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Simple Estimations of the Natural Rate of Interest and the Expected Inflation Rate

Kazuhiko NAKAHIRA
Professor, Department of
Economics, Meikai University

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INSTITUTE OF ECONOMIC RESEARCH
Chuo University
Tokyo, Japan

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Kazuhiko NAKAHIRA

Professor, Department of Economics, Meikai University

E-mail: nakahira@meikai.ac.jp

Abstract

The natural rate of interest is an important concept since it is a kind of reference for economists in that this is a significant determinant of a variety of policy rules used to characterize monetary policy. Generally, the natural rate of interest is defined as the real short-term interest rate consistent with the output equaling its potential level or the one reflecting stable prices. Estimation of the natural rate of interest is required since it cannot be observed directly. One of the simple ways of estimating the natural rate of interest is to extract the trend of the real short-term interest rate. If the central bank is inclined to raise the policy interest rate when it acknowledges an inflationary pressure or excess demand and if such a policy implementation is smoothly transmitted into adjustment processes of prices and economic activities, observed short-term real interest rate should behave closely around the evolutionary pass of the natural rate of interest. In this sense, statistically extracted trend of real short-term interest rate can be presumed as the estimated value of the natural rate of interest. We utilize the Hodrick-Prescott filter for our extraction. On the other hand, we should have the estimation of expected inflation rate in order to acquire real short-term interest rate if we follow the Fisher equation. In this paper, Kanoh (2006) procedure is adopted to estimate the expected inflation rate. Our estimation result brings us to the conclusion that the natural rate of interest in Japan is likely to show a substantial decline in recent 20 years.

Key Words: natural rate of interest; inflation expectations; Carlson-Parkin method; Hodrick-Prescott filter

JEL Classification Code: C53; E31; E32, E37

1. Introduction

The natural rate of interest is an important concept for economists since it is one of the references for monetary policy in that this rate is a significant determinant of a variety of policy rules used to characterize monetary policy. Generally, the natural rate of interest is defined as the real short-term interest rate consistent with the output equaling its potential level or the one reflecting stable prices. Estimation of the natural real interest rate is required since it cannot be observed directly.

Because of this property, positive previous studies with regard to its theoretical analysis and empirical estimation have been published. The most famous research of this kind is Laubach and Williams (2003). This study uses the IS curve, the Phillips curve, and the natural rate of interest equation as the key equations that describe the structural relationships of economy, and applies the Kalman filter technique in order to estimate some unobservable variables. Fries *et al.* (2016), Pescatori and Turunen (2016), Hakkio and Smith (2017), Holston *et al.* (2017), and Lewis and Vazquez-Grande (2017) are the other studies based on the Laubach and Williams approach. On the other hand, Lubik and Matthes (2015) adopted the Time-Varying Parameter Vector Autoregressions approach.

Another commonly used instrument for analyzing the natural rate of interest is the so-called DSGE (dynamic stochastic general equilibrium) model that clarifies structural interpretations of the system. Edge *et al.* (2008), Justiniano and Primiceri (2010), Barsky *et al.* (2014), Curdia *et al.* (2015), Del Negro *et al.* (2015, 2017), Goldby *et al.* (2015), Hristov (2016), Gerali and Neri (2017), Okazaki and Sudo (2018), and Iiboshi *et al.* (2022) are the studies based on DSGE methodology.

After the remark by Summers (2013), the issue whether the natural rate is substantially been declining (and will continue to decline) has been attracted attention. We found the researches that are related to this issue or the secular stagnation hypothesis, including Eggertsson *et al.* (2017), Rachel and Smith (2015), Sajedi and Thwaites (2016), and Summers (2014).

