Quantal Response Equilibrium vs. Cognitive Hierarchies: An Analysis of Initial Responses in an Asymmetric All-Pay Auction Experiment

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Abstract

Predicting initial responses to novel strategic situations has been a challenge in game theory. People are not as sophisticated as players assumed by solution concepts in game theory, and their initial play has a tendency to systematically deviate from equilibrium. Several behavioral models of games have been proposed to bridge a gap between initial behavior and equilibrium play. This paper fits two one-parameter behavioral models, a quantal response equilibrium (QRE) and a cognitive hierarchy (CH) model, into the first-round data of the experiment conducted by Otsubo (2013). Estimation results show that the QRE accounts better for deviations from Nash equilibrium play than the CH model.

Key Words

Initial responses; All-pay auction; Experiment; Quantal response equilibrium; Cognitive hierarchy model

Contents

1 Introduction

In game theory experiments it is a common practice to allow participants to play games repeatedly. Repetition obviously generates more data. In addition, it may help participants to understand the structure of games, learn how others play (via information feedback), and adjust their own behavior owing to experience. Previous experimental studies of learning have attempted to address how current behavior shapes future behavior and whether behavior converges to equilibrium play.[1]

Another behaviorally intriguing question is how people react to novel strategic situations. This ques-

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1 See Chapter 6 of Camerer (2003) for a comprehensive survey of the experimental studies of learning.

tion is of practical importance because real-world strategic interactions are not always repeated but often played only once. In game theory, a state is in equilibrium if players form correct beliefs of what other players do and best respond to their beliefs. Without any prior interaction with other players, how could people form correct beliefs of others' actions? The cognitive requirements for initial responses to be in equilibrium are far more stringent than for learning to converge to equilibrium (Costa-Gomes et al., 2009). Therefore, when people confront novel strategic situations, their initial responses would not be in equilibrium.

Several behavioral models of games have been proposed to account for initial responses to unprecedented strategic situations. Examples include a non-equilibrium model based on level-k thinking (Stahl & Wilson, 1994, 1995; Nagel, 1995), another non-equilibrium model that is closely related to the level-k model called a cognitive hierarchy (CH) model (Camerer et al., 2004), and an equilibrium concept that relaxes the best response assumption of Nash equilibrium called a quantal response equilibrium (QRE) (McKelvey & Palfrey, 1995). A natural question is to identify experimental games in which these models explain initial play well and reasons why they does not in other games.

This paper explores how well the QRE and the CH model account for first-round behavior in an asymmetric all-pay auction experiment conducted by Otsubo (2013).[2] In this experiment, participants played an identical all-pay auction game 60 times. The game possesses a unique Nash equilibrium in mixed strategies, and its implications were tested based on the data of the last 30 rounds so as to reduce confounding with any early-round learning and adjustment. Otsubo (2013) confirmed that the behavior of participants is consistent with the Nash equilibrium on the aggregate level and that the theoretical predictions are well supported by the data. However, participants' initial behavior was neither inquired in detail nor compared with any behavioral models of games.

This paper proceeds as follows. Section 2 gives an overview of the experiment run by Otsubo (2013) and summarizes its first-round data. Section 3 discusses the QRE and the CH model. Section 4 reports maximum likelihood estimates of their parameters. Section 5 concludes.

2 An Asymmetric All-Pay Auction Experiment

2.1 Game and Equilibrium

Otsubo (2013) ran a laboratory experiment in which participants repeatedly played the following asymmetric all-pay auction game.[3]

Two political candidates, an incumbent and a challenger, compete for elected office. Index the incumbent by i and the challenger by c. The winner receives a single, symmetrically valued prize r.[4] To win

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2 In the literature of contest theory, these models have been fitted to experimental data in an attempt to bridge the gap between observed behavior and equilibrium play. See Crawford and Iriberri (2007) and Bernard (2010) for the level-k model, Gneezy (2005) and Lim et al. (2014) for the CH model, Anderson et al. (1998), Rapoport and Amaldoss (2004), Gneezy and Smorodinsky (2006), Sheremeta (2011), Chowdhury et al. (2014), and Lim et al. (2014) for the QRE.

3 All-pay auctions (Hillman & Samet, 1987; Baye et al., 1996) have been used to model a variety of economic, social, and political contests. See Konrad (2009) for a survey of the literature of contests and Dechenaux et al. (2015) for a recent survey of experimental research on contests.

the election, each candidate simultaneously chooses her amount of irrevocable campaign spending (expenditure) e_j, $j \in \{i, c\}$, from the common set $E = \{0, 1, 2, \ldots, l\}$, where l is a common spending limit. For a candidate to be eligible for winning office, her level of campaign spending has to be at least m.[5] Hereafter, the parameters l, m, and r are assumed to be integer values such that $0 < m < l < r$.

