### Electromagnetic Plane Wave Scattering by Aperture on Thick Conducting Screen

### 1. Introduction

The problem of electromagnetic diffraction by an aperture perforated on a perfectly conducting screen has received considerable attention [1]-[11]. The solution can be applied to a practical problem, namely the diffraction by a window aperture on a building wall. Here, the model of thick conducting slit represents for the two-dimensional building window, while the rectangular hole on a thick conducting screen is a model of three-dimensional one. Solving this problem plays a crucial role in outdoor-indoor wireless communication where windows can be considered as the main propagation path for electromagnetic waves. Moreover, window diffraction analyses help mobile service providers to determine base station arrangement and plan a suitable communication frequency.

The thesis includes investigation of diffraction from a two-dimensional loaded conducting thick slit and a three-dimensional rectangular hole in a thick conducting screen. Canonical slit diffraction problems have been analyzed by other methods such as the Kobayashi potential (KP) method [2], [4] and the geometrical theory of diffraction (GTD) [1], [7]–[9]. While the KP method is formulated analytically in terms of eigenfunctions and the solution is rather exact, it may suffer a convergence problem of the series as the aperture width of the slit or hole becomes electrically large, which is usual in the radio communication application. In order to analyze the scattering of the cellular communication signals by building windows whose dimensions are pretty large compared with the wavelength, high frequency asymptotic techniques such as GTD and physical optics (PO) may be more convenient to use. GTD is known to yield accurate results in slit diffraction, if the effect of the multiple edge diffraction is included [1], [8]. However, GTD may not be applicable for the case of the rectangular holes, since the accurate diffraction coefficient of the corner is not available yet.

An efficient calculation method is required for the above practical radio wave propagation application where the scatterer can be three-dimensional and made of various materials. In the thesis, the Kirchhoff approximation (KA) is used for the analysis. In the KA formulation, the scattering field is calculated as radiation from an equivalent magnetic current which is postulated on the closing aperture of the window. The KA method has great advantages for general scattering anal-



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Figure 1: Scattering of an electromagnetic plane wave by a loaded slit in a thick conducting screen.

yses, since the formulation is simpler than that of the GTD, and easy to adapt to complicated problems. By our analysis, it has been shown that the KA method can be applied confidently for large aperture cases especially for estimating the main diffraction beam behavior. Also the simple formulation leads to a short calculation time. Accordingly, the KA method can be successfully applied to investigate the plane wave scattering by more complicated and realistic objects such as thick loaded conducting slits which can be considered as a model of window aperture with glass layers. Scattering analysis by three-dimensional rectangular holes in a thick conducting screen is also investigated in this thesis. Based on the obtained formulation, numerical calculation has been done and the results are compared with those by the GTD [7], [9] and KP method [10]. Good agreement has been found to confirm the validity of our formulation. Various numerical calculations and comparisons are done, and representative results are shown in this summary.

## 2. Analysis of plane wave scattering by loaded conducting thick slits

In this chapter, the KA method has been applied to formulate plane wave scattering by loaded conducting thick slits. The proposed model is an important canonical scattering problem to solve before being applied to practical solution of diffraction by window aperture on



Figure 2: Scattering field at each region may be considered as radiation from the equivalent sources at the apertures.

building wall. Analysis in this chapter is developed from the scattering by an empty conducting thick slit [11]. Here, the effect of the loaded layer (window glass) is also taken into account. This study proves that the KA method can be utilized to solve efficiently scattering problem not only from conducting object, but also the dielectric materials where more analyses required to obtain the scattering fields. As illustrated in Fig. 1, a plane wave with a unit amplitude:

