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The Effectiveness of Ambient Charges in a Static *n*-firm Cournot Oligopoly[1)]

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This study investigates the effectiveness of ambient charges for Nonpoint Source (NPS) pollution in an *n*-firm Cournot oligopoly framework. Previous research showed that ambient charges can decrease NPS pollution in a two-firm Cournot duopoly model. We extend this analysis to demonstrate that ambient charges can abate NPS pollution in an *n*-firm Cournot oligopoly case. We use standard market demand functions for *n*-firm Cournot cases in the same industry. The results show that a reduction in NPS pollution occurs not only in a static Cournot duopoly case, but also in a static *n*-firm Cournot oligopoly cases.

1 Introduction

This paper examines whether ambient charges can reduce Nonpoint Source (NPS) pollution in an *n*-firm Cournot oligopoly framework. NPS pollution — that is, pollution from many diffuse sources as opposed to one single source — accounts for the majority of the pollution levels today.[2)] This pollution cannot precisely be observed at the level of abatement or from the discharge of any individual suspected polluter. These unobservable features prevent us from reducing NPS pollution. In order to abate this pollution, Segerson (1988) describes ambient charges — charges based on the total amount of pollution irrespective of specific origin — as a possible instrument of NPS pollution control. In her scheme, the regulatory authority considers emission cut-off levels, not for the individual firms in an industry, but for all the firms belonging to a relevant industry. If the emission standards set by government for NPS pollution of all firms in an industry are compared based on actual levels of emissions, those firms exceeding the cut-off levels can be identified and forced to pay the same fines as each other. By imposing cut-off levels on all the firms in an industry, we can control the total amount of NPS pollution. This payment scheme can be described as ambient charges. After an study of Segerson (1988), there are many studies on ambient charges. Poe et al. (2004) and Suter et al. (2008) discuss ambient charges in terms of experimental research. Xepapadeas (2011) shows some diffferent types of schemes for firms to decrease NPS pollution and Xepapadeas also investigates

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2) Raju and Ganguli (2013) p. 77.

that ambient schemes (: ambient chrages) can abate NPS pollution efficiently. Raju and Ganguli (2013) analyze the outcome of ambient charges in a Cournot duopoly market under constant and decreasing returns to scale. Guerrini et al. (2018) examines ambient charges in a delay Cournot duopoly model. H. Sato (2017) uses ambient charges to reduce NPS pollution in a static Cournot duopoly case.

This paper extends the research conducted by H. Sato (2017) to an *n*-firm Cournot oligopoly framework, and shows that ambient charges can decrease NPS pollution in the *n*-firm Cournot oligopoly case in the same way as in a Cournot duopoly case.

This paper is organized as follows. The methodology is described in Section 2, where we demonstrate that ambient charges can reduce NPS pollution in an *n*-firm Cournot oligopoly case. In the final section, we present the concluding remarks.

2 An *n*-firm Cournot Oligopoly Model

This section demonstrates that ambient charges can reduce NPS pollution in an *n*-firm Cournot oligopoly market. It shows that this reduction occurs not only in a Cournot duopoly case, but also in general Cournot oligopoly cases. These cases have not previously been explored in this context, so we believe this research provides further insight on the effectiveness of ambient charges for reducing NPS pollution. First, we set the market demand function for firm i (i = 1, 2 …, n):

$$p=a-b\sum_{i=1}^{n} q_i, \quad for \quad i=1, 2, \ldots, n. \tag{1}$$

Assume that there are n firms in the same industry that produce a homogeneous product. In an *n*-firm Cournot oligopoly market, p stands for a product price, which is assumed to be the same in all firms in the same industry. a stands for the choke-off price, which indicates that the market price is non-zero and a positive constant. b is the slope of the inverse demand curve of Equation (1) and a positive constant. q_i is the quantity of firm i's production. $\sum_{i=1}^{n} q_i$ shows the total quantity of *n*-firm production. It is also assumed that n firms have the same production technology. Next, we set the profit function to obtain the optimum quantity of each firm's production:

$$\begin{aligned}\pi_i &= pq_i-cq_i-m\left(\sum_{i=1}^{n} e_iq_i-\bar{E}\right)\\ &=\left(a-b\sum_{i=1}^{n} q_i-c-me_i\right)q_i-m\left(\sum_{j=1, j\neq i}^{n} e_jq_j-\bar{E}\right).\end{aligned} \tag{2}$$