In this game these candidates are asymmetric in that ties are always broken in favor of the incumbent. This may happen due to, for example, officeholder benefits such as voters' status-quo bias and greater name recognition. The incumbent wins the election if $e_i \geqq e_c$ and $e_i \geqq m$ and loses otherwise. The challenger, on the other hand, wins the election if $e_c > e_i$ and $e_c \geqq m$ and loses otherwise. Formally, the incumbent's contest success function is :

$$f_i(e_i, e_c) = \begin{cases} 1 & \text{if } e_i \geqq e_c \text{ and } e_i \geqq m \\ 0 & \text{otherwise} \end{cases}$$

Similarly, the challenger's contest success function is :

$$f_c(e_i, e_c) = \begin{cases} 1 & \text{if } e_c > e_i \text{ and } e_c \geqq m \\ 0 & \text{otherwise} \end{cases}$$

For $j \in \{i, c\}$, candidate j's payoff function is given by

$$u_j(e_j, e_{-j}) = r \cdot f_j(e_j, e_{-j}) - e_j.$$

It is straightforward to show that there is no Nash equilibrium in pure strategies. Let (p_i, p_c) be a profile of mixed strategies. Otsubo (2013) proved that there exists a unique Nash equilibrium in mixed strategies (p_i^*, p_c^*) characterized by

$$p_i^*(e_i) = \begin{cases} 0 & \text{if } e_i \in \{0, \ldots, m-1\} \\ \dfrac{m+1}{r} & \text{if } e_i = m \\ \dfrac{1}{r} & \text{if } e_i \in \{m+1, \ldots, l-1\} \\ 1 - \dfrac{l}{r} & \text{if } e_i = l \end{cases} \tag{1}$$

and

$$p_c^*(e_c) = \begin{cases} 1 - \left(\dfrac{l-m}{r}\right) & \text{if } e_c = 0 \\ 0 & \text{if } e_c \in \{1, \ldots, m\} \\ \dfrac{1}{r} & \text{if } e_c \in \{m+1, \ldots, l\} \end{cases} \tag{2}$$

with associated equilibrium payoffs $r - l$ for the incumbent and 0 for the challenger.[6]

2.2 Experiment

Table 1 summarizes the experimental design of Otsubo (2013). There are two independent variables, each of which has two levels. The first is the size of the spending limit l : low ($l = 8$) versus high ($l =$

4 This prize can be thought as the amount of benefits that the winner receives during her term of office.

5 Hillman and Samet (1987) discussed theoretical implications of the minimum expenditure requirement in the context of all-pay auctions.

6 For the proof, see the supplementary material of Otsubo (2013).

Table 1: Design of the experiment

Treatment	Session	l	r	No. of participants per session	Nash	
					Incumbent	Challenger
LL	1, 2	8	15	32	5.6667	2.3333
HL	3, 4	13	15	32	7	6
LH	5, 6	8	20	32	6.25	1.75
HH	7, 8	13	20	32	8.5	4.5

13). The second is the size of the prize r: low ($r=15$) versus high ($r=20$). Therefore, the experiment has a total of four treatments: LL ($l=8$ and $r=15$), HL ($l=13$ and $r=15$), LH ($l=8$ and $r=20$), and HH ($l=13$ and $r=20$). The sixth and the seventh columns present the expected expenditures under Nash equilibrium play for the incumbent and the challenger, respectively.

There are two sessions per treatment. Each session consists of 32 participants, sixteen of whom played the game as the incumbent and the rest as the challenger. These roles were randomly assigned at the beginning of the session, and participants retained their roles throughout the session.

2.3 Initial Responses to the Game

Table 2 presents general statistics of expenditures in the first round by treatment and role. Participants exhibited a tendency to overspend except challenger-role participants in treatment HL. The mean expenditure was fairly close to the expected expenditure under equilibrium play for the incumbent, but not for the challenger. A Wilcoxon signed rank test was used to test the null hypothesis of no difference between the observed and predicted mean expenditures. For the incumbent, the null hypothesis was not rejected at any conventional levels of significance in all treatments. For the challenger, the null hypothesis was rejected at the 0.1% significant level for treatments LL, HL, and LH.[7]

Figure 1 displays side by side the relative frequency distribution of expenditures and the probability distribution specified by the unique mixed-strategy Nash equilibrium by treatment and role. There are eight panels in the figure; for example, the top-left panel shows the relative frequency distribution of expenditures for 32 incumbent-role participants and the equilibrium probabilities in treatment LL.

