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Global solutions for the Navier-Stokes equations in the rotational framework

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Abstract

The existence of global solutions for the Navier-Stokes equations with the Coriolis force is considered in the homogeneous Sobolev spaces. Without Coriolis force, it is known that the unique global solutions are obtained if the initial data is sufficiently small. In this paper, the unique global solutions are obtained for large initial data if the speed of rotation is sufficiently large.

Key words: Navier-Stokes equations, Coriolis force, Global solutions

1 Introduction

We consider the initial value problem for the Navier-Stokes equations with the Coriolis force

$$\begin{cases} \dfrac{\partial u}{\partial t} - \Delta u + \Omega e_3 \times u + (u \cdot \nabla)u + \nabla p = 0 & \text{in } \mathbb{R}^3 \times (0,\infty), \\ \operatorname{div} u = 0 & \text{in } \mathbb{R}^3 \times (0,\infty), \\ u(x,0) = u_0(x) & \text{in } \mathbb{R}^3, \end{cases} \tag{NSC}$$

where $u = u(x,t) = (u_1(x,t), u_2(x,t), u_3(x,t))$ and $p = p(x,t)$ denote the unknown velocity field and the unknown pressure of the fluid at the point $(x,t) \in \mathbb{R}^3 \times (0,\infty)$, respectively, while $u_0 = u_0(x) = (u_{0,1}(x), u_{0,2}(x), u_{0,3}(x))$ denotes the given initial

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velocity field satisfying the compatibility condition div $u_0 = 0$. Here, $\Omega \in \mathbb{R}$ is the speed of rotation around the vertical unit vector $e_3 = (0, 0, 1)$.

The purpose of this paper is to show the existence and the uniqueness of the global solutions to (NSC) in the homogeneous Sobolev spaces $\dot{H}^s(\mathbb{R}^3)$ ($s \geq 1/2$). In particular, we obtain global solutions for large initial velocity u_0 if the speed of the rotation is sufficiently fast. For the existence of global solutions to (NSC), Chemin-Desjardins-Gallagher-Grenier [5,6] proved that for any initial data $u_0 \in L^2(\mathbb{R}^2)^2 + H^{\frac{1}{2}}(\mathbb{R}^3)^3$, there exists a positive parameter Ω_0 such that for every $\Omega \in \mathbb{R}$ with $|\Omega| \geq \Omega_0$ there exists a unique global solution. Babin-Mahalov-Nicolaenko [1–3] showed the existence of global solutions and the regularity of the solutions to (NSC) for the periodic initial data with large $|\Omega|$. On the other hand, Giga-Inui-Mahalov-Saal [11] showed the existence of global solutions for small initial data $u_0 \in FM_0^{-1}(\mathbb{R}^3)^3$, where the condition of smallness is independent of the speed of the rotation Ω, and $FM_0^{-1}(\mathbb{R}^3)$ is scaling invariant to (NSC) with $\Omega = 0$. Indeed, for the solution u to (NSC) with $\Omega = 0$, let $u_\lambda(x,t) := \lambda u(\lambda x, \lambda^2 t)$ for $\lambda > 0$. Then, u_λ is also a solution to (NSC) with $\Omega = 0$ and we have $\|u_\lambda(\cdot, 0)\|_{FM_0^{-1}} = \|u(\cdot, 0)\|_{FM_0^{-1}}$ for all $\lambda > 0$. On such other results of global solutions for small initial data, Hieber-Shibata [12] studied in the Sobolev space $H^{\frac{1}{2}}(\mathbb{R}^3)$, Konieczny-Yoneda [18] studied in the Fourier-Besov space $F\dot{B}^{2-\frac{3}{p}}_{p,\infty}(\mathbb{R}^3)$ with $1 < p \leq \infty$. On the well-posedness for (NSC) with $\Omega = 0$ in the scaling invarint spaces, we refer to Fujita-Kato [7], Kato [14], Kozono-Yamazaki [19], Koch-Tataru [17]. On the local existence of solutions to (NSC), we refer to the results by Giga-Inui-Mahalov-Matsui [9,10] and Sawada [20]. In our previous result [13], we showed that it is possible to take the existence time of the solutions long for initial data $u_0 \in \dot{H}^s(\mathbb{R}^3)^3$ ($1/2 < s < 5/4$) if the speed of rotation Ω is sufficiently large.

In this paper, we establish the existence theorem on global solutions to (NSC) for the initial velocity u_0 in the homogeneous Sobolev spaces $\dot{H}^s(\mathbb{R}^3)$ ($1/2 \leq s < 3/4$). In the case $s > 1/2$, the existence of global solutions is obtained if the speed of rotation Ω is large compared with the norm of initial data $\|u_0\|_{\dot{H}^s}$. On the other hand, in the critical case $s = 1/2$, the speed $|\Omega|$ to obtain the existence of global solutions is determined by each precompact set $K \subset \dot{H}^{\frac{1}{2}}(\mathbb{R}^3)^3$, which the initial data belongs to.

We consider the following integral equation:

$$u(t) = T_\Omega(t)u_0 - \int_0^t T_\Omega(t-\tau)\mathbb{P}\nabla \cdot (u \otimes u)d\tau, \qquad \text{(IE)}$$

where $\mathbb{P} = (\delta_{ij} + R_i R_j)_{1 \leq i,j \leq 3}$ denotes the Helmholtz projection onto the divergence-free vector fields and $T_\Omega(\cdot)$ denotes the semigroup corresponding to the linear problem of (NSC), which is given explicitly by

$$T_\Omega(t)f = \mathcal{F}^{-1}\Big[\cos\Big(\Omega\frac{\xi_3}{|\xi|}t\Big)e^{-t|\xi|^2}I\widehat{f}(\xi) + \sin\Big(\Omega\frac{\xi_3}{|\xi|}t\Big)e^{-t|\xi|^2}R(\xi)\widehat{f}(\xi)\Big]$$

for $t \geq 0$ and divergence-free vector fields f. Here, I is the identity matrix in $\mathbb{R}^3$, R_j $(j = 1, 2, 3)$ is the Riesz transform and $R(\xi)$ is the skew-symmetric matrix symbol related to the Riesz transform, which is defined by

$$R(\xi) := \frac{1}{|\xi|} \begin{pmatrix} 0 & \xi_3 & -\xi_2 \\ -\xi_3 & 0 & \xi_1 \\ \xi_2 & -\xi_1 & 0 \end{pmatrix} \quad \text{for } \xi \in \mathbb{R}^3 \setminus \{0\}.$$

We refer to Hieber-Shibata [12] for the derivation of the explicit form of $T_\Omega(\cdot)$.

