修士論文要旨 (2020 年度)

導体円柱による電磁波散乱のための物理光学と幾何光学に基づく等価電流法の等価性

A Field Equivalence between Physical Optics and Geometrical Optics-Based Equivalent Current Methods for Scattering from Circular Cylinders

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1. Introduction

In the decades, electromagnetic scattering (EM) analyses are essential topics among scientifically inclined philosophers. Variety of methods have been proposed to solve the EM scattering problems efficiently.

Numerical methods, for instance the method of moment, the boundary element method, and the finite difference time domain method, could be used to solve a considered scattering problem. However, the drawback of these methods is that the computation becomes time consuming and computer-intensive at high frequencies, while asymptotic methods [1]-[3] can solve problem more efficiently by consisting in the asymptotic evaluation of the Maxwell's equations. This is specially true when dealing with electrically large geometries.

One of the high frequency methods, physical optics (PO) has been proved to be very powerful method. The method utilizes the induced current by the the incident magnetic field. This surface current flows only on the illuminated surface of the cylinder.

In Ref. [4], the high frequency scattering from edged objects has been investigated. The paper describes the equivalent currents with GO-based method, which includes equivalent electric and magnetic currents in both illuminated and shadowed portions. This formulation gives us a better description for obtaining the scattering field from non-perfect bodies. In [5], the plane wave scattering from a penetrable rectangular cylinder has been analyzed using the extended PO method. Now, a question arises as whether this method can also be applied to the smooth objects. This is the motive of the study.

In this paper, the extended PO method will be used to evaluate the scattering far-field from a circular conducting cylinder. The scattering field formulation by the equivalent currents obtained from GO field has been found to be exactly the same as one derived by the conventional PO approximation for both E and H polarizations. Accordingly, together with the previous study for edged objects [5], the field equivalence between the PO and GO-based equivalent current methods has been confirmed for scattering by smooth and edged objects.

The time-harmonic factor $e^{j\omega t}$ is assumed and suppressed throughout the context.

2. Scattering Field Formulation using Surface Equivalence Theorem

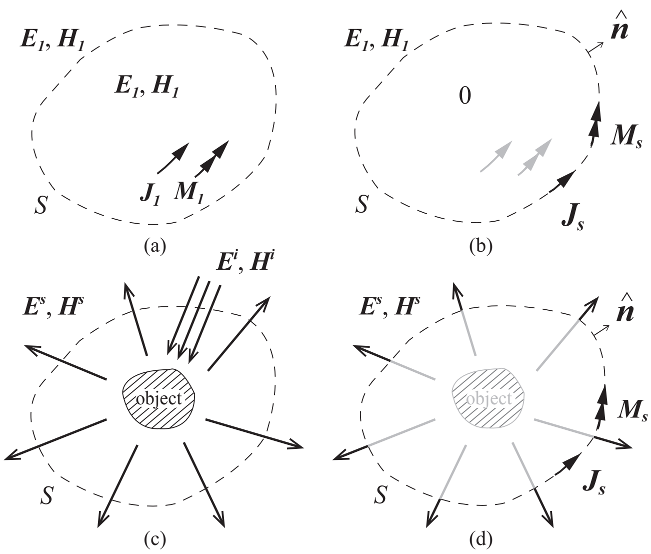

Figure 1: Field equivalence principle model. (a) Fields $\boldsymbol{E}_1, \boldsymbol{H}_1$ excited by original sources by $\boldsymbol{J}_1, \boldsymbol{M}_1$. (b) Fields $\boldsymbol{E}_1, \boldsymbol{H}_1$ excited by the equivalence surface currents $\boldsymbol{J}_s, \boldsymbol{M}_s$ on S. (c) Scattering fields $\boldsymbol{E}^{\mathrm{s}}, \boldsymbol{H}^{\mathrm{s}}$ by an object due to the incident wave $\boldsymbol{E}^{\mathrm{i}}, \boldsymbol{H}^{\mathrm{i}}$. (d) Scattering fields $\boldsymbol{E}^{\mathrm{s}}, \boldsymbol{H}^{\mathrm{s}}$ by the equivalent surface currents $\boldsymbol{J}_s, \boldsymbol{M}_s$ on S.