Table 2: Summary of general statistics regarding expenditures

Treatment	Incumbent				Challenger			
	Mean	Median	Mode	Std. Dev.	Mean	Median	Mode	Std. Dev.
LL	6.2188	7	8	1.8445	4.7188	5	8	2.9538
HL	8.125	8	13	4.0301	3.75	3	0	3.2528
LH	6.5	8	8	1.9177	3.9688	4	0	2.9998
HH	9.1875	10	13	4.2155	5.75	4.5	0	4.7247

7 R package “exactRankTests” (version 0.8.32) was used.

There are two notable findings from the figure. First, not all participants followed equilibrium play. In equilibrium, no participant should choose 0 as the incumbent and 1 as the challenger. However, one incumbent-role participant chose 0 whereas 11 challenger-role participants chose 1. Second, the number of challenger-role participants choosing 0 is significantly small. In other words, too many challenger-role participants "entered" the auction. The equilibrium probabilities of choosing 0 are 0.5333, 0.65, and 0.4 for treatments LL, LH, and HH, respectively. The corresponding observed relative frequencies are 5, 6, and 7. Then, exact p-values for the one-sided binomial test are computed as follows :

$$P(X_{\mathrm{LL}} \leqq 5) = \sum_{n=0}^{5} \binom{32}{n} (0.5333)^{n} (1-0.5333)^{32-n} \approx 1.1847 \times 10^{-5}$$

$$P(X_{\mathrm{LH}} \leqq 6) = \sum_{n=0}^{6} \binom{32}{n} (0.65)^{n} (1-0.65)^{32-n} \approx 1.0823 \times 10^{-7}$$

$$P(X_{\mathrm{HH}} \leqq 7) = \sum_{n=0}^{7} \binom{32}{n} (0.4)^{n} (1-0.4)^{32-n} \approx 2.4822 \times 10^{-2}$$

where X_t is the number of challenger-role participants choosing 0 in treatment t. A one-sided binomial test rejected the null hypothesis of equilibrium play at any conventional levels of significance for treat-

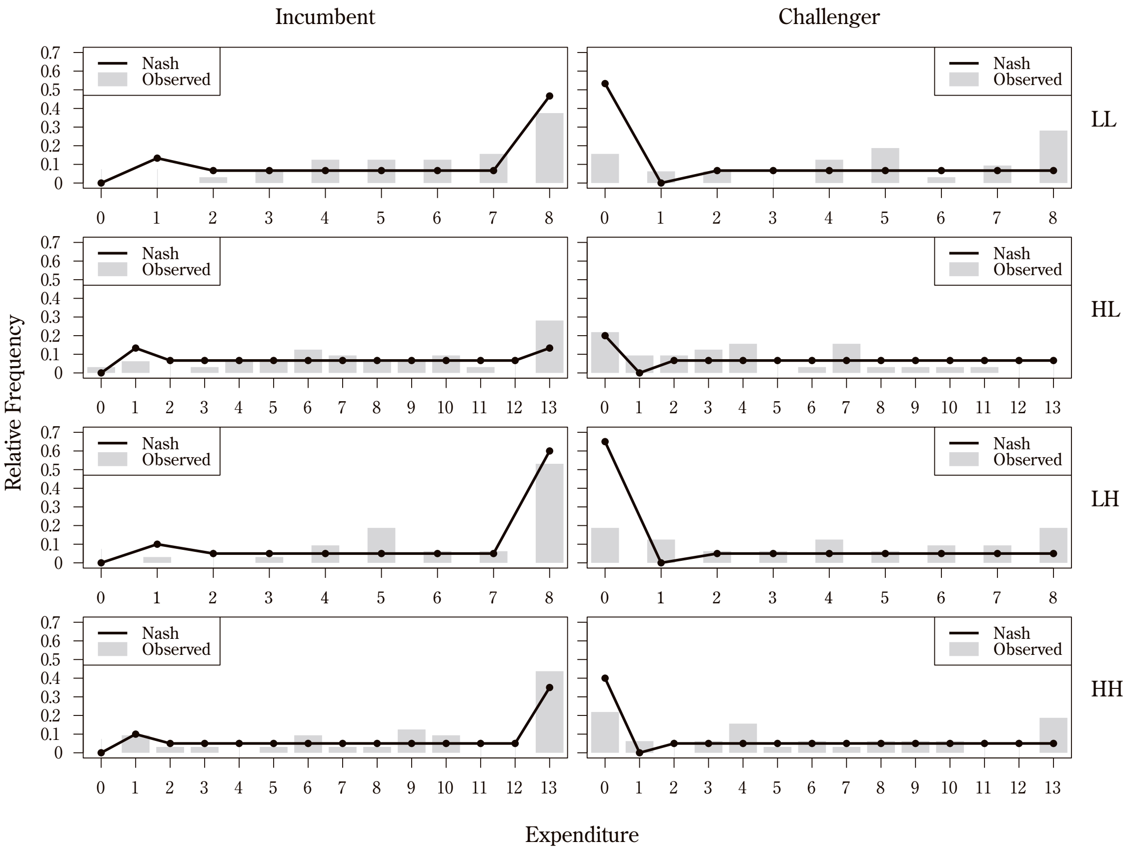

Figure 1 : Nash equilibrium and the relative frequency distribution of expenditure

ments LL and LH and at the 5% significance level for treatment HH.

