflict of interest between women and men. By contrast, it is noteworthy that a source of conflict is only heterogeneous degrees of psychic pain from contributing to child rearing and both spouses care for children in this paper. The bargaining engenders a downward bias in person $M$’s education investment and an upward tendency in person $M$’s child rearing. Person $M$ is not necessarily a man in our model.

5. Policy implications

This section presents analyses of optimal policies to eliminate or at least alleviate the inefficiency deriving from family bargaining. We examine policies of two types, which are related to child-rearing and education investment.

First, we consider subsidies for child rearing, which can be regarded as child allowances in the present setting. Letting $s$ be the subsidy rate and letting $T$ be lump-sum taxes, the payoff to person $i$ can be written as

$$U^i = c_i - T + (1 + s)G - \gamma_i a(g_i) - b(w_i) \quad (i = F, M).$$  \hspace{1cm} (21)

In this case, the subsidies do not affect the conditions for choice of education investment. However, the optimal conditions for the contribution to child rearing are modified as

$$(1 + s)\phi - \theta w_M - \theta \gamma_M a'(g_M) = 0 \quad \text{and}$$  \hspace{1cm} (22a)

20) Yakita (2017) presents an examination of implications of family policy in a two-stage Nash bargaining model, but the examination is conducted based on assumption that women and men have identical payoff functions.

21) We assume away redistribution between spouses through tax-subsidy policy.
\[(1+\delta)\phi - (1-\theta)w_F - (1-\theta)\gamma_F\alpha'(g_F) = 0. \quad (22b)\]

Line \(w_i = L_i(g_i)\) moves to the right in \((g, w)\) space of Figure 2 \((i = F, M)\). Consequently, the contributions of person \(F\) and \(M\) increase. Thereby, the level of child rearing set by the subsidy rate approximately. However, because the combination of the wage rate and public goods provision must satisfy \(w_i = W_i(g_i)\) \((i = F, M)\), the subsidy policy might not achieve the efficient solution in general.\(^{22}\) In this sense, the policy related to child rearing at home might not alleviate and might even eliminate the fertility inefficiency related to family bargaining.\(^{23}\)

Next, we consider (differentiated) taxes on education investment. The tax revenue is rebated to the person as lump-sum transfers. Letting \(\tau_i\) be the tax rate and \(R_i\) be the transfer, the payoff of person \(i\) can be written as

\[
U_i = c_i + R_i + G - \gamma_i\alpha(g_i) - (1 + \tau_i)b(w_i) \quad (i = F, M). \quad (23)
\]

The tax-transfer policy does not affect the optimal conditions for the contribution to child rearing for a given wage rate. However, the optimal conditions for education investment are

\[
(1 + \tau_M)b'(w_M) = \left(\frac{\theta'}{1-\theta} \frac{1}{w_F}\right)(U_M - U_P) + y - g_M, \quad (24)
\]

\[
(1 + \tau_F)b'(w_F) = -\left(\frac{\theta'}{\theta} \frac{1}{w_F}\right)(w_M/w_F)(U_M - U_P) + y - g_F. \quad (25)
\]

Therefore, by setting

\[
\tau_M = \left(\frac{\theta'}{1-\theta} \frac{1}{w_F}\right)(U_M - U_P)b'(w_M) < 0, \quad (26)
\]

\[
\tau_F = -\left(\frac{\theta'}{\theta} \frac{1}{w_F}\right)(w_M/w_F)(U_M - U_P)b'(w_F) > 0, \quad (27)
\]

the policy might achieve the efficient solution for fertility and education.\(^{24}\) With the tax policy of (26) and (27), conditions (24) and (25) respectively coincide with (7) and (8). Because conditions (17) and (18) hold with \(\theta = \theta(w_M/w_F)\) in this case, four equations (17),

\(^{22}\) Even if child-rearing subsidies are differentiated, the efficiency cannot be achieved because of conditions (19) and (20').

\(^{23}\) We can also show that childcare policy cannot achieve the efficiency. Empirical results related to the effects of child policy on fertility are mixed. Rindfuss et al. (2010) and Luci-Greulich and Thévenon (2013) claimed that availability of childcare increases fertility although Mason and Kuhlthau (1992) and Wong and Levine (1992) report that expansion of childcare provision might not increase fertility. In reality, as Figure 3 illustrates, many OECD countries experienced a “rebound” in fertility after child policies were reinforced from about the latter 1990s.