According to the surface equivalence theorem, assuming that EM fields $\boldsymbol{E}_1, \boldsymbol{H}_1$ excited by current sources $\boldsymbol{J}_1, \boldsymbol{M}_1$ as shown in Fig. 1(a), if these sources are enclosed by a virtual surface S then the EM field outside S can be excited by equivalent surface currents $\boldsymbol{J}_s, \boldsymbol{M}_s$ as in Fig. 1(b).

Now considering to the electromagnetic scattering problem from an object illuminated by an incident field $\boldsymbol{E}^{\mathrm{i}}, \boldsymbol{H}^{\mathrm{i}}$ in Fig. 1(c). Then following the surface equivalence theorem, the scattering fields $\boldsymbol{E}^{\mathrm{s}}, \boldsymbol{H}^{\mathrm{s}}$ outside of the virtual surface S depicted in Fig. 1(d) may be derived from the surface currents $\boldsymbol{J}_{\rm s}, \boldsymbol{M}_{\rm s}$ as

$$\boldsymbol{E}^{\rm s}(\boldsymbol{r}) = -\int_S \Big(j\omega\mu_0 \boldsymbol{J}_{\rm s}(\boldsymbol{r}')G(\boldsymbol{r},\boldsymbol{r}') + \boldsymbol{M}_{\rm s}(\boldsymbol{r}') \times \nabla' G(\boldsymbol{r},\boldsymbol{r}') + \frac{j}{\omega\varepsilon_0}\boldsymbol{J}_{\rm s}(\boldsymbol{r}')\cdot\nabla'\nabla' G(\boldsymbol{r},\boldsymbol{r}')\Big) dS, \tag{1}$$

$$\boldsymbol{H}^{\rm s}(\boldsymbol{r}) = -\int_S \Big(j\omega\varepsilon_0 \boldsymbol{M}_{\rm s}(\boldsymbol{r}')G(\boldsymbol{r},\boldsymbol{r}') - \boldsymbol{J}_{\rm s}(\boldsymbol{r}') \times \nabla' G(\boldsymbol{r},\boldsymbol{r}') + \frac{j}{\omega\mu_0}\boldsymbol{M}_{\rm s}(\boldsymbol{r}')\cdot\nabla'\nabla' G(\boldsymbol{r},\boldsymbol{r}')\Big) dS, \tag{2}$$

where ε_0 and μ_0 are the permittivity and the permeability of the free space, respectively, and ω is the angular frequency. $G(\boldsymbol{r},\boldsymbol{r}')$ denotes the free space Green's function and ∇' indicates differentiation with respect to the source coordinates.

The above scattering fields $\boldsymbol{E}^{\rm s}, \boldsymbol{H}^{\rm s}$ are exact only as equivalent currents $\boldsymbol{J}_s, \boldsymbol{M}_s$ are found. However, it is usually difficult to find the true currents and the current approximation may be used in the following.

2.1 PO Current

If the scattering object is made of a large electric conducting body, then the induced current on the object's surface may be approximated as

$$\boldsymbol{J}^{\rm PO} = \begin{cases} 2\hat{\boldsymbol{n}} \times \boldsymbol{H}^{\rm i}, & \text{on the illuminated } S, \\ 0, & \text{on the shadowed } S. \end{cases} \tag{3}$$

The PO current is exact when the scattering surface is infinitely wide and flat.

2.2 Equivalent Current Approximation

If the virtual surface S is assumed to be set right on the conducting body, then the equivalent currents $\boldsymbol{J}_{\rm s}, \boldsymbol{M}_{\rm s}$ may be easily approximated in term of the incident GO field $\boldsymbol{E}^{\rm i}, \boldsymbol{H}^{\rm i}$ and the reflected GO fields $\boldsymbol{E}^{\rm r}, \boldsymbol{H}^{\rm r}$ as

$$\boldsymbol{J}_{\rm s} = \hat{\boldsymbol{n}} \times \boldsymbol{H}^{\rm s} \simeq \begin{cases} \hat{\boldsymbol{n}} \times \boldsymbol{H}^{\rm r}, & \text{on the illuminated } S, \\ \hat{\boldsymbol{n}} \times (-\boldsymbol{H}^{\rm i}), & \text{on the shadowed } S, \end{cases} \tag{4}$$