3 Two Behavioral Models of Games

The Nash predictions account well for the initial behavior of incumbent-role participants, but not for that of challenger-role participants. This section introduces two behavioral models of games that have been used to explain initial responses to novel strategic situations.

3.1 Quantal Response Equilibrium

The first model is a quantal response equilibrium (QRE) model (McKelvey & Palfrey, 1995). In a sharp contrast to the Nash equilibrium, the QRE does not require that players select the best choice with certainty. Instead, players are more likely to select better choices and less likely to choose worse choices, according to a quantal response function that maps expected payoffs into choice probabilities. A player's expected payoffs from different choices are determined by beliefs about the other players' choices, and beliefs must match choice probabilities in equilibrium.

Let $(p_{i,\lambda}, p_{c,\lambda})$ be a strategy profile (i.e., a set of probability distributions), where for all $j \in \{i, c\}$

$$p_{j,\lambda} = (p_{j,\lambda}(0), p_{j,\lambda}(1), \ldots, p_{j,\lambda}(l)).$$

The probability distribution $p_{-j,\lambda}$ represents candidate j's beliefs about the other player $-j$'s expenditure levels. Given this probability distribution, candidate j computes her expected payoff from each of $l+1$ expenditure levels. For example, candidate j's expected payoff from choosing e_j is given by

$$u_j(e_j, p_{-j,\lambda}) = \sum_{e_{-j}\in E} p_{-j,\lambda}(e_{-j})u_j(e_j, e_{-j}).$$

Then, candidate j's probability of choosing e_j is determined by a logistic quantal response function:

$$p_{j,\lambda}(e_j) = \frac{\exp(\lambda \cdot u_j(e_j, p_{-j,\lambda}))}{\sum_{e\in E} \exp(\lambda \cdot u_j(e, p_{-j,\lambda}))},$$

where λ is an error parameter that ranges from 0 to ∞. Since there are two players, each of who has $l+1$ strategies, there are $2(l+1)$ logistic quantal response functions. Then, for a given λ, a QRE is defined as a strategy profile $(p^*_{i,\lambda}, p^*_{c,\lambda})$ such that for all $j \in \{i, c\}$

$$p^*_{j,\lambda}(e_j) = \frac{\exp(\lambda \cdot u_j(e_j, p^*_{-j,\lambda}))}{\sum_{e\in E} \exp(\lambda \cdot u_j(e, p^*_{-j,\lambda}))} \text{ for all } e_j \in E.$$

Two comments on λ are in order. First, the QRE is a function of λ; as the value of λ varies, QRE probabilities also vary. Second, λ describes the degree of rationality. As λ goes to 0, $p^*_{j,\lambda}(e_j)$ converges to $\frac{1}{l+1}$ (i.e., uniform randomization). On the other hand, as λ approaches ∞, $p^*_{j,\lambda}(e_j)$ converges to $p^*_j(e_j)$ (i.e., Nash equilibrium probabilities). Table 3 presents QRE and Nash equilibrium probabilities in treatment LL. The QRE is numerically computed for each of five different values of λ; $\lambda \in \{0, 0.1, 0.5, 2, 10\}$. When $\lambda = 0$, the QRE is just a discrete uniform distribution; both roles choose each strategy with equal probability. As λ gets larger (e.g., $\lambda = 10$), the QRE is almost identical with the Nash equilibrium of the game.

Table 3 : QRE and Nash equilibrium probabilities in treatment LL

Role	Expenditure	QRE (λ)					Nash
		0	0.1	0.5	2	10	
Incumbent	0	0.1111	0.0765	0.0127	0.0000	0.0000	0.0000
	1	0.1111	0.0914	0.0495	0.0716	0.1192	0.1333
	2	0.1111	0.0956	0.0577	0.0675	0.0671	0.0667
	3	0.1111	0.1006	0.0645	0.0668	0.0667	0.0667
	4	0.1111	0.1067	0.0715	0.0668	0.0667	0.0667
	5	0.1111	0.1143	0.0809	0.0671	0.0667	0.0667
	6	0.1111	0.1240	0.0983	0.0693	0.0667	0.0667
	7	0.1111	0.1368	0.1439	0.0848	0.0678	0.0667
	8	0.1111	0.1541	0.4208	0.5061	0.4792	0.4667
Challenger	0	0.1111	0.0917	0.1487	0.4123	0.5240	0.5333
	1	0.1111	0.0930	0.0992	0.0558	0.0000	0.0000
	2	0.1111	0.0965	0.0872	0.0647	0.0628	0.0667
	3	0.1111	0.1008	0.0816	0.0663	0.0666	0.0667
	4	0.1111	0.1061	0.0803	0.0666	0.0667	0.0667
	5	0.1111	0.1126	0.0832	0.0668	0.0667	0.0667
	6	0.1111	0.1210	0.0926	0.0677	0.0667	0.0667
	7	0.1111	0.1318	0.1175	0.0734	0.0668	0.0667
	8	0.1111	0.1465	0.2097	0.1262	0.0797	0.0667