\(^{24}\) We assume here that \(\tau_M > -1\). Otherwise, the after-tax-and-subsidy marginal effort to achieve a wage rate becomes negative.
(18), (24), and (25) with the education tax/subsidy (26) and (27) determine four variables, \( w_i \) and \( g_i \) \((i = F, M)\). If the system has a unique solution, the solution is Pareto efficient with \( \theta (w_M/w_F) = \bar{\theta} \). The education tax rate is negative for person \( M \) and positive for person \( F \). By imposing a positive tax on person \( F \)'s education investment and by subsidizing person \( M \)'s education, the government might achieve the efficiency conditions. The signs of the taxes related to education investment of women and men are expected to be opposite.

Therefore, we obtain the following results related to policies:

Proposition 3: *Fertility inefficiency caused by bargaining might be eliminated by taxes on education-investment expenditures of person \( F \) and \( M \): the tax rate on person \( F \) should be positive, whereas the tax rate on person \( M \) is negative. Child policies might not alleviate the inefficiency.*

The result implies that if children are family public goods, then government should rely on education policy rather than family/child policy to alleviate and eliminate the fertility inefficiency caused by family bargaining. With fertility inefficiency caused by strategic behaviors, the tax and subsidy policy related to education investment would be socially desirable. The result of the differentiated taxes on education investment between genders is novel.

At this stage of argument, it is also noteworthy that because of the quasi-linearity of payoff function (1), income redistribution between spouses in a lump-sum manner does not affect education investment and fertility decisions of families in the setup of this paper. However, for instance, the transfers from men to women increase mothers’ consumption, which is expected to include more goods for children than fathers’ consumption.

6. Concluding remarks

We analyzed the effects of family bargaining on the number of children as family public goods in a collective model which assumes that the family maximizes the weighted average of the payoffs of its spouses. If the respective bargaining powers of spouses depend on the gender wage ratio, then family bargaining exerts a downward bias to fertility, i.e., fertility inefficiency in the sense that the fertility rate is lower than the

25) Calculation of the tax rate might be difficult in reality. The policy might involve intra-family transfers.

26) We have not discussed the components of parents’ consumption in this paper.

efficient fertility rate. The effects of bargaining also raise the education level of a spouse who is more tolerable to child rearing at home and pull down the education level of the other spouse. In many economically developed countries, governments undertake various family and child policies to raise the fertility rate because the fertility rate is likely to be lower than the replacement rate. However, the result in this study predicts that if inefficiency derives from such strategic behaviors of individuals, then such family and child policies might not alleviate fertility inefficiency. By contrast, education policy can achieve the efficient fertility rate.

We can present a remark: It is noteworthy that our result does not mean that investment in education must always be taxed or subsidized. Taxation of education spending can be justified only when the strategic behaviors of individuals cause inefficient allocation. Without commitment, spouses want to obtain advantageous benefits in bargaining family public goods provision by bearing additional efficiency costs of excessive education activities.\(^{28}\)

Appendices

A: Derivation of the second-order condition for family welfare maximization

Using the first-order conditions for family welfare maximization \((5)\)–\((8)\), we obtain

\[
-\frac{\partial w_M}{\partial g_M} - \gamma_M a' d g_M = 0, \quad (5)
\]

\[
-\frac{\partial w_F}{\partial g_F} - \gamma_F a' d g_F = 0, \quad (6)
\]

\[
-\frac{\partial g_M}{\partial w_M} - b a' = 0 \quad \text{and} \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad
\]

from which we have the following Hessian matrix:

\[
\begin{pmatrix}
-\gamma_M a'(g_M) & 0 & -1 & 0 \\
0 & -\gamma_F a'(g_F) & 0 & -1 \\
-1 & 0 & -b'(w_M) & 0 \\
0 & -1 & 0 & -b'(w_F)
\end{pmatrix}.
\]

\[\text{(A1)}\]