The marginal cost of a firm i is a positive constant c. e_i represents the ratio of each firm's emissions. Firm i emits pollutants e_iq_i in connection with these productions. The government can measure an industry's total emission quantity as $\sum_{i=1}^{n} e_iq_i$ The environmental standard is $\bar{E}$, and it is provided exogenously. Segerson (1988) shows that if

$\sum_{i=1}^{n} e_i q_i \geq \bar{E}$, then the government will levy all firms the same penalties, amounting to m times the difference between the total emission quantity and the environmental standard. This difference allows us to provide the form of automatic, required payments in an n-firm Cournot oligopoly model. Differentiating Equation (2) for q_i, we obtain the best response function for an n-firm Cournot oligopoly case:

$$\frac{\partial \pi_i}{\partial q_i} = a - 2bq_i - b\sum_{k=1, k\neq i}^{n} q_k - c - me_i = 0. \tag{3}$$

In Equation (3), n firms maximize their profits. The second-order condition of Equation (2) for q_i is:

$$\frac{\partial^2 \pi_i}{\partial q_i^2} = -2b < 0. \tag{4}$$

Equation (4) shows that the second-order condition for the profit maximization is always satisfed because $b>0$. Next, we show the existence of n-firm Cournot equilibrium for Equation (3), which consists of simultaneous equations for n firms. A series of these simultaneous equations is summarized in vector form A, q and B, which are:

$$Aq = B. \tag{5}$$

where

$$A = (A_{ij})_{(n,\,n)}, \quad with \quad A_{ii} = 2b \quad and \quad A_{ij} = b \quad for \quad i \neq j,$$
$$q = (q_i)_{(n,\,1)}, \quad B = (a - c - me_i)_{(n,\,1)}.$$

Now, we calculate the existence of n-firm Cournot equilibrium for Equation (5). Here, we can consider $|A| \neq 0$[3]:

$$A^{-1} = (x_{ij})_{(n,\,n)}. \tag{6}$$

Considering the identity matrix I for $A^{-1}A$, we obtain the following vector form:

$$q = A^{-1}B. \tag{7}$$

Calculating Equation (7) for the existence of A^{-1},[4] we can write as:

$$\begin{pmatrix} q_1 \\ q_2 \\ \vdots \\ q_i \\ \vdots \\ q_n \end{pmatrix} = \frac{1}{(n+1)b} \begin{pmatrix} n & -1 & -1 & \dots & \dots & -1 \\ -1 & n & -1 & \dots & \dots & -1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & n & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & -1 & \dots & \dots & n \end{pmatrix} \cdot \begin{pmatrix} a-c-me_1 \\ a-c-me_2 \\ \vdots \\ a-c-me_i \\ \vdots \\ a-c-me_n \end{pmatrix} \tag{8}$$

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3) See *Appendix A*. 1.
4) See *Appendix A*. 2.

Arranging Equation (8), we obtain the optimum quantity of production for n firms:

$$q_i{}^* = \frac{a - c - nme_i + \sum_{j=1,\, j \neq i}^{n} me_j}{(n+1)b}. \tag{9}$$

Moreover, we define the industrial emission function $E(m)$ in an n-firm market, which is:

$$E(m) = \sum_{i=1}^{n} e_i q_i{}^*. \tag{10}$$

Substituting Equation (9) into Equation (10), we obtain[5), 6)]

$$E(m) = \frac{1}{(n+1)b} \left[(a-c)\sum_{i=1}^{n} e_i - m\left(\sum_{i=1}^{n-1} \sum_{j=2}^{n} (e_i - e_j)^2 + \sum_{i=1}^{n} e_i^2\right)\right]$$
$$for \quad i,\ j \in N, \quad i \neq j. \tag{11}$$

Differentiating Equation (11) for $E(m)$, we obtain:

$$E'(m) = \frac{-1}{(n+1)b} \left(\sum_{i=1}^{n-1} \sum_{j=2}^{n} (e_i - e_j)^2 + \sum_{i=1}^{n} e_i^2\right) \quad for \quad i,\ j \in N,\ i \neq j. \tag{12}$$

Equation (12) indicates the derivative of NPS pollution in an n-firm Cournot oligopoly case. n is a positive constant, so $E'(m)$ is negative. Hence, we obtain:

Theorem 1.

$$E'(m) < 0.$$

This theorem shows that when we impose ambient charges on an n-firm Cournot oligopoly market, the total amount of NPS pollution is abated. This explanation suggests that ambient charges are effective in reducing the quantity of NPS pollution in an n-firm Cournot oligopoly case in the same way as in a two-firm Cournot duopoly case.