$$\boldsymbol{M}_{\rm s} = \boldsymbol{E}^{\rm s} \times \hat{\boldsymbol{n}} \simeq \begin{cases} \boldsymbol{E}^{\rm r} \times \hat{\boldsymbol{n}}, & \text{on the illuminated } S, \\ (-\boldsymbol{E}^{\rm i}) \times \hat{\boldsymbol{n}}, & \text{on the shadowed } S. \end{cases} \tag{5}$$

3. Scattering Field from a Circular Conducting Cylinder

In order to confirm the scattering formulation equivalence between the conventional PO and the GO-based

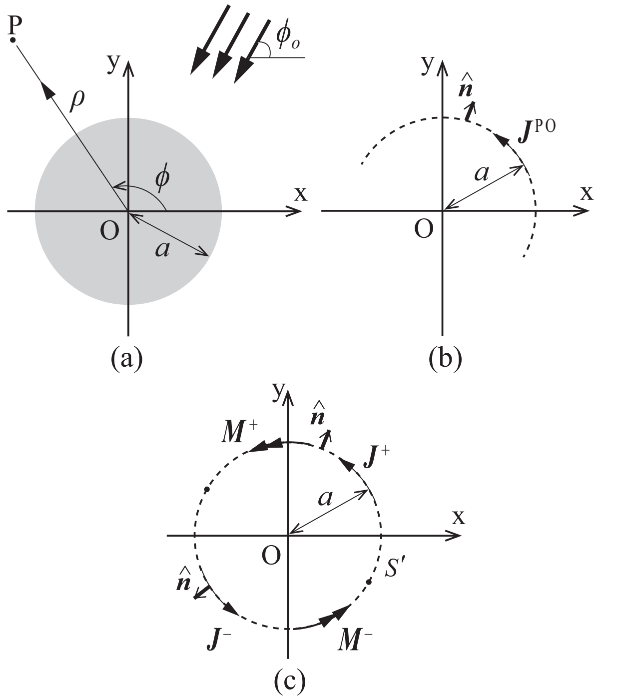

Figure 2: Scattering from a circular conducting cylinder. (a) The circular cylinder illuminted by a plane wave. (b) PO current $\boldsymbol{J}^{\rm PO}$ by the incident field on the illuminated surface. (c) Equivalent currents $\boldsymbol{J}^+, \boldsymbol{J}^-, \boldsymbol{M}^+, \boldsymbol{M}^-$ derived by GO fields.

equivalent current methods for smooth objects, let formulate a plane wave scattering from a circular conducting cylinder. Figure 2(a) shows a two dimensional circular conducting cylinder of radius a illuminated by a plane wave with an incident angle of ϕ_0. Because of the symmetry of the scatterer, the incident angle is assumed as $0 \le \phi_0 \le \pi/2$ without losing the generality. The scattering formulation may be separated into two polarizations.

3.1 E Polarization

The E polarized incident plane wave may be written in the polar coordinate as

$$E_z^{\rm i} = E_0 e^{jk\rho\cos(\phi-\phi_0)}, \tag{6}$$

$$H_\rho^{\rm i} = -\frac{1}{j\omega\mu_0}\frac{1}{\rho}\frac{\partial E_z^{\rm i}}{\partial\phi}, \ H_\phi^i = \frac{1}{j\omega\mu_0}\frac{\partial E_z^{\rm i}}{\partial\rho}. \tag{7}$$

3.1.1 PO Approximation

PO current $\boldsymbol{J}^{\rm PO}$ on the illuminated surface may be represented as Fig. 2(b). From the incident wave, the current $\boldsymbol{J}^{\rm PO}$ can be found for $-\pi/2 \le \phi - \phi_0 \le \pi/2$ from Eq. (3) as

$$\boldsymbol{J}^{\rm PO}(\phi) = 2\hat{\boldsymbol{\rho}} \times \boldsymbol{H}^{\rm i}\big|_{\rho=a} = 2\sqrt{\frac{\varepsilon_0}{\mu_0}}E_0\cos(\phi-\phi_0)e^{jka\cos(\phi-\phi_0)}\hat{\boldsymbol{z}}. \tag{8}$$