3.2 Cognitive Hierarchy Model

The second model is a non-equilibrium model called a cognitive hierarchy (CH) model (Camerer et al., 2004). The QRE requires that players form correct beliefs of what other players do and stochastically best respond to these beliefs. In a sharp contrast to the QRE, the CH model assumes that players form *incorrect* beliefs about other players' actions and best respond to these beliefs.[8]

The CH model assumes that players are using h steps of reasoning with a frequency distribution of steps $f(h)$. Following Camerer et al. (2004), the distribution $f_\tau(h)$ is assumed to be Poisson with mean τ. Thus, the frequency of players doing h steps of reasoning is given by

$$f_\tau(h) = \frac{e^{-\tau}\tau^h}{h!} \quad h = 0, 1, 2, \ldots.$$

The CH model also assumes that k-step players have an accurate belief about the relative frequencies of players who are doing *less* steps of reasoning than they are. For example, 2-step players believe that other players are doing either 0 step or 1 step of reasoning.[9] Thus, k-step players believe that the

8 To be precise, players doing 0 step of reasoning do not best respond; instead, they uniformly randomize over the entire strategy space.

relative frequency of h-step players follows an upper truncated Poisson distribution given by

$$g_{k,\tau}(h)=\begin{cases}\dfrac{f_\tau(h)}{\sum_{q=0}^{k-1}f_\tau(q)}, & h\leqq k-1\\ 0, & h\geqq k.\end{cases}$$

Suppose that player j is doing k steps of reasoning. Let $P_j^k(e_j)$ be the probability that player j doing k steps of reasoning chooses pure strategy e_j. Then, given the k-step player's belief about how many steps of reasoning other players are doing, $g_{k,\tau}(0)$, $g_{k,\tau}(1)$, . . . , player j's expected payoff from choosing e_j is given by

$$\sum_{e_{-j}\in E}u_j(e_j,e_{-j})\left[\sum_{h=0}^{k-1}g_{k,\tau}(h)P_{-j}^h(e_{-j})\right]$$

The CH model assumes the following hierarchical process of reasoning:

- Suppose that player j is doing 1 step of reasoning. Player j believes that other players $-j$ are doing 0 step of reasoning. Following Camerer et al. (2004), it is assumed that the 0-step player chooses each strategy with equal probability, i.e., $P_{-j}^0(e_{-j})=\dfrac{1}{l+1}$ for all $e_{-j}\in E$. Given $g_{1,\tau}(0)$, the 1-step player can find the best response to the other players (0-step players). That is, $P_j^1(e_j^*)=1$ if and only if

$$e_j^*=\arg\max_{e_j\in E}\sum_{e_{-j}\in E}u_j(e_j,e_{-j})\left[g_{1,\tau}(0)P_{-j}^0(e_{-j})\right]$$

  or randomizing equally over strategies that yield the highest expected payoff.
- Suppose that player j is doing 2 steps of reasoning. Player j believes that other players $-j$ are doing either 0 step or 1 step of reasoning. Given $g_{2,\tau}(0)$ and $g_{2,\tau}(1)$, player j can find a best response. That is, $P_j^2(e_j^*)=1$ if and only if

$$e_j^*=\arg\max_{e_j\in E}\sum_{e_{-j}\in E}u_j(e_j,e_{-j})\left[\sum_{h=0}^{1}g_{2,\tau}(h)P_{-j}^h(e_{-j})\right]$$

  or randomizing equally over strategies that yield the highest expected payoff.
- Recursively applying this process derives the best responses of players doing different steps of reasoning. Since it is implausible to assume that players can exercise infinite steps of reasoning, this paper limits the maximum step of reasoning to $k=5$.[10] Then, the frequency of players doing h steps of reasoning is given by:

$$f_\tau(h)=\begin{cases}\dfrac{e^{-\tau}\tau^h}{h!} & h=0,1,2,3,4\\ 1-\sum_{v=0}^{h-1}f_\tau(v) & h=5\end{cases}$$

To get a better picture of how this iterative process works, the following demonstrates how to de-

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9 This example highlights a major difference between the CH and level-k models. The Level-k model assumes that players doing $k\geqq 1$ steps of reasoning naively believe that others are doing exactly one step below. For example, 1-step players believe that other players are doing 0 step of reasoning, 2-step players believe that other players are doing 1 step of reasoning, and so on.