3 Conclusion

This paper examines the effectiveness of ambient charges decreasing NPS pollution in a Cournot oligopoly market. We also show the abatement of ambient charges in a two-firm Cournot duopoly case. In this case, the total emission function is:

$$E(m) = \sum_{i=1}^{2} e_i q_i{}^* = \frac{(a-c)(e_1+e_2) - m((e_1-e_2)^2 + (e_1)^2 + (e_2)^2)}{3b}. \tag{13}$$

5) We define $\sum_{i=m}^{m} a_i \equiv a_m$.
6) See *Appendix A*. 3 for the calculations for Equation (11).

$\sum_{i=1}^{2} e_i q_i^* (= e_1 q_1^* + e_2 q_2^*)$ can be denoted as a function $E(m)$, which indicates the total amount of NPS pollution in a two-firm Cournot duopoly case. By differentiating it, the decrease in pollution is expressed as $E'(m)$, which is:

$$E'(m) = -\frac{(e_1 - e_2)^2 + (e_1)^2 + (e_2)^2}{3b} < 0. \tag{14}$$

The outcome of Equation (14) is, finally, the same one as Sato (2017)'s formula[7] of two-firm Cournot duopoly model. This conclusion also identifies the outcome of an two-firm Cournot duopoly case at which ambient charges can abate NPS pollution. Hence, we show that this paper generalizes H. Sato (2017).

This paper is only a static model, and does not consider dynamic models. In order to extend this research, we must examine dynamic models to more precisely outline the point at which the NPS pollution can be reduced.

Appendix

A.1 Calculation for the nonsingularity of $|A|$

The determinant $|A|$ of A in an n-firm Cournot oligopoly model can be defined as:

$$|A| = \begin{vmatrix} 2b & b & \cdots & b \\ b & 2b & \cdots & b \\ \vdots & \vdots & \ddots & b \\ b & b & \cdots & 2b \end{vmatrix}. \tag{A.1}$$

The determinant $|A|$ becomes as follows;

$$|A| = \overbrace{\begin{vmatrix} 2b & b & \cdots & b & b \\ b & 2b & \cdots & b & b \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ b & b & \cdots & 2b & b \\ b & b & \cdots & b & 2b \end{vmatrix}}^{n}. \tag{A.2}$$

We can calculate the determinant in (A.2) by using the formulas of the determinant.[8]

$$|A| = b^n \overbrace{\begin{vmatrix} 2 & 1 & \cdots & 1 & 1 \\ 1 & 2 & \cdots & 1 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & \cdots & 2 & 1 \\ 1 & 1 & \cdots & 1 & 2 \end{vmatrix}}^{n} = b^n \overbrace{\begin{vmatrix} 1 & 0 & \cdots & 0 & 1 \\ -1 & 1 & \cdots & 0 & 1 \\ 0 & -1 & \cdots & 0 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 1 \\ 0 & 0 & \cdots & -1 & 2 \end{vmatrix}}^{n}. \tag{A.3}$$

By calculating the cofacter expantion of the n-th raw of the determinant in Equation (A.3), we obtain

7) See *Appendix A*. 4.
8) Refer to Chiang and Wainwright (2005), and Nikaido (1961).

$$|A| = b^n\ [2(-1)^{(n+n)} \overbrace{\begin{vmatrix} 1 & 0 & \dots & 0 \\ -1 & 1 & \dots & 0 \\ 0 & -1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{vmatrix}}^{n-1} + (-1)(-1)^{(n+(n-1))} \overbrace{\begin{vmatrix} 1 & 0 & \dots & 1 \\ -1 & 1 & \dots & 1 \\ 0 & -1 & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{vmatrix}}^{n-1}]. \tag{A.4}$$