Then the scattering electric field can be obtained from Eq. (1) by integrating the PO current in Eq. (8) over the illuminated surface as

$$E_z^{\mathrm{PO}} = -j\omega\mu_0 \int_{\phi_0-\pi/2}^{\phi_0+\pi/2} J_z^{\mathrm{PO}}(\phi')G(\rho,\phi;a,\phi')a\ d\phi', \quad (9)$$

$$G(\boldsymbol{\rho},\boldsymbol{\rho}') = \frac{1}{4j}\mathrm{H}_0^{(2)}(k|\boldsymbol{\rho}-\boldsymbol{\rho}'|), \quad (10)$$

where $\mathrm{H}_0^{(2)}(\chi)$ denotes the zero-th order Hankel function of the second kind.

Assuming $k\rho \gg ka$, one may use the asymptotic approximation of the Hankel function for a large argument [2, 3]. Then Eq. (9) becomes with a new variable $\varphi = \phi' - \phi_0$ as

$$\begin{aligned} E_z^{\mathrm{PO}} &\sim -2jkaE_0C(k\rho) \\ &\cdot \int_{-\pi/2}^{\pi/2} \cos\varphi e^{jka[\cos\varphi+\cos(\varphi+\phi_0-\phi)]}\ d\varphi, \end{aligned} \quad (11)$$

$$C(\chi) = \sqrt{\frac{1}{8\pi\chi}}e^{-j\chi-j\pi/4}. \quad (12)$$

3.1.2 Equivalent Current Approximation

Figure 2(c) describes the equivalent currents derived by the GO fields. These currents on the cylinder surface can be found from Eqs. (4),(5). For the illuminated region $(0 < |\phi-\phi_0| < \pi/2)$,

$$\begin{aligned} \boldsymbol{J}^+(\phi) &= \hat{\boldsymbol{\rho}} \times \boldsymbol{H}^{\mathrm{r}}\big|_{\rho=a} \\ &= \sqrt{\frac{\varepsilon_0}{\mu_0}}E_0\cos(\phi-\phi_0)e^{jka\cos(\phi-\phi_0)}\hat{\boldsymbol{z}}, \end{aligned} \quad (13)$$

$$\boldsymbol{M}^+(\phi) = \boldsymbol{E}^{\mathrm{r}} \times \hat{\boldsymbol{\rho}}\big|_{\rho=a} = -E_0e^{jka\cos(\phi-\phi_0)}\hat{\boldsymbol{\phi}}, \quad (14)$$

and for the shadow region $(\pi/2 < |\phi-\phi_0| < \pi)$,

$$\begin{aligned} \boldsymbol{J}^-(\phi) &= \hat{\boldsymbol{\rho}} \times (-\boldsymbol{H}^{\mathrm{i}})\big|_{\rho=a} \\ &= -\sqrt{\frac{\varepsilon_0}{\mu_0}}E_0\cos(\phi-\phi_0)e^{jka\cos(\phi-\phi_0)}\hat{\boldsymbol{z}} \\ &= -\boldsymbol{J}^+(\phi), \end{aligned} \quad (15)$$

$$\begin{aligned} \boldsymbol{M}^-(\phi) &= (-\boldsymbol{E}^{\mathrm{i}}) \times \hat{\boldsymbol{\rho}}\big|_{\rho=a} \\ &= -E_0e^{jka\cos(\phi-\phi_0)}\hat{\boldsymbol{\phi}} = \boldsymbol{M}^+(\phi). \end{aligned} \quad (16)$$