$$\phi_u^i = e^{-ik(x\cos\theta_0 + z\sin\theta_0)} \tag{1}$$

impinges upon a slit perforated on an infinitely long perfectly conducting thick screen with incident angle  $\theta_0$ . The width and thickness of the slit are a and b, respectively, and k is the free space wavenumber. Inside the slit, there exists a medium layer of permittivity  $\varepsilon_r$  and permeability  $\mu_r$  and its thickness is  $b_2 - b_1$ .  $\phi_y^i$  represents for  $E_{y}^{i}(H_{y}^{i})$  for E(H) polarization. In order to determine the scattering contributions  $\phi_y^s$ , the KA method is utilized here. According to the KA method, the scattering fields can be obtained by the radiation from equivalent magnetic current sources on the closing aperture of the slit as in Fig. 2. Equivalent magnetic currents are also applied to calculate the fields penetrating inside the slit. Here, the reflection coefficient  $R_m$  and transmission coefficients  $T_m$  represent for the loaded layer effect on the scattering field construction. Accordingly, the scattering field in the upper half-space (z > 0) is given by a summation of the primary and secondary upper scattering field  $\phi_1^{\rm s} + \phi_{11}^{\rm s}$ , and the one in the lower half-space (z < -b) is by  $\phi_2^{\rm s}$ .

Representative numerical results for the scattering far fields by thick loaded conducting slit obtained by using the derived formulas are shown in Fig. 3. The aperture widths are set to be ka = 30, 7, full-slit loaded layer with thickness  $kb_1 = 0, kb_2 = kb = 2$ , the complex relative permitivity  $\varepsilon_r = 3 + i4$  represents for the lossy loaded layer, the complex relative permeability  $\mu_r = 1$ , and



Figure 3: Comparison of the far-field patterns (width variation) of KA, KP and GTD methods. E polarization.  $\theta_0 = 40^\circ$ ,  $kb_1 = 0$ ,  $kb_2 = kb = 2$ ,  $\varepsilon_r = 3 + i4$ ,  $\mu_r = 1$ . (a) ka = 30. (b) ka = 7.

the incident angle  $\theta_0 = 40^{\circ}$  is chosen. As can be seen from the figure, in general, the sharper and stronger the scattering patterns obtained when the aperture becomes wider, especially at the main lobe. One observes that the main lobes direct the corresponding reflected and incident shadow boundary directions. When the loaded medium has a loss (denoted by the imaginary part in loaded material parameters), the scattering fields in the both half-spaces become weak due to the decay inside the dielectric material. The effect of the loaded layer inside the slit to the scattering field is given by comparison with the case of empty slit. The KA method is proved that to be accurate especially for electrically large aperture by good agreement with the results derived by the GTD and the KP methods.

# 3. Analysis of plane wave scattering by rectangular hole in a thick conducting screen

The KA method is extended in this chapter to derive the scattering field from the three-dimensional rectangular hole in a thick conducting screen. This is a necessary expansion since one knows that the typical scatterer in our life is in three-dimensional. In combination with the previous dielectric material effect analysis, the KA



Figure 4: Scattering of an electromagnetic plane wave by a rectangular hole in a thick conducting screen.  $(\theta_0, \phi_0)$ denote the angles of incidence.



Figure 5: Scattering in the upper and lower half-spaces excited by equivalent magnetic currents.

method is able to solve more practical three-dimensional scattering problems, for example three-dimensional window scattering with glass effect, dielectric screen, or multiple apertures. These all are important aspects to construct an effective practical outdoor-indoor radio wave propagation model. Figure 4 shows a plane wave impinging upon a rectangular hole perforated on a thick conducting screen. The length, width and thickness of the hole are a, b and c, respectively. The incident plane wave with an arbitrary polarization may be decomposed into the transverse electric (TE) and the transverse magnetic (TM) components with respect to the incident plane as

$$\boldsymbol{E}^{i} = (E^{TM} \hat{\boldsymbol{\theta}}_{0} + E^{TE} \hat{\boldsymbol{\phi}}_{0}) e^{i \boldsymbol{k}^{i} \cdot \boldsymbol{r}}, \qquad (2)$$

$$\boldsymbol{H}^{\mathrm{i}} = \sqrt{\frac{\varepsilon_0}{\mu_0}} (-E^{\mathrm{TM}} \hat{\boldsymbol{\phi}}_0 + E^{\mathrm{TE}} \hat{\boldsymbol{\theta}}_0) e^{i \boldsymbol{k}^{\mathrm{i}} \cdot \boldsymbol{r}}, \quad (3)$$