Arranging Equation (A.4) by using the formulas of the determination of (A.4), we obtain

$$|A| = b^n\ [2 + (n-1)] = (n+1)b^n > 0. \tag{A.5}$$

A.2 Calculation for the inverse matrix of A

Here, we calculate A^{-1}, which is the inverse matrix of A. In order to show A^{-1}, we use Equations (6) and $A^{-1} A = I$. We pick up an i's row of A^{-1} and all of the columns of A in Equation $A^{-1} A = I$, and calculate simultaneous equations:

$$\begin{cases} 2bx_{i1} + bx_{i2} + \dots + bx_{in} = 0 & \text{(B.1 a)} \\ bx_{i1} + 2bx_{i2} + \dots + bx_{in} = 0 & \text{(B.1 b)} \\ \vdots & \\ bx_{i1} + bx_{i2} + 2bx_{ii-1} + bx_{ii} + \dots + bx_{in} = 0 & \text{(B.1 c)} \\ bx_{i1} + bx_{i2} + bx_{ii-1} + 2bx_{ii} + bx_{ii+1} \dots + bx_{in} = 1 & \text{(B.1 d)} \\ bx_{i1} + bx_{i2} + bx_{ii-1} + bx_{ii} + 2bx_{ii+1} \dots + bx_{in} = 0 & \text{(B.1 e)} \\ \vdots & \\ bx_{i1} + bx_{i2} + \dots + 2bx_{in} = 0. & \text{(B.1 f).} \end{cases}$$

Subtracting Equation (B.1 a) from Equation (B.1 b),

$$bx_{i1} - bx_{i2} = 0.$$

where

$$x_{i1} = x_{i2}. \tag{B.2}$$

Likewise, subtracting another upper equation from another lower one in the previous simultaneous equations, we obtain:

$$x_{ia} = x, \quad for \quad i, a \in N, \quad a = 1, 2, \dots, n, \quad i \neq a. \tag{B.3}$$

Substituting Equation (B.3) into Equations (B.1 a), (B.1 b), (B.1 c), (B.1 d), (B.1 e) and (B.1 f), we obtain a pair of simultaneous equations:

$$\begin{cases} nbx + bx_{ii} = 0. & \text{(B.4 a)} \\ (n-1)bx_{i1} + 2bx_{ii} = 1. & \text{(B.4 b)} \end{cases}$$

from which,

$$x = \frac{-1}{b} + x_{ii}. \tag{B.5}$$

Substituting Equation (B.5) into Equation (B.4 a), we obtain:

$$x_{ii} = \frac{n}{(n+1)b}. \tag{B.6}$$

Hence, we obtain:

$$x = -\frac{1}{(n+1)b}. \tag{B.7}$$

Q.E.D.

A.3 Arrangement for Equation (11)

Substituting Equation (9) into Equation (10), we obtain:

$$E(m)=\frac{1}{(n+1)b}\ [(a-c)(e_1+e_2+\ldots+e_n)-m(e_1\ (ne_1-e_2-\ldots-e_n)$$
$$+e_2\ (-e_1+ne_2-\ldots-e_n)+\ldots+e_n\ (-e_1-e_2-\ldots+ne_n))].$$

Arranging this equation, we obtain:

$$E(m)=\frac{1}{(n+1)b}\ [(a-c)\sum_{i=1}^{n}\ e_i-m((i-1)\sum_{i=1}^{n}\ e_i^2-2\sum_{i=1}^{n-1}\ \sum_{j=2}^{n}\ e_ie_j)+\sum_{i=1}^{n}\ e_i^2]. \tag{C.1}$$

Transforming (C.1), we obtain Equation (11).

Q.E.D.

A.4 The formula of H. Sato (2017) in order to abate NPS pollution

H. Sato (2017) shows that the optimum quantity of the industrial emission in a two-firm Cournot equilibrium is Equation (D.1):

$$e_1q_1{}^*+e_2q_2{}^*=\frac{(a-c)(e_1+e_2)-2m(e_1\ e_2-e_1^2-e_2^2)}{3b}\ . \tag{D.1}$$

By differentiating Equation (D.1) by m, we obtain

$$E'(m)=\frac{2(e_1\ e_2-e_1^2-e_2^2)}{3b}\ <0. \tag{D.2}$$

H. Sato shows that $E'(m)$ is negative in Equaiton (D.2) if $e_1>0$ and $e_2>0$ are given in Equation (D.2).[9] As a result, the value of $E'(m)$ in H. Sato (2017) is negative as Proposition 1 in this paper.

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9) The proof of (D. 2), see Proof in H. Sato (2017) pp. 6–7.

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