10 Camerer et al. (2004) reported that assuming a τ value of 1.5 could give reliable predictions for many games. If $\tau=1.5$, the frequency of players doing 5 steps or more is negligibly small; $f_\tau(5)\approx 0.014$ and $f_\tau(h)$ is negligibly small for $h>5$. $k=5$ is reasonably high.

rive best responses in treatment LL under the assumption of $\tau = 3$.

Step-1 Players : Consider a $k = 1$ step incumbent. Because $g_{1,3}(0) = 1$, the incumbent believes that the challenger is a 0-step player, that is, the challenger chooses each expenditure with equal probability, $P_c^0(e_c) = \frac{1}{9}$ for all $e_c \in E$. Given this belief, the 1-step incumbent can compute the expected payoff from choosing e_i, which is given by

$$\sum_{e_c \in E} u_i(e_i, e_c)\left[\underbrace{1}_{g_{1,3}(0)} \times \underbrace{\frac{1}{9}}_{P_c^0(e_c)}\right].$$

Table 4 shows expected payoffs for the CH model with $\tau = 3$ in treatment LL. The nine values from the third row to the 11th row in the third column of the table represent the 1-step incumbent's expected payoffs from nine different expenditure levels, respectively. The value in bold, 7, is the largest expected payoff of the 1-step incumbent, and the corresponding expenditure $e_i^* = 8$ is the best response (i.e., $P_i^1(8) = 1$). Similarly, a $k = 1$ step challenger believes that the incumbent is a 0-step player, that is, the incumbent chooses each expenditure with equal probability. It is straightforward to compute the challenger's expected payoffs, which are presented as the nine values from the 12th row to the 20th row in the third column of the table. Choosing $e_c^* = 8$ yields the highest expected payoff to the 1-step challenger.

Table 4 : Expected payoffs for the CH model with $\tau = 3$ in treatment LL

Role	Expenditure	Number of Steps of Reasoning (k)				
		1	2	3	4	5
Incumbent	0	0	0	0	0	0
	1	2.3333	−0.1667	**7.3333**	**9.6410**	7.4478
	2	3	−0.75	6.5294	8.7692	**9.6412**
	3	3.6667	−1.3333	5.7255	7.8974	8.7430
	4	4.3333	−1.9167	4.9216	7.0256	7.8448
	5	5	−2.5	4.1176	6.1538	6.9466
	6	5.6667	−3.0833	3.3137	5.2821	6.0483
	7	6.3333	−3.6667	2.5098	4.4103	5.1501
	8	**7**	**7**	7	7	7
Challenger	0	0	**0**	**0**	0	0
	1	0.6667	−0.5833	−0.8039	−0.8718	−0.8982
	2	1.3333	−1.1667	−1.6078	**3.4487**	**5.4173**
	3	2	−1.75	−2.4118	2.5769	4.5191
	4	2.6667	−2.3333	−3.2157	1.7051	3.6209
	5	3.3333	−2.9167	−4.0196	0.8333	2.7226
	6	4	−3.5	−4.8235	−0.0385	1.8244
	7	4.6667	−4.0833	−5.6275	−0.9103	0.9262
	8	**5.3333**	−4.6667	−6.4314	−1.7821	0.0280

Thus, the 1-step challenger's best response is to choose $e_c^* = 8$ for sure (i.e., $P_c^1(8) = 1$).

Step-2 Players : Because

$$g_{2,3}(0) = \frac{f_3(0)}{f_3(0) + f_3(1)} = 0.25$$

$$g_{2,3}(1) = \frac{f_3(1)}{f_3(0) + f_3(1)} = 0.75$$

a step-2 incumbent believes that the challenger is a 0-step player with probability $g_{2,3}(0) = 0.25$ and a 1-step player with probability $g_{2,3}(1) = 0.75$. That is, the incumbent believes that $P_c^0(e_c) = \frac{1}{9}$ for all $e_c \in E$ with probability 0.25 and $P_c^1(8) = 1$ with probability 0.75. Given this belief, the 1-step incumbent can compute the expected payoff from choosing e_i, which is given by

$$\sum_{e_c \in E} u_i(e_i, e_c) \left[\sum_{h=0}^{1} g_{2,3}(h) P_c^h(e_c) \right].$$

The nine values from the third row to the 11th row in the fourth column of Table 4 represent the 2-step incumbent's expected payoffs from nine different expenditure levels, respectively. The results show that the 2-step incumbent's best response is to choose $e_i^* = 8$ for sure (i.e., $P_i^2(8) = 1$). Similarly, a 2-step challenger believes that the incumbent is a 0-step player with probability $g_{2,3}(0) = 0.25$ and a 1-step player with probability $g_{2,3}(1) = 0.75$. Given this belief, the 1-step challenger can compute the expected payoff from choosing e_c, which is given by

$$\sum_{e_i \in E} u_c(e_i, e_c) \left[\sum_{h=0}^{1} g_{2,3}(h) P_i^h(e_i) \right].$$

The nine values from the 12th row to the 20th row in the fourth column of the table represent the 2-step challenger's expected payoffs from nine different expenditure levels, respectively. The results indicate that the 2-step challenger should choose $e_c^* = 0$ for sure (i.e., $P_c^2(0) = 1$).