where  $\mathbf{k}^{i}$  (with  $|\mathbf{k}^{i}| = k = \omega \sqrt{\varepsilon_{0} \mu_{0}}$ ),  $\mathbf{r}$ ,  $\varepsilon_{0}$  and  $\mu_{0}$  represent the free space incident wave number vector, the position vector to the observation point, the free space permittivity, and permeability, respectively. Similar to Sect. 2., the KA formulation here allows scattering field



Figure 6: Comparison of far-field patterns  $(E_{\phi})$  in  $\theta$  variation of the thick screen cases for TE polarization in the incident plane  $(\phi = \phi_0, \pi + \phi_0), \theta_0 = \pi/6, \phi_0 = \pi/4, ka = kb = 30$ . (a)  $kc = \sqrt{6ka}$ . (b)  $kc = 2\sqrt{6ka}$ .

to be derived from radiations from the equivalent magnetic current sources on the virtually closed apertures, as shown in Fig. 5 [12].

Representative numerical results for the scattering far fields by rectangular hole in thick conducting screen obtained by using the derived formulas are shown in Fig. 6. One can observe the influence of the three-dimensional model with different thickness parameters on scattering patterns in the figure, for example in case of aperture width ka = kb = 30, arbitrary incident angle in three-dimensional space  $\theta_0 = \pi/6, \ \phi_0 = \pi/4$  for TE polarization as in the figure. The pattern in the upper region is independent of the screen's thickness since the upper scattering fields excited by  $M_1^+$  contain no information on the screen's thickness due to the KA method. Two representative thickness parameters are chosen here to show clearly the incident beam reflection through the hole on the conducting wall. The incident plane wave partly impinges in the hole and experiences the internal reflection at the internal hole's wall. Here, one may trace the internal bouncing by geometrical optics (GO) beams to predict the reflection direction. In case of  $kc = \sqrt{6ka}$ , the single GO beam reflection occurs in Fig. 6(a), and the double bouncing GO beam propagation can be observed in Fig. 6(b) for  $kc = 2\sqrt{6ka}$ .

The single and double bouncing phenomenon also occurs at various screen thicknesses depend on the incident angle. The incident plane wave in the hole's aperture is changed into the waveguide modes according to the KA derivation, the lower scattering patterns confirm that the modal re-radiation field correctly illustrates the GO beam bouncing prediction. The corresponding results by the KP method are included for the comparison, and the agreement can be observed especially at the main reflected and transmitted directions. In parallel, other techniques to validate the accuracy of the formulation is considering the results in some special cases, comparing the special results with theoretical prediction. For example, the scattering fields in a circumstance of infinitely thin screen has been formulated analytically. Also, the relation between two-dimensional and three-dimensional scattering formulation has been derived. Accordingly, one can estimate the scattering field of the two-dimensional result from the corresponding three-dimensional one, or vise versa by the conversion.

#### 4. Conclusion

In this thesis, plane wave scattering by a thick conducting loaded slit (two-dimensional building window with glass) and a rectangular hole in a thick conducting screen (three-dimensional building window) have been formulated by the KA method. The scattering field can be obtained by the radiation from equivalent magnetic current sources postulated on the apertures. Equivalent magnetic currents are also applied to calculate the penetrating fields inside the window and the subsequently transmitted field in the lower region. In comparison with the GTD and the KP method, good agreement has been observed. Since the KA method is rather efficient to compute numerically for large aperture cases, one can extend our formulation to more practical scattering prediction, such as multiple windows or dielectric wall. It may also be applied to inverse scattering problems to estimate the physical parameters such as the thickness of the wall and the dimension of the aperture from the scattering field. Also, the scattering field approximation by the KA method in combination with wave propagation calculation by shooting bouncing rays (SBR) method may lead to an efficient wave propagation simulation. These aspects may be studied in a future.

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