Theorem 1.1 *Let $\Omega \in \mathbb{R} \setminus \{0\}$, and let s, p and θ satisfy*

$$\frac{1}{2} < s < \frac{3}{4}, \quad \frac{1}{3} + \frac{s}{9} < \frac{1}{p} < \frac{2}{3} - \frac{s}{3}, \tag{1.1}$$

$$\frac{s}{2} - \frac{1}{2p} < \frac{1}{\theta} < \frac{5}{8} - \frac{3}{2p} + \frac{s}{4}, \quad \frac{3}{4} - \frac{3}{2p} \leq \frac{1}{\theta} < 1 - \frac{2}{p}. \tag{1.2}$$

Then, there exists a positibe constant $C = C(s, p, \theta) > 0$ such that for any initial velocity field $u_0 \in \dot{H}^s(\mathbb{R}^3)^3$ with

$$\|u_0\|_{\dot{H}^s} \leq C|\Omega|^{\frac{s}{2}-\frac{1}{4}} \quad \text{and} \quad \operatorname{div} u_0 = 0, \tag{1.3}$$

there exists a unique global solution $u \in C([0,\infty), \dot{H}^s(\mathbb{R}^3))^3 \cap L^\theta(0,\infty; \dot{H}^s_p(\mathbb{R}^3))^3$ to (NSC).

Remark 1.2 The existence of global solutions for small initial data $u_0 \in \dot{H}^{\frac{1}{2}}(\mathbb{R}^3)^3$ were shown by Hieber-Shibata [12]. The size condition (1.3) on initial data can be regarded as a continuous extension of that in $\dot{H}^{\frac{1}{2}}(\mathbb{R}^3)^3$. Indeed, Hieber-Shibata [12] assumed the smallness condition $\|u_0\|_{H^{\frac{1}{2}}} \leq \delta$ for some $\delta > 0$, which corresponds to our condition (1.3) with $s = 1/2$.

Remark 1.3 The space $L^{\theta_0}(0,\infty; \dot{H}^{s_0}_{p_0}(\mathbb{R}^3))$ is scaling invariant to (NSC) in the case $\Omega = 0$ if θ_0, s_0 and p_0 satisfy

$$\frac{2}{\theta_0} + \frac{3}{p_0} = 1 + s_0. \tag{1.4}$$

On the first condition of (1.2), we see that

$$\frac{2}{\theta} + \frac{3}{p} < \frac{5}{4} + \frac{s}{2} < 1 + s \quad \text{if } s > \frac{1}{2}.$$

Therefore, the space $L^\theta(0,\infty; \dot{H}^s_p(\mathbb{R}^3))$ in Theorem 1.1 includes more regular functions than those in the scaling invariant spaces.

By Theorem 1.1 for the case $s > 1/2$, it is possible to obtain global solutions for initial data $u_0 \in \dot{H}^s(\mathbb{R}^3)^3$ if Ω satisfies

$$|\Omega| > C\|u_0\|_{\dot{H}^{\frac{1}{2}}}^{\frac{2}{s-\frac{1}{2}}}. \tag{1.5}$$

Therefore, the speed $|\Omega|$ of rotation to obtain global solutions is determined by the each bounded set in $\dot{H}^s(\mathbb{R}^3)$ if $s > 1/2$. We next consider the critical case $s = 1/2$.

Theorem 1.4 *For any $u_0 \in \dot{H}^{\frac{1}{2}}(\mathbb{R}^3)^3$ with $\operatorname{div} u_0 = 0$, there exists $\omega = \omega(u_0) > 0$ such that for any $\Omega \in \mathbb{R}$ with $|\Omega| > \omega$, there exists a unique global solution u to* (NSC) *in $C([0,\infty), \dot{H}^{\frac{1}{2}}(\mathbb{R}^3))^3 \cap L^4(0,\infty; \dot{H}_3^{\frac{1}{2}}(\mathbb{R}^3))^3$.*

Remark 1.5 The space $L^4(0,\infty; \dot{H}_3^{\frac{1}{2}}(\mathbb{R}^3))$ in Theorem 1.6 is scaling invariant space in the case $\Omega = 0$ since $\theta_0 = 4$, $s_0 = 1/2$ and $p_0 = 3$ satisfy (1.4).

Since the condition (1.5) breaks down in the case $s = 1/2$, it is not clear whether the Coriolis parameter Ω to obtain global solutions for initial data $u_0 \in \dot{H}^{\frac{1}{2}}(\mathbb{R}^3)^3$ can be characterized by the norm of initial data $\|u_0\|_{\dot{H}^{\frac{1}{2}}}$ such as (1.5). To overcome this difficulty, we consider a class of precompact subsets in $\dot{H}^{\frac{1}{2}}(\mathbb{R}^3)^3$.