This process continues until finding the best responses of step-5 players. Table 5 summarizes the best responses of players with five different reasoning levels when $\tau = 3$. Notice that best responses depend largely on the value of τ. If the value of τ is relatively large as in the current example, then there would be a variation in best responses for players doing more steps of reasoning. On the other hand, if the value of τ is very small (e.g., $\tau = 0.1$), then the population of players turns out to be quite homogeneous regarding the depth of reasoning ; the frequency of players doing 0 step of reasoning is quite

Table 5 : Best responses for the CH model with $\tau = 3$ in treatment LL

Number of Steps of Reasoning (k)	Incumbent	Challenger
1	8	8
2	8	0
3	1	0
4	1	2
5	2	2

high. As a result, there is little variation in best responses for players doing higher steps of reasoning.

4 Estimation and Model Comparison

The model parameters λ and τ were estimated using maximum likelihood techniques.[11] Table 6 shows log-likelihoods and maximum likelihood estimates of λ and τ separately for each treatment. 90% confidence intervals for these estimates were constructed by bootstrapping (Efron & Tibshirani, 1993).[12] The log-likelihoods for the unique mixed-strategy Nash equilibrium were also computed for comparison with the QRE and the CH model.[13] The log-likelihoods indicate that the QRE outperformed the CH and the Nash equilibrium in all treatments.

Table 7 presents observed and expected mean expenditures by treatment and role.[14] Two discernible features are found in this table. First, the expected expenditures under the CH model do not differ between the two roles in treatments LL and LH. This happens because the estimated values of τ are so small that both roles doing k ($k > 0$) steps of reasoning should choose the same expenditure level as their best responses, namely $e_i = e_c = 8$ for all $k > 0$. Second, the QRE minimizes the difference between observed and expected mean expenditures in almost all cases. There are two exceptional cases; the difference is minimized under the CH model for the challenger in treatment LL and under the Nash equilibrium for the challenger in treatment HL.

Table 6 : Log-likelihoods and maximum likelihood estimates of λ and τ

Treatment	Nash	QRE		CH	
	Log-Likelihood	Log-Likelihood	λ [90% C.I.]	Log-Likelihood	τ [90% C.I.]
LL	−156.9644	−125.1891	0.4856 [0.3111, 0.7565]	−129.9179	0.2799 [0.1493, 0.6667]
HL	−194.5664	−164.7086	1.3460 [0, 2.4708]	−166.6543	0.1100 [0.0072, 0.3576]
LH	−167.0454	−119.5485	0.4560 [0.2753, 0.8058]	−127.0528	0.3275 [0.1910, 1.2222]
HH	−165.5561	−146.7512	1.6697 [0.7923, 3.6998]	−148.1882	0.4881 [0.2781, 0.7316]

11 This paper implicitly assumes that all participants assigned to the same treatment have an identical parameter value, regardless of which role they play in the experiment.

12 For each treatment, 10000 bootstrap samples were generated from the first-round data.

13 Each player has one non-equilibrium strategy; $e_i = 0$ for the incumbent and $e_c = 1$ for the challenger. The presence of non-equilibrium strategies makes the log-likelihood undefined. In order to avoid this issue, a noisy Nash model (NNM), first introduced by McKelvey and Palfrey (1998), was employed as a substitute for Nash equilibrium. The NNM assumes that players use the Nash equilibrium with probability γ and a uniform randomization over all pure strategies with probability $1-\gamma$. As $\gamma \to 1$, the NNM converges to the Nash equilibrium. The log-likelihoods for the Nash equilibrium in Table 3 are in fact those for the NNM with $\gamma = 0.9999$.

14 The expected mean expenditures are computed using the estimated values of λ and τ presented in Table 3.

Table 7：Observed and expected mean expenditures

Treatment	Incumbent				Challenger			
	Mean	Nash	QRE	CH	Mean	Nash	QRE	CH
LL	6.2188	5.6667	5.9893	4.9766	4.7188	2.3333	4.3412	4.9766
HL	8.125	7	7.3719	7.1770	3.75	6	6.5150	7.1039
LH	6.5	6.25	6.7455	5.1172	3.9688	1.75	4.0789	5.1172
HH	9.1875	8.5	9.0980	9.0104	5.75	4.5	5.3349	7.8843

