

High Frequency Scattering Analysis by Dielectric Edged Objects via Surface Equivalence Theorem

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1. Introduction

Electromagnetic scattering describes and explains the field behavior in the object when an incident electromagnetic wave illuminates an object. The electromagnetic scattering can be studied by looking at the solutions of Maxwell's equations, but the exact solutions exist only for a limited number of canonical shapes. In which, electromagnetic scattering by edged objects is one of the important problems, and these analyses can be applied for the propagation and diffraction estimation.

While some exact solutions [1]–[3] are available to estimate the scattering field by a simply shape object composed of a simple material constitution, some numerical methods [4]–[6] have enable for objects of small size compared with wavelength. However, these numerical methods may have a problem to apply for the scattering by electrically large objects because of unrealistic execution time and heavy memory requirement. Hence, there is a need to develop an approximate approach that provides fast and acceptably accurate results for electrically large objects. Several asymptotic techniques such as the geometrical theory of diffraction (GTD) [7], [8] and the physical optics (PO) [9], [10] can be efficiently used for high frequency scattering problems from the object made of conducting materials. However, when scattering objects are composed of dielectric materials, the problem becomes difficult to solve, since there is no reliable solution available for estimating the diffraction by dielectric/magnetic objects [11]. While there are some improved versions of PO are been proposed to obtain more accurate scattering fields like the physical theory of diffraction (PTD) [12] and modified PO, the extension to the scattering by dielectric objects is not clear yet. This question is just a motivation of our present study, in which equivalent electric and magnetic currents to the scattering problems have been discussed.

In this thesis, equivalent current method which is based on the surface equivalence theorem [13] has been used to solve the scattering by dielectric edged objects. According to the surface equivalence theorem, the scattering field from a conducting body may be formulated as the corresponding radiation from equivalent currents on a postulated surface enclosing the scattering body. When the equivalent electric and magnetic currents on the surface of the scattering object are obtained from the reflected/incident GO rays, and the radiation integrals due to these equivalent currents are evaluated asymptotically,

the results are found to coincide with those by the PO formulation [15]. Since our formulation matches with PO for the conducting case, and can be extended to the cases when the scattering objects are dielectric, this method may be called as *extended PO* method, in which the equivalent currents are obtained from the reflected/transmitted and incident GO rays.

Since the incident wave illuminates on the dielectric body, it excites the reflected wave at the illuminated surfaces and transmitted wave inside the scattering body. Then this original transmitted wave continues to excite the internal reflected and outgoing transmitted waves due to the multiple bouncing effects, and these outgoing transmitted waves radiate again from the body. The effect of these internal bouncing waves can also be considered in our formulation. While equivalent current formulation in terms of reflected fields from the dielectric objects has already been utilized to obtain the specular reflected field for estimating their dielectric constants [14], the accuracy of the scattering field in non-specular reflection direction has not been discussed fully yet. The numerical results by the dielectric rectangular cylinders are calculated and compared with those HFSS simulation. A good agreement has been observed to confirm the validity of this method.

In the following discussion, the time-harmonic factor $e^{-i\omega t}$ is assumed and suppressed throughout the text.

2. Surface Equivalence Theorem

According to the surface equivalence theorem, the fields outside an imaginary closed surface are obtained by placing, over the closed surface, suitable electric and magnetic currents that satisfy the boundary conditions. In case of the conducting objects, if one assumes that the radius of curvature of the surface is large compared with the wavelength, then the scattering fields \mathbf{E}^s , \mathbf{H}^s may be given by the reflected GO fields \mathbf{E}^r , \mathbf{H}^r for the illuminated surface of the object. For the shadowed surface, one could expect that the total fields \mathbf{E} , \mathbf{H} would be almost null, therefore the scattering fields must behave to cancel the original incident fields \mathbf{E}^i , \mathbf{H}^i . Then, one shall assume $\mathbf{E}^s = -\mathbf{E}^i$, and $\mathbf{H}^s = -\mathbf{H}^i$. Accord-