Theorem 1.6 *Let K be an arbitrary precompact set in $\dot{H}^{\frac{1}{2}}(\mathbb{R}^3)^3$. Then, there exists $\omega(K) > 0$ such that for any $\Omega \in \mathbb{R}$ with $|\Omega| > \omega(K)$ and for any $u_0 \in K$ with $\operatorname{div} u_0 = 0$, there exists a unique global solution u to* (NSC) *in $C([0,\infty), \dot{H}^{\frac{1}{2}}(\mathbb{R}^3))^3 \cap L^4(0,\infty; \dot{H}_3^{\frac{1}{2}}(\mathbb{R}^3))^3$.*

Remark 1.7 For the original Navier-Stokes equations

$$\begin{cases} \dfrac{\partial u}{\partial t} - \Delta u + (u \cdot \nabla)u + \nabla p = 0 & \text{in } \mathbb{R}^3 \times (0,\infty), \\ \operatorname{div} u = 0 & \text{in } \mathbb{R}^3 \times (0,\infty), \\ u(x,0) = u_0(x) & \text{in } \mathbb{R}^3, \end{cases} \tag{NS}$$

it is known by the results of Brezis [4], Giga [8] and Kozono [15] that the existence time T of local solutions for initial data in $L^r(\mathbb{R}^3)$ $(3 < r < \infty)$ and $L^3(\mathbb{R}^3)$ is determined by the each bounded set B in $L^r(\mathbb{R}^3)$ $(3 < r < \infty)$ and the each precompact set K in $L^3(\mathbb{R}^3)$, respectively. Note that the space $L^3(\mathbb{R}^3)$ is a scaling critical space to (NS). On the other hand, the sufficint speed Ω to obtain global solutions is determined by the bounded sets and precompact sets in Theorem 1.1 and Theorem 1.6, respectively. Therefore, our theorems can be regarded as a counterpart of such results from the viewpoint of the Coriolis parameter Ω for the existence of global solutions.

On the existence of local solutions in $\dot{H}^s(\mathbb{R}^3)$, it is also expected that the sufficient speed Ω to obtain local solutions is determined by the existence time $T > 0$, and each bounded set for the case $s > 1/2$ or each precompact set for the case $s = 1/2$. In our previous result [13], we considered the case $s > 1/2$ and showed that the existence time $T > 0$ satisfies $T \geq c|\Omega|^{\alpha} \|u_0\|_{\dot{H}^s}^{-\beta}$ with some constants $c, \alpha, \beta > 0$. By this result, we see that for the time $T > 0$ and the bounded set B in $\dot{H}^s(\mathbb{R}^3)$, the sufficient speed Ω to obtain local solutions is determined by T and B if $s > 1/2$. In the case $s = 1/2$, the following is our theorem for local solutions.

Theorem 1.8 *For any $T > 0$ and precompact set K in $\dot{H}^{\frac{1}{2}}(\mathbb{R}^3)$, there exists $\omega = \omega(T, K) > 0$ such that for any $\Omega \in \mathbb{R}$ with $|\Omega| > \omega$ and for any $u_0 \in K$ with* $\operatorname{div} u_0 = 0$, *there exists a unique local solution u to* (NSC) *in* $C([0, T), \dot{H}^{\frac{1}{2}}(\mathbb{R}^3))^3 \cap L^4(0, T; \dot{H}_3^{\frac{1}{2}}(\mathbb{R}^3))^3$.

Remark 1.9 For any precompact set K in $\dot{H}^{\frac{1}{2}}(\mathbb{R}^3)$, the constant $\omega(T, K) > 0$ in Theorem 1.8 is increasing and bounded with respect to $T > 0$. Indeed, $\omega(T, K) > \omega(\widetilde{T}, K)$ if $T > \widetilde{T}$ since a local solution on the time interval $[0, T)$ is also a solution on $[0, \widetilde{T})$. By Theorem 1.6 for global solutions, it suffices to take $|\Omega|$ sufficiently large to obtain global solutions and the lower bound $\omega(T, K)$ for local solutions does not diverge to infinity as $T \to \infty$.

This paper is organized as follows. In Section 2, we introduce propositions to prove theorems which are on linear estimates for the semigroup $T_{\Omega}(\cdot)$ and the bilinear estimate. In Section 3, we prove Theorem 1.1, Theorem 1.6 and Theorem 1.8.

2 Preliminaries

In what follows, we denote by $C > 0$ various constants and by $0 < c < 1$ various small constants. In order to introduce propositions to prove theorems, let us recall the definition of the homogeneous Besov spaces in brief. Let ϕ be a radial smooth function satisfying

$$\operatorname{supp} \widehat{\phi} \subset \{\, \xi \in \mathbb{R}^3 \,|\, 2^{-1} \leq |\xi| \leq 2 \,\}, \quad \sum_{j \in \mathbb{Z}} \widehat{\phi}(2^{-j}\xi) = 1 \quad \text{for any } \xi \in \mathbb{R}^3 \setminus \{0\}.$$

Let $\{\phi_j\}_{j \in \mathbb{Z}}$ be defined by

$$\phi_j(x) := 2^{3j} \phi(2^j x) \quad \text{for } j \in \mathbb{Z}, x \in \mathbb{R}^3.$$

Then, for $s \in \mathbb{R}, 1 \leq p, q \leq \infty$, the homogeneous Besov space $\dot{B}^s_{p,q}(\mathbb{R}^3)$ is defined by the set of all tempered distributions $f \in \mathcal{S}'(\mathbb{R}^3)$ with

$$\|f\|_{\dot{B}^s_{p,q}} := \left\| \left\{ 2^{sj} \|\phi_j * f\|_{L^p(\mathbb{R}^3)} \right\}_{j \in \mathbb{Z}} \right\|_{\ell^q(\mathbb{Z})} < \infty.$$

Lemma 2.1 *[13] Let $2 \leq p \leq \infty$. There exists $C > 0$ such that*

$$\|\mathcal{F}^{-1} e^{\pm i \frac{\xi_3}{|\xi|} \Omega t} \mathcal{F} f\|_{\dot{B}^0_{p,2}} \leq C \left\{ \frac{\log(e + |\Omega| t)}{1 + |\Omega| t} \right\}^{\frac{1}{2}(1 - \frac{2}{p})} \|f\|_{\dot{B}^{3(1-\frac{2}{p})}_{\frac{p}{p-1}, 2}} \tag{2.1}$$

for all $\Omega \in \mathbb{R}$, $t > 0$, $f \in \dot{B}^{3(1-\frac{2}{p})}_{\frac{p}{p-1},2}(\mathbb{R}^3)$.