Then the scattering electric far-field due to the equivalent electric currents can be derived from Eq. (1) by integrating along the cylinder surface with the Green's function. Since $\boldsymbol{J}^+(\phi)$ is one half of $\boldsymbol{J}^{\mathrm{PO}}$, thus the corresponding scattering field $E_z^{\mathrm{J+}}$ is found to be exactly one half of E_z^{PO} in Eq. (11). Thus

$$E_z^{\mathrm{J+}} = \frac{1}{2}E_z^{\mathrm{PO}}, \quad (17)$$

$$\begin{aligned} E_z^{\mathrm{J-}} &= -j\omega\mu_0 \int_{\phi_0+\pi/2}^{\phi_0+3\pi/2} J_z^-(\phi')G(\rho,\phi;a,\phi')a\ d\phi' \\ &\approx jkaE_0C(k\rho) \\ &\cdot \int_{\pi/2}^{3\pi/2} \cos\varphi e^{jka[\cos\varphi+\cos(\varphi+\phi_0-\phi)]}\ d\varphi. \end{aligned} \quad (18)$$

Similarly, the corresponding scattering electric far-field due to the equivalent magnetic currents can be found from Eq. (1) as

$$\begin{aligned} E_z^{\mathrm{M}} &= E_z^{\mathrm{M+}} + E_z^{\mathrm{M-}} \\ &= \int_0^{2\pi} M_\phi(\phi')\frac{\partial}{\partial\rho'}G(\rho,\phi;\rho',\phi')\big|_{\rho'=a}a\ d\phi' \\ &\approx -jkaE_0C(k\rho) \\ &\cdot \int_0^{2\pi} \cos\varphi e^{jka[\cos\varphi+\cos(\varphi+\phi_0-\phi)]}\ d\varphi. \end{aligned} \quad (19)$$

Summing up Eqs. (17),(18), and (19) yields a cancellation of $E_z^{\mathrm{J-}}$ and $E_z^{\mathrm{M-}}$ in the shadow region $(\pi/2 < \varphi < 3\pi/2)$, and the final result of the scattering field becomes exactly the same as the far field formulation by PO in Eq. (11), namely

$$\begin{aligned} E_z^{\mathrm{s}} &= E_z^{\mathrm{J+}} + E_z^{\mathrm{J-}} + E_z^{\mathrm{M+}} + E_z^{\mathrm{M-}} \\ &= E_z^{\mathrm{J+}} + E_z^{\mathrm{M+}} = E_z^{\mathrm{PO}}. \end{aligned} \quad (20)$$

3.2 H Polarization

A similar result can be derived for the H polarization case, in which the incident TM plane wave can be written as

$$H_z^{\mathrm{i}} = H_0e^{jk\rho\cos(\phi-\phi_0)}. \quad (21)$$

3.2.1 PO Approximation

From the incident plane wave, one can derive the PO current $\boldsymbol{J}^{\mathrm{PO}}$ from Eq. (3) on the illuminated surface as depicted in Fig. 2(b), as

$$\boldsymbol{J}^{\mathrm{PO}}(\phi) = 2\hat{\boldsymbol{\rho}} \times \boldsymbol{H}^{\mathrm{i}}\big|_{\rho=a} = -2H_0e^{jka\cos(\phi-\phi_0)}\hat{\boldsymbol{\phi}}, \quad (22)$$

for $-\pi/2 \le \phi-\phi_0 \le \pi/2$. The scattering magnetic far-field can be obtained by integrating the PO current as Eq. (2) over the illuminated surface . Then one finds the scattering radiation integral by the PO approximation as [6]

$$\begin{aligned} H_z^{\mathrm{PO}} &\sim 2jkaH_0C(k\rho) \\ &\cdot \int_{-\pi/2}^{\pi/2} \cos(\varphi+\phi_0-\phi)e^{jka[\cos\varphi+\cos(\varphi+\phi_0-\phi)]}\ d\varphi. \end{aligned} \quad (23)$$

3.2.2 Equivalent Current Approximation

The equivalent currents $\boldsymbol{J}^{\pm}, \boldsymbol{M}^{\pm}$ on the cylinder surface can be found as