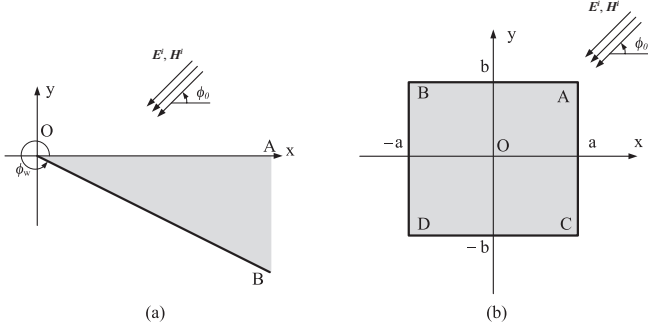


Figure 1: Scattering by plane waves \mathbf{E}^i , \mathbf{H}^i for incident angle $0 < \phi_0 < \pi/2$. (a) Diffraction by a wedge. (b) Scattering by a rectangular cylinder.

ingly, equivalent currents may be approximated as:

$$\mathbf{J}_s = \hat{\mathbf{n}} \times \mathbf{H}^s \simeq \begin{cases} \hat{\mathbf{n}} \times \mathbf{H}^r & \text{on illuminated } S, \\ \hat{\mathbf{n}} \times (-\mathbf{H}^t) & \text{on shadowed } S, \end{cases} \quad (1)$$

$$\mathbf{M}_s = \mathbf{E}^s \times \hat{\mathbf{n}} \simeq \begin{cases} \mathbf{E}^r \times \hat{\mathbf{n}} & \text{on illuminated } S, \\ (-\mathbf{E}^i) \times \hat{\mathbf{n}} & \text{on shadowed } S. \end{cases} \quad (2)$$

For the dielectric objects, the situation becomes more complicated than the conducting case in Eqs. (1) and (2). When the incident wave impinges on the illuminated surfaces, it excites the reflected wave (\mathbf{E}^r , \mathbf{H}^r) and the transmitted waves (\mathbf{E}^t , \mathbf{H}^t). Therefore, the scattering fields (\mathbf{E}^s , \mathbf{H}^s) on the illuminated region are given by the reflected waves (\mathbf{E}^r , \mathbf{H}^r) and the transmitted waves (\mathbf{E}^t , \mathbf{H}^t), if any. On the shadowed region, one has to consider the transmitted waves (\mathbf{E}^t , \mathbf{H}^t), then the scattering fields would be $\mathbf{E}^s = -\mathbf{E}^i + \mathbf{E}^t$, $\mathbf{H}^s = -\mathbf{H}^i + \mathbf{H}^t$.

Accordingly, the equivalent current may be approximated as

$$\mathbf{J}_s = \begin{cases} \hat{\mathbf{n}} \times (\mathbf{H}^r + \mathbf{H}^t) & \text{on illuminated } S, \\ \hat{\mathbf{n}} \times (-\mathbf{H}^i + \mathbf{H}^t) & \text{on shadowed } S, \end{cases} \quad (3)$$

$$\mathbf{M}_s = \begin{cases} (\mathbf{E}^r + \mathbf{E}^t) \times \hat{\mathbf{n}} & \text{on illuminated } S, \\ (-\mathbf{E}^i + \mathbf{E}^t) \times \hat{\mathbf{n}} & \text{on shadowed } S. \end{cases} \quad (4)$$

3. High Frequency Scattering Analysis by Physical Optics Approximation

In this chapter, the physical optics (PO) approximation are applied to estimate electromagnetic scattering from the conducting wedge and rectangular cylinder. Physical optics (PO) approximation is a method which is known to have pretty accurate and can be efficiently used for high frequency scattering problems from the object made of conducting materials. According to the PO approximation, if the scattering objects are large compared with the wavelength, the PO currents \mathbf{J}^{PO} are approximated from the incident magnetic field as

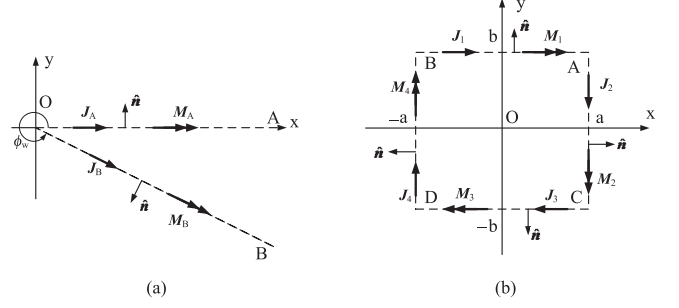


Figure 2: Equivalent electric and magnetic currents \mathbf{J}_s , \mathbf{M}_s approximated by surface equivalence theorem. (a) On the wedge surfaces. (b) On the cylinder surfaces.