Lemma 2.2 *Let* $1 < q \le 2 \le p < \infty$ *satisfy* $1/q \ge 1 - 1/p$. *Then, there exists* $C > 0$ *such that*

$$\|T_\Omega(t)f\|_{L^p} \le Ct^{-\frac{3}{2}(\frac{1}{q}-\frac{1}{p})}\left\{\frac{\log(e+|\Omega|t)}{1+|\Omega|t}\right\}^{\frac{1}{2}(1-\frac{2}{p})}\|f\|_{L^q} \tag{2.2}$$

for all $\Omega \in \mathbb{R}, t > 0, f \in L^q(\mathbb{R}^3)$.

Proof. By the continuous embedding $\dot{B}^0_{p,2}(\mathbb{R}^3) \hookrightarrow L^p(\mathbb{R}^3)$ and (2.1), we have

$$\|T_\Omega(t)f\|_{L^p} \le C\|T_\Omega(t)f\|_{\dot{B}^0_{p,2}} \le C\left\{\frac{\log(e+|\Omega|t)}{1+|\Omega|t}\right\}^{\frac{1}{2}(1-\frac{2}{p})}\|e^{t\Delta}f\|_{\dot{B}^{3(1-\frac{2}{p})}_{\frac{p}{p-1},2}}.$$

And we have from Lemma 2.2 in [16] and the continuous embedding $L^q(\mathbb{R}^3) \hookrightarrow \dot{B}^0_{q,2}(\mathbb{R}^3)$

$$\|e^{t\Delta}f\|_{\dot{B}^{3(1-\frac{2}{p})}_{\frac{p}{p-1},2}} \le Ct^{-\frac{3}{2}(\frac{1}{q}-\frac{1}{p})}\|f\|_{\dot{B}^0_{q,2}} \le Ct^{-\frac{3}{2}(\frac{1}{q}-\frac{1}{p})}\|f\|_{L^q(\mathbb{R}^3)}.$$

Therefore, we obtain (2.2). ❑

Proposition 2.3 *[13] Let* $2 < p < 6$, $2 < \theta < \infty$ *satisfy*

$$\frac{3}{4} - \frac{3}{2p} \le \frac{1}{\theta} < 1 - \frac{2}{p}.$$

Then, there exists $C > 0$ *such that*

$$\|T_\Omega(\cdot)f\|_{L^\theta(0,\infty;L^p)} \le C|\Omega|^{-\frac{1}{\theta}+\frac{3}{4}(1-\frac{2}{p})}\|f\|_{L^2}$$

for all $\Omega \in \mathbb{R} \setminus \{0\}$, $f \in L^2(\mathbb{R}^3)$.

Proposition 2.4 *For every* $f \in \dot{H}^{\frac{1}{2}}(\mathbb{R}^3)$, *it holds that*

$$\lim_{|\Omega|\to\infty} \|T_\Omega(\cdot)f\|_{L^4(0,\infty;\dot{H}^{\frac{1}{2}}_3)} = 0. \tag{2.3}$$

Proof. Since $\mathcal{S}(\mathbb{R}^3)$ is dense in $\dot{H}^{\frac{1}{2}}(\mathbb{R}^3)$, there exists $\{f_N\}_{N=1}^\infty \subset \mathcal{S}(\mathbb{R}^3)$ such that $f_N \to f$ in $\dot{H}^{\frac{1}{2}}(\mathbb{R}^3)$. Then, we have from Proposition 2.3

$$\begin{aligned}\|T_\Omega(\cdot)f\|_{L^4(0,\infty;\dot{H}^{\frac{1}{2}}_3)} &\le \|T_\Omega(\cdot)(f_N - f)\|_{L^4(0,\infty;\dot{H}^{\frac{1}{2}}_3)} + \|T_\Omega(\cdot)f_N\|_{L^4(0,\infty;\dot{H}^{\frac{1}{2}}_3)} \\ &\le C\|f_N - f\|_{\dot{H}^{\frac{1}{2}}} + \|T_\Omega(\cdot)f_N\|_{L^4(0,\infty;\dot{H}^{\frac{1}{2}}_3)}.\end{aligned} \tag{2.4}$$

On the second term of the last right hand side, we have from (2.1), (2.2) and Lebesgue's dominated convergence theorem

$$
\begin{aligned}
\|T_\Omega(\cdot)f_N\|_{L^4(0,1;\dot{H}^{\frac{1}{2}}_3)} &\le C\Big\|\Big\{\frac{\log(e+|\Omega|t)}{1+|\Omega|t}\Big\}^{\frac{1}{2}(1-\frac{2}{3})}\|f_N\|_{\dot{B}^{\frac{1}{2}+3(1-\frac{2}{3})}_{\frac{3}{2},2}}\Big\|_{L^4(0,1)} \\
&= C\Big\|\Big\{\frac{\log(e+|\Omega|t)}{1+|\Omega|t}\Big\}^{\frac{1}{6}}\Big\|_{L^4(0,1)}\|f_N\|_{\dot{B}^{\frac{3}{2}}_{\frac{3}{2},2}} \\
&\to 0 \quad \text{as } |\Omega|\to\infty,
\end{aligned}
\tag{2.5}
$$

$$
\begin{aligned}
\|T_\Omega(\cdot)f_N\|_{L^4(1,\infty;\dot{H}^{\frac{1}{2}}_3)} &\le C\Big\|t^{-\frac{3}{2}(\frac{2}{3}-\frac{1}{3})}\Big\{\frac{\log(e+|\Omega|t)}{1+|\Omega|t}\Big\}^{\frac{1}{2}(1-\frac{2}{3})}\|f_N\|_{\dot{H}^{\frac{1}{2}}_{\frac{3}{2}}}\Big\|_{L^4(1,\infty)} \\
&\le C\Big\|t^{-\frac{1}{2}}\Big\{\frac{\log(e+|\Omega|t)}{1+|\Omega|t}\Big\}^{\frac{1}{6}}\Big\|_{L^4(1,\infty)}\|f_N\|_{\dot{H}^{\frac{1}{2}}_{\frac{3}{2}}} \\
&\to 0 \quad \text{as } |\Omega|\to\infty.
\end{aligned}
\tag{2.6}
$$