$$\begin{aligned}\boldsymbol{J}^{+}(\phi) &= \hat{\boldsymbol{\rho}} \times \boldsymbol{H}^{\mathrm{r}}\big|_{\rho=a} \\ &= -H_0 e^{jka\cos(\phi-\phi_0)}\hat{\boldsymbol{\phi}} = \frac{1}{2}\boldsymbol{J}^{\mathrm{PO}}(\phi), \qquad (24)\\ \boldsymbol{M}^{+}(\phi) &= \boldsymbol{E}^{\mathrm{r}} \times \hat{\boldsymbol{\rho}}\big|_{\rho=a} \\ &= -\sqrt{\frac{\mu_0}{\varepsilon_0}} H_0 \cos(\phi-\phi_0) e^{jka\cos(\phi-\phi_0)}\hat{\boldsymbol{z}}, \quad (25)\\ \boldsymbol{J}^{-}(\phi) &= \hat{\boldsymbol{\rho}} \times (-\boldsymbol{H}^{\mathrm{i}})\big|_{\rho=a} \\ &= H_0 e^{jka\cos(\phi-\phi_0)}\hat{\boldsymbol{\phi}} = -\boldsymbol{J}^{+}(\phi), \qquad (26)\\ \boldsymbol{M}^{-}(\phi) &= (-\boldsymbol{E}^{\mathrm{i}}) \times \hat{\boldsymbol{\rho}}\big|_{\rho=a} \\ &= -\sqrt{\frac{\mu_0}{\varepsilon_0}} H_0 \cos(\phi-\phi_0) e^{jka\cos(\phi-\phi_0)}\hat{\boldsymbol{z}}. \quad (27)\end{aligned}$$

The equivalent electric current on the illuminated surface $\boldsymbol{J}^{+}(\phi)$ is found to be one half of $\boldsymbol{J}^{\mathrm{PO}}$ in Eq. (22) like E polarization, while the equivalent magnetic currents in both illuminated and shadowed regions are found to be the same each other. Thus the scattering far-field due to the equivalent electric currents can be found as

$$\begin{aligned}H_z^{\mathrm{J}} &= H_z^{\mathrm{J}+} + H_z^{\mathrm{J}-} \\ &\approx jkaH_0 C(k\rho) \\ &\cdot \left[\int_{-\pi/2}^{\pi/2} \cos(\varphi+\phi_0-\phi) e^{jka[\cos\varphi+\cos(\varphi+\phi_0-\phi)]}\, d\varphi \right. \\ &\left. - \int_{\pi/2}^{3\pi/2} \cos(\varphi+\phi_0-\phi) e^{jka[\cos\varphi+\cos(\varphi+\phi_0-\phi)]}\, d\varphi \right], \qquad (28)\end{aligned}$$

and the far-field due to the magnetic equivalent current

$$\begin{aligned}H_z^{\mathrm{M}} &= -j\omega\varepsilon_0 \int_0^{2\pi} M_\phi(\phi') G(\rho,\phi;\rho',\phi')\big|_{\rho'=a} a\, d\phi' \\ &\approx jkaH_0 C(k\rho) \\ &\cdot \int_0^{2\pi} \cos(\varphi+\phi_0-\phi) e^{jka[\cos\varphi+\cos(\varphi+\phi_0-\phi)]}\, d\varphi. \qquad (29)\end{aligned}$$

Then the total scattering magnetic far-field is given by

$$\begin{aligned}H_z^{\mathrm{s}} &= H_z^{\mathrm{J}} + H_z^{\mathrm{M}} \\ &= 2jkaH_0 C(k\rho) \\ &\cdot \int_{-\pi/2}^{\pi/2} \cos(\varphi+\phi_0-\phi) e^{jka[\cos\varphi+\cos(\varphi+\phi_0-\phi)]}\, d\varphi. \qquad (30)\end{aligned}$$

This is found to be exactly the same as one by the PO approximation in Eq. (23).

4. Conclusion

In this study, field equivalence between PO and GO-based equivalent current methods has been shown for the plane wave scattering by a circular conducting cylinder for both E and H polarizations. While the conventional PO current is postulated only for non-penetrable conducting objects, our formulation using the GO-based equivalent currents can be easily applied to the penetrable objects such as dielectric and/or magnetic bodies [5]. The accuracy of the method should be checked for such penetrable objects in a future investigation.

Acknowledgment

I appreciated greatly that MEXT and Chuo University have provided me great opportunity to study in Japan. I would also like to take this opportunity to express my gratitude to Prof. Hiroshi Shirai for his supervision of this research. And this work has been supported partially by from JSPS Grant.

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