$\mathbf{J}^{PO} = 2\hat{\mathbf{n}} \times \mathbf{H}^i$ and flow on the physically illuminated surfaces. Then, the scattering field from this object may be derived by integrating these PO currents with the free-space Green's function. Since PO formulation involves the information of the incident wave and the scattering object's surface, it is easy to construct the equivalent currents for the scattering field.

This chapter starts by applying the PO method to derive diffraction field from a conducting wedge as shown in Fig. 1(a) (Section 3.1). Here, the scattering field is obtained from only one PO current flowing on the illuminated surface OA. The results include the edge diffracted field and the GO field which gives a reflected field in the illuminated region or a field to cancel the incident field in the shadow region. In Section 3.2, the conducting rectangular cylinder has been considered. In this case, the incident wave impinges on the cylinder at two surfaces AB and AC as shown in Fig. 1(b), and excites two PO currents on these illuminated surfaces. Then, the total scattering fields are given by summing up the fields radiated from these PO currents.

4. High Frequency Scattering Analysis by Equivalent Current Method

In this chapter, the surface equivalence theorem are applied to derive the scattering fields by a wedge and a rectangular cylinder. According to the surface equivalence theorem in Section 2, the scattering field by the objects are approximated by the radiation from the equivalent currents calculated by the reflected/transmitted GO rays. In Section 4.1, the scattering fields by the conducting wedge and rectangular cylinder are formulated to compare with those obtained from the PO in Chapter 3. Then, the scattering fields by the dielectric wedge and rectangular cylinder are considered in Section 4.2.

4.1 Scattering by a Conducting Edged Object

In this section, the scattering fields by a conducting wedge and rectangular cylinder are formulated. According to the surface equivalence theorem, the corresponding equivalent electric and magnetic currents \mathbf{J}_s , \mathbf{M}_s are calculated the GO rays on a postulated surface enclosing the scattering object. While currents \mathbf{J}_s , \mathbf{M}_s on the illuminated surface are approximated by the GO reflected rays \mathbf{E}^r , \mathbf{H}^r , those on the shadow surface are obtained from the incident GO rays $-\mathbf{E}^i$, $-\mathbf{H}^i$. Then, the total scattering field are given by summing up the all contributions radiated from these equivalent currents.

Firstly, the diffraction by a conducting wedge is considered. The equivalent currents \mathbf{J}_A , \mathbf{M}_A at surface OA are approximated by the GO reflected wave, and currents \mathbf{J}_B , \mathbf{M}_B at surface OB are calculated from the minus incident wave as shown in Fig. 2(a). Then, the total scattering field is given by summing up four contributions derived from these currents. While the diffracted component of this result is found to match with the one obtained by the PO approximation in Section 3.1, the internal field inside the wedge is found to be asymptotically zero due to the virtue of the surface equivalent theorem. For the conducting rectangular cylinder, eight equivalent currents $\mathbf{J}_1 \sim \mathbf{J}_4$ and $\mathbf{M}_1 \sim \mathbf{M}_4$ are approximated by the GO reflected and incident rays as shown in Fig. 2(b). When the resulting scattering fields radiating from these currents are combined, the final result is found to be exactly the same as the result obtained by the PO in Section 3.2.

Therefore, from our derivation, one concludes that the results by surface equivalence theorem match with those obtained by the PO in Chapter 3, which utilized the induced electric currents on the illuminated physical surfaces. Then, when applying this method to estimate the scattering by the dielectric objects, one expects that this method could have the similar accuracy as PO formulation.

4.2 Scattering by a Dielectric Edged Object

In this section, the outside scattering formulations for the dielectric wedge and rectangular cylinder have been derived. Here, the situation becomes more complicated than the conducting case in Section 4.1. When the incident plane wave impinges on the illuminated surfaces of a dielectric object, it excites the reflected wave at the illuminated surfaces and transmitted waves inside the cylinder. In addition, the original transmitted wave excites the internal reflected and transmitted waves due to the multiple bouncing effects and they radiate again from the body. The intensity of these transmitted waves \mathbf{E}^t , \mathbf{H}^t depends on the dielectric constant and the dimension of the scattering objects, and be-