Therefore, we obtain (2.3) by (2.4), (2.5), (2.6) and the convergence $f_N \to f$ in $\dot{H}^{\frac{1}{2}}(\mathbb{R}^3)$ as $N\to\infty$. ❑

Proposition 2.5 *Let $2<p<3$ and $6/5<q<2$ satisfy*

$$
1-\frac{1}{p}\le\frac{1}{q}<\frac{1}{3}+\frac{1}{p}, \tag{2.7}
$$

$$
\max\Big\{0,\frac{1}{2}-\frac{3}{2}\Big(\frac{1}{q}-\frac{1}{p}\Big)-\frac{1}{2}\Big(1-\frac{2}{p}\Big)\Big\}<\frac{1}{\theta}\le\frac{1}{2}-\frac{3}{2}\Big(\frac{1}{q}-\frac{1}{p}\Big). \tag{2.8}
$$

Then, there exists $C>0$ such that

$$
\Big\|\int_0^t T_\Omega(t-\tau)\mathbb{P}\nabla f(\tau)d\tau\Big\|_{L^\theta(0,\infty;\dot{H}^s_p)} \le C|\Omega|^{-\{\frac{1}{2}-\frac{3}{2}(\frac{1}{q}-\frac{1}{p})-\frac{1}{\theta}\}}\|f\|_{L^{\frac{\theta}{2}}(0,\infty;\dot{H}^s_q)} \tag{2.9}
$$

for all $s\in\mathbb{R}$, $\Omega\in\mathbb{R}\setminus\{0\}$, $f\in L^{\frac{\theta}{2}}(0,\infty;\dot{H}^s_q(\mathbb{R}^3))$.

Proof. We only consider the case $s=0$ for simplicity since the case $s\neq0$ is treated similarly. By Lemma 2.2, we have

$$
\begin{aligned}
&\Big\|\int_0^t T_\Omega(t-\tau)\mathbb{P}\nabla f(\tau)d\tau\Big\|_{L^\theta(0,\infty;L^p)} \\
&\le C\Big\|\int_0^t (t-\tau)^{-\frac{1}{2}-\frac{3}{2}(\frac{1}{q}-\frac{1}{p})}\Big\{\frac{\log(e+|\Omega||t-\tau|)}{1+|\Omega||t-\tau|}\Big\}^{\frac{1}{2}(1-\frac{2}{p})}\|f(\tau)\|_{L^q}d\tau\Big\|_{L^\theta(0,\infty)}.
\end{aligned}
$$

In the case $1/\theta = 1/2 - 3(1/q - 1/p)/2$, we have from Hardy-Littlewood-Sobolev's inequality

$$\begin{aligned}
&\Big\| \int_0^t (t-\tau)^{-\frac{1}{2}-\frac{3}{2}(\frac{1}{q}-\frac{1}{p})} \Big\{ \frac{\log(e+|\Omega||t-\tau|)}{1+|\Omega||t-\tau|} \Big\}^{\frac{1}{2}(1-\frac{2}{p})} \|f(\tau)\|_{L^q} d\tau \Big\|_{L^\theta(0,\infty)} \\
&\le \Big\| \int_0^t (t-\tau)^{-\frac{1}{2}-\frac{3}{2}(\frac{1}{q}-\frac{1}{p})} \|f(\tau)\|_{L^q} d\tau \Big\|_{L^\theta(0,\infty)} \\
&\le C\|f\|_{L^{\frac{\theta}{2}}(0,\infty;L^q)}.
\end{aligned}$$

In the case $1/\theta < 1/2 - 3(1/q - 1/p)/2$, we have from Hausdorff-Young's inequality with $1/\theta = 2/\theta + 1/r - 1$

$$\begin{aligned}
&\Big\| \int_0^t (t-\tau)^{-\frac{1}{2}-\frac{3}{2}(\frac{1}{q}-\frac{1}{p})} \Big\{ \frac{\log(e+|\Omega||t-\tau|)}{1+|\Omega||t-\tau|} \Big\}^{\frac{1}{2}(1-\frac{2}{p})} \|f(\tau)\|_{L^q} d\tau \Big\|_{L^\theta(0,\infty)} \\
&\le \Big\| t^{-\frac{1}{2}-\frac{3}{2}(\frac{1}{q}-\frac{1}{p})} \Big\{ \frac{\log(e+|\Omega|t)}{1+|\Omega|t} \Big\}^{\frac{1}{2}(1-\frac{2}{p})} \Big\|_{L^r(0,\infty)} \|f\|_{L^{\frac{\theta}{2}}(0,\infty;L^q)} \\
&= C|\Omega|^{\frac{1}{\theta}-\frac{1}{2}+\frac{3}{2}(\frac{1}{q}-\frac{1}{p})} \|f\|_{L^{\frac{\theta}{2}}(0,\infty;L^q)}.
\end{aligned}$$

Therefore, we obtain (2.9). □

Proposition 2.6 *There exists a positive constant C such that*

$$\Big\| \int_0^t T_\Omega(t-\tau)\nabla f(\tau) d\tau \Big\|_{L^\infty(0,\infty;\dot{H}^s)\cap L^4(0,\infty;\dot{H}^s_3)} \le C\|f\|_{L^2(0,\infty;\dot{H}^s)} \tag{2.10}$$

for all $s \in \mathbb{R}$, $\Omega \in \mathbb{R}$, $f \in L^2(0,\infty;\dot{H}^s(\mathbb{R}^3))$.