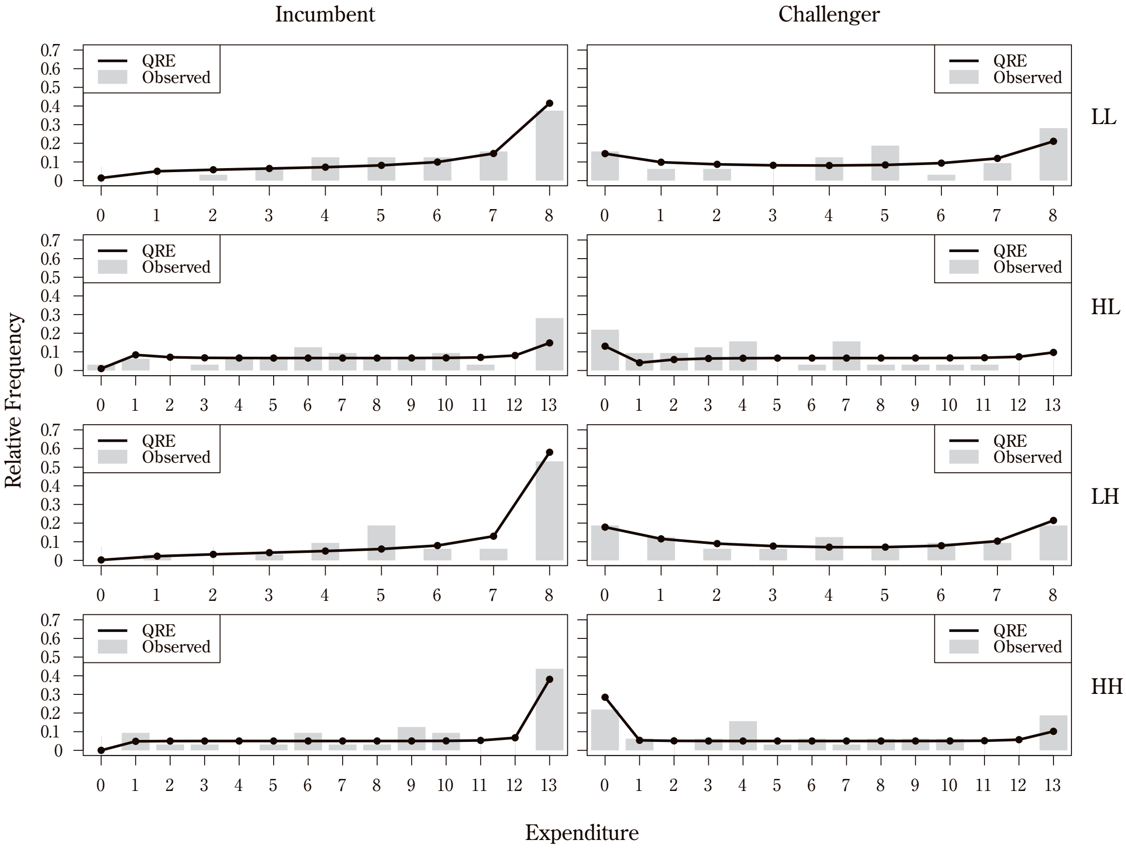

Figure 2：QRE and the relative frequency distribution of expenditure

Figure 2 presents side by side the relative frequency distribution of expenditures and the QRE probabilities by treatment and role. A comparison with Figure 1 reveals that overall the resulting QRE probabilities capture deviations from the Nash equilibrium very well. This observation hints that replacing deterministic best responses with stochastic ones would be one approach for future development of an alternative theory that better accounts for deviations from Nash equilibrium play, particularly in early rounds of games.

5 Conclusion

This paper used the first-round data of the all-pay auction experiment conducted by Otsubo (2013) to examine (i) if the Nash equilibrium of the game accounts for the first round behavior and (ii) how well alternative models of games capture deviations, if any, from Nash equilibrium play. Least surprisingly, the results show that the Nash equilibrium is not a good predictor for the first-round behavior. As alternative models, the QRE and the CH model were considered, and their parameters were estimated using maximum likelihood methods. Of these two models, the QRE does better in all treatments.[15]

One direction for future research is to consider the QRE model that allows for λ-heterogeneity. The QRE considered in this paper assumes the common λ for all players. This means that all players have the same payoff responsiveness. Yet, people obviously differ in many dimensions (e.g., skill, taste, risk attitude, level of rationality), which result in a wide variety of individual behavioral patterns that defy a simple classification. For a recent development of heterogeneity in the QRE framework, see Chapter 4 of Goeree at al. (2016).

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15 This paper is intended to examine how well the QRE and the CH model *explain* (i.e., fit) the in-sample initial play of the all-pay auction experiment and therefore is silent about how well these behavioral models *predict* initial play in games. Wright and Leyton-Brown (2017) have recently performed rigorous comparisons of behavioral models intended to predict initial play of unrepeated, simultaneous-move games.

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