One statistical simple method of estimating the natural rate of interest is to extract the trend component of real short-term interest rate. If the central bank is inclined to raise the policy interest rate when it acknowledges an inflationary pressure or excess demand and if such a policy implementation is smoothly transmitted into adjustment processes of prices and economic activities, short-term real interest rate should behave closely around the evolutionary pass of the natural rate of interest. In this sense, statistically extracted trend of real short-term interest rate can be regarded as the estimated value of the natural rate of interest. We utilize the Hodrick-Prescott filter for our extraction. On the other hand, we should have the estimation of expected inflation rate in order to acquire real short-term interest rate if we follow the Fisher equation. In our research, Kanoh (2006) procedure, a variant of the Carlson and Parkin (1975) method, is adopted to estimate the expected inflation rate.

The contribution of our paper is as follows. We utilize the Kanoh (2006) procedure, hardly ever used in empirical research, for the empirical research when the expected inflation rate is estimated in order to acquire real short-term interest rate, and it uses the inference of the real rate of interest through the Hodrick-Prescott filter. Through its analysis, we found a substantial decline of Japan's natural rate of interest in recent years.

The rest of this paper is organized as follows. Section 2 presents the theoretical derivation of the natural rate of interest. Section 3 summarizes the way of estimation of expected inflation rate through the Carlson-Parkin (1975) and the Kanoh (2006) procedures. Section 4 sets out the empirical investigation of the natural rate of interest by utilizing the Hodrick-Prescott filter. Lastly, Section 5 is devoted to the concluding remarks.

2. Theoretical Derivation of Natural Rate of Interest[1]

The utility function is given by

$$\mathrm{U} = \int_0^\infty (u(c)e^{-\rho t}dt), u'(c) > 0, u''(c) < 0. \qquad (1)$$

U: utility (from present to infinite future)

u: utility at each period

c: real consumption at each period of representative agent

ρ: rete of time preference

Household's Budget Constraint is as follows.

$$\dot{k} = \omega + (r - n)k - c. \qquad (2)$$

k: capital stock per capita

n: population real growth rate

ω: wage rate

r: real interest rate

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[1] The explanation in section 2 is basically based on Murata (2022) and Iwata *et al.* (2018).

By the equations (1) and (2), we have the Hamiltonian:

$$\mathrm{H} = u(c)e^{-\rho t} + \lambda[\omega + (r-n)k - c]. \qquad (3)$$

λ: shadow price of income

F.O.C.

$$\lambda = u'(c)e^{-\rho t} \qquad (4)$$

$$\dot{\lambda} = -(r-n)\lambda \qquad (5)$$

Transversality condition is expressed by

$$\lim_{n\to\infty}[\lambda(t)k(t)] = 0. \qquad (6)$$

Taking the time derivative of (4) and plug it into (5), we get

$$r = -\left[\frac{u''(c)c}{u'(c)}\right]\left(\frac{\dot{c}}{c}\right) + n + \rho. \qquad (7)$$

If we assume constant substitution elasticity of consumption with respect to the utility function,

$$u(c) = \frac{c^{(1-\theta)}-1}{1-\theta} \qquad (8)$$

is realized, and equation (7) should be the following form:

$$r = \theta\left(\frac{\dot{c}}{c}\right) + n + \rho. \qquad (9)$$

Since per capita consumption growth rate should be equal to per capita capital stock growth rate when the economy is on the balanced growth path, and we have:

$$\left(\frac{\dot{c}}{c}\right) = \left(\frac{\dot{K}}{K}\right) - n. \qquad (10)$$

K: capital stock

Production function (homogeneous of degree one) is

$$Y = F(AL, K) = ALf(\tilde{k}). \qquad (11)$$

Y: GDP

A: rate of technological progress (or Total Factor Productivity)

L: labor Input

AL: effective labor

Capital per unit of effective labor and GDP per unit of effective labor are expressed as follows.

$$\tilde{k} = \frac{K}{AL} \qquad (12)$$

$$\tilde{y} = \frac{Y}{AL} \qquad (13)$$

Capital per unit of effective labor should be constant on the balanced growth path, and considering (10) in addition to (12), we have

$$\left(\frac{\dot{c}}{c}\right) = \left(\frac{\dot{A}}{A}\right). \qquad (14)$$

If we define the interest rate on the balanced growth path as the natural rate of interest, r^*, we have it by considering (10) and (14),

$$r^* = \theta\left(\frac{\dot{A}}{A}\right) + n \tag{15}$$

where θ is the coefficient of relative risk aversion.