Proof. For simplicity, we show (2.10) in the case $s = 0$ since it is possible to treat the case $s \neq 0$ similarly. On the $L^\infty(0,\infty;L^2)$ norm, we have from Plancherel's theorem and Hölder's inequality

$$\begin{aligned}
\Big\| \int_0^t T_\Omega(t-\tau)\nabla f(\tau) d\tau \Big\|_{L^2} &\le C \Big\| \int_0^t e^{-(t-\tau)|\xi|^2} |\xi| |\widehat{f}(\tau)| d\tau \Big\|_{L^2} \\
&\le C \Big\| \big\| e^{-(t-\tau)|\xi|^2} \big\|_{L^2_\tau(0,t)} |\xi| \big\| \widehat{f}(\tau) \big\|_{L^2_\tau(0,t)} \Big\|_{L^2} \\
&\le C\|\widehat{f}\|_{L^2(0,\infty;L^2)} \\
&= C\|f\|_{L^2(0,\infty;L^2)}.
\end{aligned} \tag{2.11}$$

On the $L^4(0,\infty;L^3(\mathbb{R}^3))$ norm, we have from (2.2) and Hardy-Littlewood-Sobolev's inequality

$$\begin{aligned}
\Big\| \int_0^t T_\Omega(t-\tau)\nabla f(\tau) d\tau \Big\|_{L^4(0,\infty;L^3(\mathbb{R}^3))} &\le C \Big\| \int_0^t (t-\tau)^{-\frac{1}{2}-\frac{3}{2}(\frac{1}{2}-\frac{1}{3})} \|f(\tau)\|_{L^2} d\tau \Big\|_{L^4(0,\infty)} \\
&\le C\|f\|_{L^2(0,\infty;L^2)}.
\end{aligned} \tag{2.12}$$

Therefore, we obtain (2.10) by (2.11) and (2.12). □

Lemma 2.7 *Let s, p satisfy*

$$0 \le s < 3, \quad \frac{s}{3} < \frac{1}{p} < \frac{1}{2} + \frac{s}{6},$$

and let q satisfy

$$\frac{1}{q} = \frac{2}{p} - \frac{s}{3}.$$

Then, there exists $C > 0$ such that

$$\|fg\|_{\dot{H}^s_q} \le C\|f\|_{\dot{H}^s_p}\|g\|_{\dot{H}^s_p}. \tag{2.13}$$

Proof. Let r satisfy $1/q = 1/p + 1/r$. In the Sobolev spaces, it is known that

$$\|fg\|_{\dot{H}^s_q} \le C\|f\|_{\dot{H}^s_p}\|g\|_{L^r} + C\|f\|_{L^r}\|g\|_{\dot{H}^s_p}.$$

By the continuous embedding $\dot{H}^s_p(\mathbb{R}^3) \hookrightarrow L^r(\mathbb{R}^3)$, we obtain (2.13). □

3 Proof of theorems

We prove Theorem 1.1, Theorem 1.6 and Theorem 1.8. The proof of Theorem 1.4 is omitted since it is shown in the similar way to that of Theorem 1.6.

Proof of Theorem 1.1. Since the assumption on θ and p in Proposition 2.3 is satisfied by (1.1) and (1.2), there exists $C_0 > 0$ such that

$$\|T_\Omega(\cdot)u_0\|_{L^\theta(0,\infty;\dot{H}^s_p)} \le |\Omega|^{-\frac{1}{\theta}+\frac{3}{4}(1-\frac{2}{p})}C_0\|u_0\|_{\dot{H}^s}.$$

Let $\Psi(u)$ and Y be defined by

$$\Psi(u)(t) := T_\Omega(t)u_0 - \int_0^t T_\Omega(t-\tau)\mathbb{P}\nabla\cdot(u\otimes u)(\tau)d\tau, \tag{3.1}$$

$$Y := \{\, u \in L^\theta(0,\infty;\dot{H}^s_p(\mathbb{R}^3))^3 \mid \|u\|_{L^\theta(0,\infty;\dot{H}^s_p)} \le 2C_0|\Omega|^{-\frac{1}{\theta}+\frac{3}{4}(1-\frac{2}{p})}\|u_0\|_{\dot{H}^s}, \operatorname{div} u = 0 \,\},$$

$$d(u,v) := \|u-v\|_{L^\theta(0,\infty;\dot{H}^s_p)}.$$

Let q satisfy $1/q = 2/p - s/3$. Since the assumptions on s, p, q and θ in Proposition 2.5 and Lemma 2.7 are satisfied by (1.1) and (1.2), for any $u, v \in Y$, we have from

Proposition 2.3, Proposition 2.5 and Lemma 2.7

$$
\begin{aligned}
\|\Psi(u)\|_{L^\theta(0,\infty;\dot H^s_p)} &\le C_0|\Omega|^{-\frac{1}{\theta}+\frac{3}{4}(1-\frac{2}{p})}\|u_0\|_{\dot H^s} + C|\Omega|^{\frac{1}{\theta}-\frac{1}{2}+\frac{3}{2}(\frac{1}{q}-\frac{1}{p})}\|u\otimes u\|_{L^{\frac{\theta}{2}}(0,\infty;\dot H^s_q)}\\
&\le C_0|\Omega|^{-\frac{1}{\theta}+\frac{3}{4}(1-\frac{2}{p})}\|u_0\|_{\dot H^s} + C|\Omega|^{\frac{1}{\theta}-\frac{1}{2}+\frac{3}{2}(\frac{1}{q}-\frac{1}{p})}\|u\|^2_{L^\theta(0,\infty;\dot H^s_p)}\\
&\le C_0|\Omega|^{-\frac{1}{\theta}+\frac{3}{4}(1-\frac{2}{p})}\|u_0\|_{\dot H^s} + C_1|\Omega|^{\frac{1}{\theta}-\frac{1}{2}+\frac{3}{2}(\frac{1}{q}-\frac{1}{p})+2\{-\frac{1}{\theta}+\frac{3}{4}(1-\frac{2}{p})\}}\|u_0\|^2_{\dot H^s}\\
&\le C_0|\Omega|^{-\frac{1}{\theta}+\frac{3}{4}(1-\frac{2}{p})}\|u_0\|_{\dot H^s} + C_1|\Omega|^{-\frac{s}{2}+\frac{1}{4}}|\Omega|^{-\frac{1}{\theta}+\frac{3}{4}(1-\frac{2}{p})}\|u_0\|^2_{\dot H^s},
\end{aligned}
\tag{3.2}
$$