Therefore, we have the following inequalities from the second-order (sufficient) conditions:

\[
-\gamma_M a'(g_M) < 0, \quad \text{(A2)}
\]

\[
\gamma_M a'(g_M)\gamma_F a'(g_F) > 0, \quad \text{(A3)}
\]

\[\text{28)}\] In a family bargaining model with perfect intergenerational altruism, Yakita (2018) shows that it is optimal with plausible model parameters that both female education and fertility increase along economic development.
\[
\begin{vmatrix}
- \gamma_M \alpha'(g_M) & 0 & -1 \\
0 & - \gamma_F \alpha'(g_F) & 0 \\
-1 & 0 & - b^*(w_F)
\end{vmatrix}
\]

\[= - \gamma_F \alpha'(g_F)[\gamma_M \alpha'(g_M)b^*(w_F) - 1] < 0, \quad (A4)\]

\[
\begin{vmatrix}
- \gamma_M \alpha'(g_M) & 0 & -1 & 0 \\
0 & - \gamma_F \alpha'(g_F) & 0 & -1 \\
-1 & 0 & - b^*(w_M) & 0 \\
0 & -1 & 0 & - b^*(w_F)
\end{vmatrix}
\]

\[= [1 - \gamma_M \alpha'(g_M)b^*(w_M)][1 - \gamma_F \alpha'(g_F)b^*(w_F)] > 0. \quad (A5)\]

From (A4) and (A5), one obtains \(1 - \gamma, \alpha'(g_i)b^*(w_i) < 0 \) \((i = F, M)\).

**B: Proof of** \(U^M < U^F\)

If \(\gamma_M = \gamma_F\), then we have the same strategies \(w_M = w_F\) and \(g_M = g_F\) because women and men are identical. Presuming that \(\gamma_F\) declines but \(\gamma_M\) does not change, then the changes in the person \(F\)'s strategy (Nash response in this case) must satisfy the following conditions:

\[
\phi - w_F - \gamma_F \alpha'(g_F) = 0 \quad \text{and} \quad \gamma - g_F - b'(w_F) = 0. \quad (A6)\]

It is noteworthy that both persons \(F\) and \(M\) have the same optimal strategy if \(\gamma_M = \gamma_F\). Evaluating the effects of a change in \(\gamma_F\) at \(\gamma_M = \gamma_F\), we have

\[
\begin{pmatrix}
-1 & - \gamma_F \alpha' \\
- b^* & -1
\end{pmatrix}
\begin{bmatrix}
\frac{dw_F}{d\gamma_F} \\
\frac{dg_F}{d\gamma_F}
\end{bmatrix}
= \begin{pmatrix}
a' \\
0
\end{pmatrix}. \quad (A7)
\]

From (A7) we obtain

\[
\frac{dw_F}{d\gamma_F} = H^{-1}(-a') \quad \text{and} \quad (A8)
\]

\[
\frac{dg_F}{d\gamma_F} = H^{-1} b^* a', \quad (A9)
\]

where \(H = (1 - \gamma_F a' b^*) < 0\) from the second-order condition. Therefore, we have \(dw_F/d\gamma_F > 0\) and \(dg_F/d\gamma_F < 0\). Inserting these into the person \(F\)'s payoffs and using (A6), we obtain the following.

\[
\frac{dU^F}{d\gamma_F} = -(w_F - \phi + \gamma_F a') \frac{dg_F}{d\gamma_F} + (\gamma - g_F - b'(w_F)) \frac{dw_F}{d\gamma_F} - a(g_F)
\]

\[= - a(g_F) < 0. \quad (A10)\]

Consequently, if the psychic cost to person \(F\) declines, then the payoff of person \(F, U^F\), becomes higher. It is noteworthy that we ignore the response from person \(M\). The increased contribution of person \(F\) increases family public goods (i.e., the number of children). Therefore, it benefits the payoff of person \(M\). However, because person \(M\) is not expected to reduce the contribution by an equal amount, the sign of (A10) will not be reversed even if considering the response of person \(M\). Therefore, \(U^M < U^F\) when \(\gamma_M > \gamma_F\).
Acknowledgments

The author thanks Real Arai, Yun Ho Chung, Dirk Bethmann, Susumu Imai, Jun-ichi Itaya, Shaoying Ma, Atsushi Miyake, Atsue Mizushima, Kaoru Ueda, Masaya Yasuoka, and seminar participants at the Western Economic Association International (WEAI) 93rd Annual Conference, the Kansai Macroeconomics Study Group (KMSG), the 2017 International Conference of Korean Economic and Business Association (KEBA), Hokkaido University, Nagoya City University, and Nanzan University for their helpful comments and suggestions. Financial support from the Japan Society for the Promotion of Science KAKENHI Grant No. 16H03635 is gratefully acknowledged.

References

Klaus, P., Strulik, H. 2014. Gender equity and the escape from poverty. Center for European Governance


(Professor, Faculty of Economics, Nanzan University, Dr. of Economics)