Because GDP per unit of effective labor should be constant on the balanced growth path, g: potential growth rate (or GDP growth rate on the balanced growth path), becomes equal to the growth rate of the Total Factor Productivity and the population growth rate. Therefore, we have

$$g = \frac{\dot{A}}{A} + n. \tag{16}$$

Furthermore, if we consider a dynasty with population growth, equation (15) becomes

$$r^* = \theta\left(\frac{\dot{A}}{A}\right) + \rho. \tag{17}$$

In this case, the relation described below is realized as an approximation if there is no population growth:

$$r^* = g + \rho. \tag{18}$$

3. Inflation Expectations and its Estimations

3.1. Survey Data for the Estimation

The purpose of our study is to estimate the natural rate of interest. However, we should have the estimation of expected inflation rate in order to obtain the real short-term interest rate by following the Fisher equation. (In short, the Fisher equation describes that the real interest rate equals the nominal interest rate minus the expected inflation rate.)

To examine the formation process of inflation through the estimation of inflation expectations by using survey data enables us to implement our concrete research without any specific economic models. In General, there are two types of survey data on inflation expectations: qualitative and quantitative. In the qualitative surveys, respondents answer in a qualitative manner to a question such as, “Do you think that price level (or inflation) will have gone up (or down) one year from now?” The data on inflation expectations given by this kind of survey are presented in a qualitative statistic form indicating whether the majority of the polled respondents expect that price level in the future will rise, remain constant, or decline. In other words, the survey of this kind explores the general tendency of the expectation of the future price level or inflation. On the other hand, respondents answer to the question in a quantitative manner in the case of a quantitative survey. The problem is that it is hard to acquire an exact point forecast of the inflation expectation because quantitative surveys may have some defects. For instance, this kind of direct measure has an inclination to generate some measurement and sampling errors. From this point of view, it is worth making use of the qualitative surveys along with the special way of quantifying qualitative data.

3.2. The Carlson-Parkin Method for Quantifying Qualitative Survey Data and The Kanoh (2006) Methodology

A process of quantifying qualitative survey data is required to examine expectation of inflation as mentioned in the previous section. The data given by a qualitative survey, however, often brings us to some difficulties. For instance, the respondents only indicate whether price level (or inflation) will "rise", "fall" or "remain unchanged" for a certain period ahead in the survey, so the data do not include a mean value since they are qualitative. Several techniques, such as the Carlson-Parkin method, the balance method, and the regression method, have been proposed to deal with these problems. The Carlson and Parkin (1975) method[2] is a typical probability approach for the expected inflation estimation. It postulates that the qualitative answer in the survey follows an individual probability distribution that is statistically independent of other respondents' and normally distributed with finite mean and variance. Thus, the mean of the distribution is supposed to be reported by the respondent. The Carlson-Parkin methodology assumes that respondents at time t form an inflation expectation for time $t+1$ in their answers. The joint probability distribution $f(\mathrm{x}_{t+1}|\Omega_t)$ is derived by the aggregation of the individual subjective probability distributions, where Ω_t is the information set at time t and x_{t+1} is the percentage change in price level in the future for the period t to $t+1$. This distribution is presumed to have finite first- and second-order moments, and we can express the inflation expectation for the period $t+1$ as $E[\mathrm{x}_{t+1}|\Omega_t] = \pi_{t+1}^e$. Further, we regard that there exists an interval $(-\delta_t, \delta_t)$ around 0 ($\delta_t > 0$) in order that the answer "no change" will lies within this interval. With this threshold, δ_t, survey participants are supposed to report the expectation of price change in the following manner:

"prices up" if $\pi_{t+1}^e > \delta_t$, (19)

"prices down" if $\pi_{t+1}^e < -\delta_t$, (20)

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[2] The explanation of the Carlson and Parkin (1975) model given here is not exactly the same as the original procedure but the modified one. The modified version in this section is of wide applications, and introduced by some papers, for example, Henzel and Wollmershäuser (2006), Oral (2013), and Scheufele (2011).

$$\text{“no change” if } -\delta_t \leq \pi^e_{t+1} \leq \delta_t. \tag{21}$$

The answer in the survey can be interpreted as the result of the individual probability distribution over the possible future value of the variable in consideration and as a sampling from some aggregate distribution. Therefore, the percentage of the responses of “prices up” (denoted by UP_t) and “prices down” (denoted by $DOWN_t$,) can be transformed into the associated population values:

$$UP_t = 1 - \Phi\left(\frac{\delta_t - \mu_t}{\sigma_t}\right), \tag{22}$$

$$DOWN_t = \Phi\left(\frac{-\delta_t - \mu_t}{\sigma_t}\right), \tag{23}$$

where Φ is the cumulative distribution function of the standard normal distribution, and μ_t and σ_t are respectively the mean and standard deviation of the aggregate distribution of inflation expectation. By these two equations, we have

$$a_t = \Phi^{-1}(1 - UP_t) = \left(\frac{\delta_t - \mu_t}{\sigma_t}\right), \tag{24}$$

$$b_t = \Phi^{-1}(DOWN_t) = \left(\frac{-\delta_t - \mu_t}{\sigma_t}\right), \tag{25}$$

where Φ^{-1} is the inverse function of Φ. Further, μ_t and $\sigma_t$can be described as

$$\mu_t = -\delta_t\left(\frac{a_t + b_t}{a_t - b_t}\right), \tag{26}$$

$$\sigma_t = 2\delta_t\left(\frac{1}{a_t - b_t}\right), \tag{27}$$

if we have δ_t. One of the simple ways to have a plausible value of δ_t is to assume constant δ (i.e. $\delta_t = \delta$ for some δ) and

$$\sum_{t=1}^{T} \pi_t = \sum_{t=1}^{T} \mu_t, \tag{28}$$

where π_t is the observed inflation rate. With these assumptions, we have

$$\delta = -\frac{\sum_{t=1}^{T} \pi_t}{\sum_{t=1}^{T}\left(\frac{a_t + b_t}{a_t - b_t}\right)}. \tag{29}$$

Plugging this δ into (26) and (27) instead of δ_t, we obtain μ_t (expected inflation) and σ_t (standard deviation).

It is often insisted that the basic (or traditional) Carlson-Parkin method have some problems. For example, the thresholds might be asymmetric between the expectations of “prices up” and “prices down”. In this aspect, Kanoh (2006)[3] proposes the procedure that can realize two kinds of threshold by modifying the basic model: δ_1 for “prices up” and δ_2 for “prices down.” The modification by Kanoh (2006) is as follows.

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[3] Kanoh (2006) brings up several procedures for the inference of inflation expectations. The procedure applied in this paper is one of them.

The respondents are supposed to indicate an expectation of change in prices as:

"prices up" if $\pi_{t+1}^{e} > \delta_1$, (30)

"prices down" if $\pi_{t+1}^{e} < \delta_2$, (31)

"no change" if $\delta_2 \leq \pi_{t+1}^{e} \leq \delta_1$. (32)