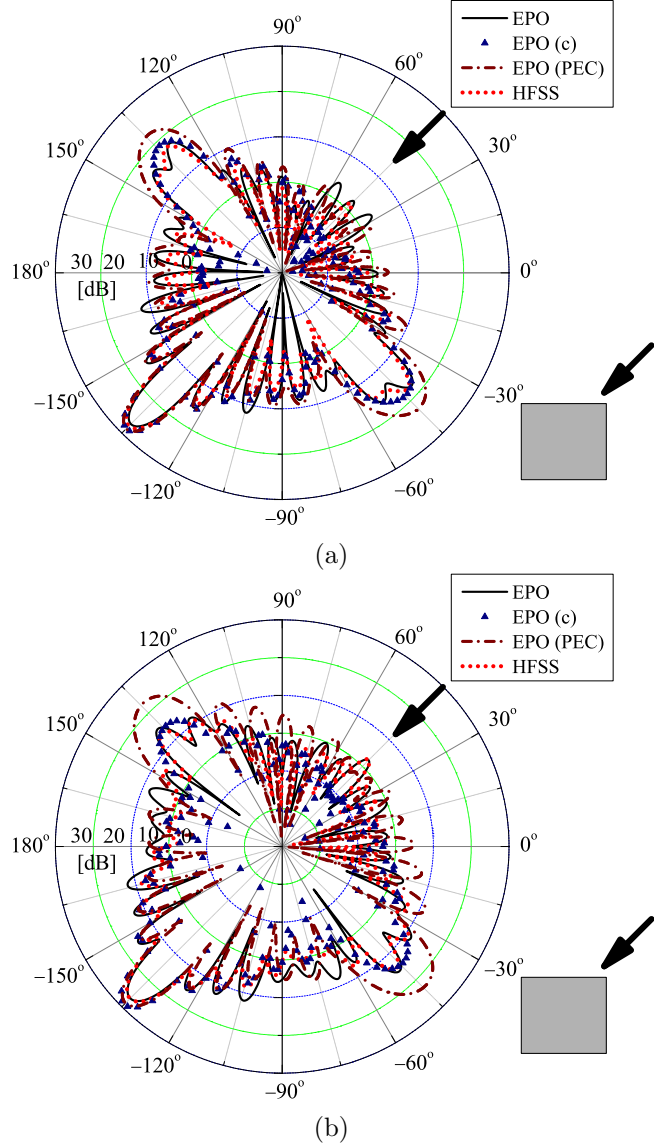


Figure 3: Radiation pattern of a rectangular dielectric cylinder. $ka = kb = 15$, $\phi_0 = 45^\circ$. $\epsilon_r = 6 + 0.1i$. (a) E polarization. (b) H polarization.

comes weaker when the number of the internal reflection increases. Therefore, the contribution from these transmitted waves should be treated carefully. The total scattering field becomes the summation of the primary contribution obtained from the reflected and incident GO rays and multiple bouncing transmitted rays.

When formulating the scattering fields by a dielectric wedge, the expressions of the equivalent currents \mathbf{J}_s , \mathbf{M}_s and the radiation field formulations due to the multiply bouncing transmitted waves are derived. However, in the case of the dielectric rectangular cylinder, because of the finite dimension, one also notice that the internal bouncing rays eventually experience the reflection at the side interfaces. So that the integration range varies when the number of the internal reflection changes. Therefore,

the expressions of the equivalent currents and the radiation fields become more complicated.

In order to estimate the validity of our formulation, some numerical results from the scattering by the dielectric rectangular cylinder are computed and compared with those by the HFSS simulation. The good agreements between our method and HFSS simulation have been observed in the forward and specular reflection directions to verify the accuracy of our method. For the nearly conducting case, one observes that our results coincide with those derived by the PEC case. As the loss of the dielectric material decreases, the lobes in the specular reflection directions become smaller and more oscillatory due to the stronger interference between the multiply bouncing transmitted waves as seen in Fig. 3.

5. Conclusion

A new high frequency approximation method to analyze the electromagnetic scattering by dielectric edged objects has been proposed. This method is based on the assumption that scattering far field is generated by the equivalent electric and magnetic currents calculated from reflected/transmitted GO rays.

From our derivation, one finds that the scattering formulas for the conducting edge objects are *exactly* the same as those obtained from the physical optics approximation. Accordingly, one expects that the field formulation as a radiation integral due to these equivalent sources has the similar accuracy as PO formulation. In case of dielectric objects, the proposed method can give a good scattering estimation as one includes the contributions from the multiple bouncing effect. It is also easy to extend our calculation to three dimensional scattering problems and applicable for the scattering estimation from more general edged objects.

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