$$
\begin{aligned}
&\|\Psi(u)-\Psi(v)\|_{L^\theta(0,\infty;\dot H^s_p)}\\
&= \Big\|\int_0^t T_\Omega(t-\tau)\mathbb{P}\nabla\cdot\{u\otimes(u-v)(\tau)+(u-v)\otimes v(\tau)\}d\tau\Big\|_{L^\theta(0,\infty;\dot H^s_p)}\\
&\le C|\Omega|^{\frac{1}{\theta}-\frac{1}{2}+\frac{3}{2}(\frac{1}{q}-\frac{1}{p})}\|u\otimes(u-v)+(u-v)\otimes v\|_{L^{\frac{\theta}{2}}(0,\infty;\dot H^s_q)}\\
&\le C|\Omega|^{\frac{1}{\theta}-\frac{1}{2}+\frac{3}{2}(\frac{1}{q}-\frac{1}{p})}\Big(\|u\|_{L^\theta(0,\infty;\dot H^s_p)}+\|v\|_{L^\theta(0,\infty;\dot H^s_p)}\Big)\|u-v\|_{L^\theta(0,\infty;\dot H^s_p)}\\
&\le C_2|\Omega|^{\frac{1}{\theta}-\frac{1}{2}+\frac{3}{2}(\frac{1}{q}-\frac{1}{p})-\frac{1}{\theta}+\frac{3}{4}(1-\frac{2}{p})}\|u_0\|_{\dot H^s}\|u-v\|_{L^\theta(0,\infty;\dot H^s_p)}\\
&= C_2|\Omega|^{\frac{1}{4}+\frac{3}{2q}-\frac{3}{p}}\|u_0\|_{\dot H^s}\|u-v\|_{L^\theta(0,\infty;\dot H^s_p)}\\
&= C_2|\Omega|^{-\frac{s}{2}+\frac{1}{4}}\|u_0\|_{\dot H^s}\|u-v\|_{L^\theta(0,\infty;\dot H^s_p)}.
\end{aligned}
$$

If Ω, u_0 satisfy

$$
C_1|\Omega|^{-\frac{s}{2}+\frac{1}{4}}\|u_0\|_{\dot H^s}\le C_0,\quad C_2|\Omega|^{-\frac{s}{2}+\frac{1}{4}}\|u_0\|_{\dot H^s}\le\frac{1}{2},
$$

then, it is possible to apply Banach's fixed point theorem in Y and we obtain $u\in Y$ with

$$
u(t)=T_\Omega(t)u_0-\int_0^t T_\Omega(t-\tau)\mathbb{P}\nabla\cdot(u\otimes u)d\tau.
$$

Here, we show that the solution $u\in Y$ satisfies $u(t)\in\dot H^s(\mathbb{R}^n)^3$ for all $t\ge 0$. On the linear part, it is easy to see that $T_\Omega(t)u_0\in\dot H^s(\mathbb{R}^3)^3$ for any $t\ge 0$. On the nonlinear part, let $1/q=2/p-s/3$ and we have from Lemma 2.2, Lemma 2.7 and Hölder's inequality

$$
\begin{aligned}
\Big\|\int_0^t T_\Omega(t-\tau)\mathbb{P}\nabla\cdot(u\otimes u)(\tau)d\tau\Big\|_{\dot H^s} &\le C\int_0^t(t-\tau)^{-\frac{1}{2}-\frac{3}{2}(\frac{1}{q}-\frac{1}{2})}\|(u\otimes u)(\tau)\|_{\dot H^s_q}d\tau\\
&\le C\int_0^t(t-\tau)^{-\frac{1}{2}-\frac{3}{2}(\frac{1}{q}-\frac{1}{2})}\|u(\tau)\|^2_{\dot H^s_p}d\tau\\
&\le C\Big\|(t-\cdot)^{-\frac{1}{2}-\frac{3}{2}(\frac{1}{q}-\frac{1}{2})}\Big\|_{L^{\frac{\theta}{\theta-2}}(0<\tau<t)}\Big\|\|u(\tau)\|^2_{\dot H^s_p}\Big\|_{L^{\frac{\theta}{2}}(0,\infty)}\\
&\le Ct^{\frac{\theta-2}{\theta}[1-\frac{\theta}{\theta-2}\{-\frac{1}{2}-\frac{3}{2}(\frac{1}{q}-\frac{1}{2})\}]}\|u\|^2_{L^\theta(0,\infty;\dot H^s_p)}.
\end{aligned}
\tag{3.3}
$$

Here, we note on the integrability at $\tau=t$ that

$$
\frac{\theta}{\theta-2}\Big\{\frac{1}{2}+\frac{3}{2}\Big(\frac{1}{q}-\frac{1}{2}\Big)\Big\}<1\quad\text{if and only if}\quad\frac{1}{\theta}<\frac{5}{8}-\frac{3}{2p}+\frac{s}{4}.
$$

Therefore, we obtain $u(t) \in \dot{H}^s(\mathbb{R}^n)^3$ and we also see $u \in C([0,\infty), \dot{H}^s(\mathbb{R}^3))^3$. □