For the inferences of the mean and the variance of the expectations, the additional assumption

$$\sum_{t=1}^{T} \sigma_t^2 = \sum_{t=1}^{T} (\pi_t - \bar{\pi})^2, \tag{33}$$

where $\bar{\pi}$ is the observed average inflation rate, is required. Then, equations (26) and (27) become

$$\mu_t = \left(\frac{a_t\delta_2 - b_t\delta_1}{a_t - b_t}\right), \tag{34}$$

$$\sigma_t = \left(\frac{\delta_1 - \delta_2}{a_t - b_t}\right), \tag{35}$$

if we assume $\delta_{1t} = \delta_1$ and $\delta_{2t} = \delta_2$ for constants δ_1 and δ_2. After some manipulations, we obtain[4]

$$\delta_1 = \frac{1}{T}\left(\sum_{t=1}^{T} \pi_t + \sum_{t=1}^{T} \frac{a_t}{a_t - b_t} \sqrt{\frac{\sum_{t=1}^{T} (\pi_t - \bar{\pi})^2}{\sum_{t=1}^{T} \left(\frac{1}{a_t - b_t}\right)^2}} \right), \tag{36}$$

$$\delta_2 = \frac{1}{T}\left(\sum_{t=1}^{T} \pi_t + \sum_{t=1}^{T} \frac{b_t}{a_t - b_t} \sqrt{\frac{\sum_{t=1}^{T} (\pi_t - \bar{\pi})^2}{\sum_{t=1}^{T} \left(\frac{1}{a_t - b_t}\right)^2}} \right). \tag{37}$$

By the substation of δ_1 and δ_2 described in (36) and (37) into (34) and (35), we have μ_{t} (expected inflation) and σ_t (standard deviation).

Kanoh (2006) procedure is one of the variants of the Carlson-Parkin methodology, but it has hardly been used for the empirical research with a few exceptions.

3.3. Consumer Confidence Survey

The consumer confidence survey in Japan (conducted by the Economic and Social Research Institute of the Cabinet Office)[5] asks the respondents to assess the general situation and expectation about the economy[6]. It is one of the applicable data sources for empirical research with the Carlson-Parkin-type

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[4] Kanoh (2006) narratively shows the steps of deriving δ_1 and δ_2 without any concrete calculation processes. However, with some manipulations applying assumptions and conditions given in his paper, we have equations (36) and (37).

[5] See "http://www.esri.cao.go.jp/en/stat/shouhi/shouhi_kaisetsu-e.html" for details.

[6] The noticeable points of the consumer confidence survey are as follows. (a) The survey of "price expectations a year ahead" is conducted across three categories: "all households", "all households except one-person households", and "one-person households." (b) From May 2004 to February 2007, the survey was conducted by telephone in months other than June, September, December, and March; the survey was conducted by direct-visit and self-completion

Table 1: Example of the Survey Result of "Price Expectations a Year Ahead"

(Unit:%)

		Go down			Stay the same	Go up			Don't know
		greater than or equal to -5%	less than -5% to greater than or equal to -2%	less than -2%	about 0%	less than 2%	greater than or equal to 2% to less than 5%	greater than or equal to 5%	
2012	Jul	1.7	2.6	4.4	19.2	19.1	30.0	16.2	6.9
	Aug	1.3	2.1	5.6	21.5	22.1	26.6	14.0	6.8
	Sep	1.1	1.9	5.0	18.5	23.2	29.2	14.9	6.2
	Oct	0.8	2.5	4.4	17.0	25.4	31.0	13.6	5.3
	Nov	0.7	1.9	5.6	20.4	25.0	27.7	13.3	5.3
	Dec	0.7	2.4	6.3	20.8	24.1	26.2	13.6	5.8

(Source: http://www.esri.cao.go.jp/en/stat/shouhi/shiken_summary_e.html)

approach. Monthly data of the survey are available from April 2004 onward. Concretely, the qualitative data obtained from the section "price expectations a year ahead" in the survey is applied to our estimation of inflation expectations since the respondents give their expectations of future price level as "go down," "stay the same," "go up," or "don't know". The example of the survey result is displayed in Table 1.

3.4 Estimation of Consumer's Inflation Expectations

Monthly data of consumer price index, seasonally non-adjusted[7], change from the previous year, (excluding fresh food, whole Japan, total) spanning the period from April 2004 to September 2022[8] is applied as the rate of inflation (π_t) to the derivation of the expected inflation rate (π_t^e) by the Kanoh (2006) procedure.

The qualitative data obtained from the consumer confidence survey for "all households" are used for our estimation. The ratios of the three items in "go down" and "go up" are combined into the totals of "go down" and "go up", respectively, for our estimation. Further, the "don't know" answers are eliminated

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questionnaires in June, September, December, and March. (c) Since April 2013, the survey has been conducted by mail. In addition, the number of sample households has been increased. Therefore, we are obliged to use discontinuous survey data in our sample period.

[7] The reason of adopting seasonally non-adjusted series (instead of seasonally adjusted series) of consumer price index (change from the previous year) is as follows. We also have to use the data given by the consumer confidence survey which are not seasonally adjusted. In order to have a consistency of the format between the two kinds of data series, seasonally non-adjusted series of consumer price index is applied.

[8] The data on "consumer price index" were retrieved from the "Portal Site" of Official Statistics of Japan operated by the Ministry of Internal Affairs and Communications, Statistics Bureau, Director-General for Policy Planning (Statistical Standards) & Statistical Research and Training Institute (in English) "http://www.e-stat.go.jp/SG1/estat/eStatTopPortalE.do".

Figure 1: Estimated Expected Inflation Rate based on Kanoh (2006) Methodology by utilizing the Japan's "consumer confidence survey" (unit: %)

from the original data set. In addition, the total of the ratios for each month often exceeds 100% due to round-off errors. To deal with this problem, the ratios are re-adjusted by proportional distribution based on our revised total sum of the ratios.

Figure 1 reports the estimated expected inflation rates by the Kanoh (2006) procedure with $\widehat{\delta_1} = -0.5025$ and $\widehat{\delta_2} = -1.2511$. The estimated $\widehat{\delta_1}$ might not be favorable in the sense that it is inconsistent with the ideal combination of the signs ($\widehat{\delta_1} > 0, \widehat{\delta_2} < 0$). However, our estimated expected inflation rate can be applied to our estimation since at least one of the two thresholds satisfies the requirement. The estimated expected inflation rate is fluctuated through our sample period from the lowest estimated value $-0.916867(\%)$ for December 2009 to the highest $3.852244(\%)$ for July 2022. The estimated values are used to the inference of the natural rate of interest in Japan.

4. The Natural Rate of Interest and its Estimation

4.1. Basics of the Hodrick-Prescott Filter

To understand behavior of time series data, we often try to decompose the series into a trend and a cyclical component. In this context, the "trend" means the long-term growth of the series and the "cycle" is the deviation of the series from that trend. The Hodrick-Prescott Filter is one of the widely used econometric tools to remove the cyclical component of a time series from raw data. In other words, this filter is a kind of method to acquire a smooth estimate of the long-term trend component of a series.

Basic idea of this filter depends on the decomposition of time series. Specifically, this is usually a two-sided linear filter[9] for calculating the smoothed series $\boldsymbol{s}$ and $\boldsymbol{y}$ through minimizing the variance of $\boldsymbol{y}$ around $\boldsymbol{s}$, subject to the penalty parameter that restricts the second difference of $\boldsymbol{s}$. In short, the Hodrick-Prescott Filter sets *s* in order to minimize the value of the equation:

$Min_{\{s_t\}}(\sum_{t=1}^{T}(y_t - s_t)^2 + \lambda \sum_{t=2}^{T-1}[(s_{t+1} - s_t) - (s_t - s_{t-1})]^2)$,