Proof of Theorem 1.6. Let $\delta > 0$ be an arbitrary positive number to be determined later. Since K is precompact in $\dot{H}^{\frac{1}{2}}(\mathbb{R}^3)^3$, the closure of K is compact. Hence there exist a natural number $N(\delta, K)$ and $\{f_j\}_{j=1}^{N(\delta,K)} \subset \dot{H}^{\frac{1}{2}}(\mathbb{R}^3)^3$ such that

$$K \subset \cup_{j=1}^{N(\delta,K)} B(f_j, \delta),$$

where $B(f,\delta)$ denotes a ball in $\dot{H}^{\frac{1}{2}}(\mathbb{R}^3)^3$ with center being f and radius δ. By Proposition 2.4, there exists $\omega_0(\delta, K) > 0$ such that we have

$$\sup_{j=1,2,\cdots N(\delta,K)} \|T_\Omega(\cdot) f_j\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})} \le \delta$$

for all $\Omega \in \mathbb{R}$ with $|\Omega| > \omega_0(\delta, K)$. Then, for any $f \in K$, there exists $j \in \{1, 2, \cdots, N(\delta, K)\}$ such that $f \in B(f_j, \delta)$ and we have from Proposition 2.3

$$\begin{aligned}
\|T_\Omega(\cdot) f\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})} &\le \|T_\Omega(\cdot)(f_j - f)\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})} + \|T_\Omega(\cdot) f_j\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})} \\
&\le C\|f_j - f\|_{\dot{H}^{\frac{1}{2}}} + \delta \\
&\le C\delta.
\end{aligned}$$

Therefore, there exists a positive constant $C_1 > 0$

$$\sup_{f\in K} \|T_\Omega(\cdot) f\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})} \le C_1\delta \tag{3.4}$$

for all $\Omega \in \mathbb{R}$ with $|\Omega| > \omega_0(\delta, K)$. Then, let the space X be defined by

$$\begin{aligned}
X &:= \{\, u \in C([0,\infty), \dot{H}^{\frac{1}{2}}(\mathbb{R}^3))^3 \mid \|u\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})} \le 2C_1\delta,\ \operatorname{div} u = 0 \,\}, \\
d(u,v) &:= \|u - v\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})}.
\end{aligned}$$

Let Ψ be defined by (3.1). For any $u \in X$, we have from Proposition 2.6, Lemma 2.7 and Hölder's inequality

$$\begin{aligned}
\|\Psi(u)\|_{L^\infty(0,\infty;\dot{H}^{\frac{1}{2}})} &\le C\|u_0\|_{\dot{H}^{\frac{1}{2}}} + C\|u \otimes u\|_{L^2(0,\infty;\dot{H}^{\frac{1}{2}})} \\
&\le C\|u_0\|_{\dot{H}^{\frac{1}{2}}} + C\Big\|\|u\|^2_{\dot{H}_3^{\frac{1}{2}}}\Big\|_{L^2(0,\infty)} \\
&\le C\|u_0\|_{\dot{H}^{\frac{1}{2}}} + C\|u\|^2_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})}.
\end{aligned} \tag{3.5}$$

We also have from Proposition 2.6, Lemma 2.7 and Hölder's inequality

$$\begin{aligned}
\|\Psi(u)\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})} &\le \|T_\Omega(\cdot) u_0\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})} + C\|u \otimes u\|_{L^2(0,\infty;\dot{H}^{\frac{1}{2}})} \\
&\le \|T_\Omega(\cdot) u_0\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})} + C\Big\|\|u\|^2_{\dot{H}_3^{\frac{1}{2}}}\Big\|_{L^2(0,\infty)} \\
&\le \|T_\Omega(\cdot) u_0\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})} + C_2\|u\|^2_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})}.
\end{aligned} \tag{3.6}$$

Similarly, we also have for $u, v \in X$

$$\|\Psi(u) - \Psi(v)\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})} \le C_3\Big(\|u\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})} + \|v\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})}\Big)\|u - v\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})}. \tag{3.7}$$

Here, since δ is an arbitrary positive number, let $\delta > 0$ satisfy

$$\delta < \min\Big\{\frac{1}{4C_1C_2}, \frac{1}{8C_1C_3}\Big\},$$

where C_1, C_2 and C_3 is the constants in (3.4), (3.6) and (3.7), respectively. Then, we have from (3.4), (3.5), (3.6) and (3.7)

$$\begin{aligned} \|\Psi(u)\|_{L^\infty(0,\infty;\dot{H}^{\frac{1}{2}})} &< \infty \\ \|\Psi(u)\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})} &\le 2C_1\delta, \\ \|\Psi(u) - \Psi(v)\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})} &\le \frac{1}{2}\|u - v\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})}, \end{aligned}$$

for all $u, v \in X$, $\Omega \in \mathbb{R}$ with $|\Omega| > \omega_0(\delta, K)$. Therefore, it is possible to apply Banach's fixed point theorem to obtain the global solutions. □

Proof of Theorem 1.8. By the same argument to the precompact set K in $\dot{H}^{\frac{1}{2}}(\mathbb{R}^3)$ as that of proof of Theorem 1.6, we see that for any $T > 0$ and $\delta > 0$, there exist $\omega(T, K) > 0$ and $C_1 > 0$ such that

$$\sup_{f\in K}\|T_\Omega(\cdot)f\|_{L^4(0,T;\dot{H}_3^{\frac{1}{2}})} \le C_1\delta,$$

for all $\Omega \in \mathbb{R}$ with $|\Omega| > \omega(T, K)$. Then, we can obtain the similar estimate as (3.5), (3.6) and (3.7) in which time interval $(0, \infty)$ is replaced with $(0, T)$. It is possible to apply Banach's fixed point theorem in the space

$$\begin{aligned} X &:= \{\, u \in C([0, T), \dot{H}^{\frac{1}{2}}(\mathbb{R}^3))^3 \mid \|u\|_{L^4(0,T;\dot{H}_3^{\frac{1}{2}})} \le 2C_1\delta,\ \operatorname{div} u = 0 \,\}, \\ d(u, v) &:= \|u - v\|_{L^4(0,T;\dot{H}_3^{\frac{1}{2}})} \end{aligned}$$

and obtain local solutions. □

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