where $\boldsymbol{y_t}$ for $\mathrm{t} = 1,2,\ldots,\mathrm{T}$ means the logarithms of given time series variable, $\boldsymbol{s_t}$ is a trend component and λ means a penalty parameter (or smoothing parameter). The series $\boldsymbol{y_t}$ consists of $\boldsymbol{s_t}$, $\boldsymbol{c_t}$, and $\boldsymbol{\epsilon_t}$. In short, $\boldsymbol{y_t} = \boldsymbol{s_t} + \boldsymbol{c_t}$ where $\boldsymbol{c_t}$ is a cyclical component. The first term in the parenthesis describes the summation of the squared deviations of (the logarithms of) a given time series variable and a trend component. The second term displays the product of a smoothing parameter and the summation of the squares of the second differences of the trend component. In other aspects, the second term penalizes variations in the rate of growth of the trend components, that is, the larger the level of λ, the higher is the penalty. With respect to the value of this penalty parameter (or smoothing parameter), Hodrick and Prescott (1997) suggest $\lambda = 1600$ in the case of quarterly data series. On the other hand, Ravn and Uhlig (2002) insist that λ should be varied by the fourth power of the frequency observation ratio. Concretely, for annual data, λ should be $1600/4^4 = 6.25$, for example. Actually, $\lambda = 100$ for yearly data and $\lambda = 14400$ in the case of monthly data are commonly adopted.

It is often suggested that the estimated trend by utilizing the Hodrick-Prescott Filter is relatively more sensitive than the ones by other filters when it faced with transitory shocks and short-term fluctuations at the end of the sample period of estimation. Baxter and King (1999) suggest that this property derives suboptimal feature of the Hodrick-Prescott Filter. Considering this problem, it is practical to eliminate the estimated values by the Hodrick-Prescott Filter for the first two years and the last two years (in the case of yearly data) in the research.

The Hodrick-Prescott Filter is often explained from the aspect of frequency domain. With regard to this standpoint, the penalty parameter (or smoothing parameter), λ, can be associated with the cut-off frequency of the filter, in short, the frequency at which it dampens the impact of the cyclical component

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[9] A one-sided version is explained in some previous studies, but this paper is concentrated on a two-sided one.

of the original series. Concerning this property, Nilsson and Gyomai (2011) proposes the defect of the Hodrick-Prescott Filter. On the other hand, Hamilton (2018) points out the drawback of the Hodrick-Prescott Filter in that it generates series with spurious dynamic relations without the basis in the underlying data-generation process. However, the Hodrick-Prescott Filter is broadly adopted because of its usability.

4.2. Estimation of the Natural Rate of Interest

Since the natural interest rate is not directly observable, its estimation is required. One of the statistical ways of estimating the natural rate of interest is to extract the trend component from estimated short-term real interest rate. If the central bank is inclined to raise the policy interest rate when it acknowledges an inflationary pressure or excess demand and if such a policy implementation is smoothly transmitted into adjustment processes of prices and economic activities, observed short-term real interest rate should behave closely around the evolutionary pass of the natural rate of interest. In this sense, the value that is obtained by the extraction of the trend component from real short-term interest rate can be regarded as the estimated natural rate of interest. For the estimation of expected inflation rate in order to construct the real short-term interest rate, the Kanoh (2006) procedure is adopted as explained in the previous section. Concretely, we adopted the following formula by following the Fisher equation.

estimated real short-term interest rate = short-term nominal interest rate – expected inflation rate
= observed uncollateralized overnight call rate[10] – expected inflation rate estimated by Kanoh (2006) procedure

We utilize the Hodrick-Prescott filter for the extraction of the trend component from the estimated real short-term interest rate by setting the penalty parameter (smoothing parameter), λ, as 14400 in accordance with the so-called “frequency power rule” of Ravn and Uhlig (2002). Estimation is conducted by using monthly data spanning the period April 2004 to September 2022 to make consistent with the estimation of expected inflation.

Figure 2 displays the estimated natural rate of interest. By looking at this figure, we figure out the fact that the estimated natural rate of interest shows a substantial decline in recent 20 years although it is fluctuated through our sample period. In particular, it takes negative value from May 2012, and estimated about $-2.3295(\%)$ in September 2022. If this is true, the Japanese central bank is faced with the great

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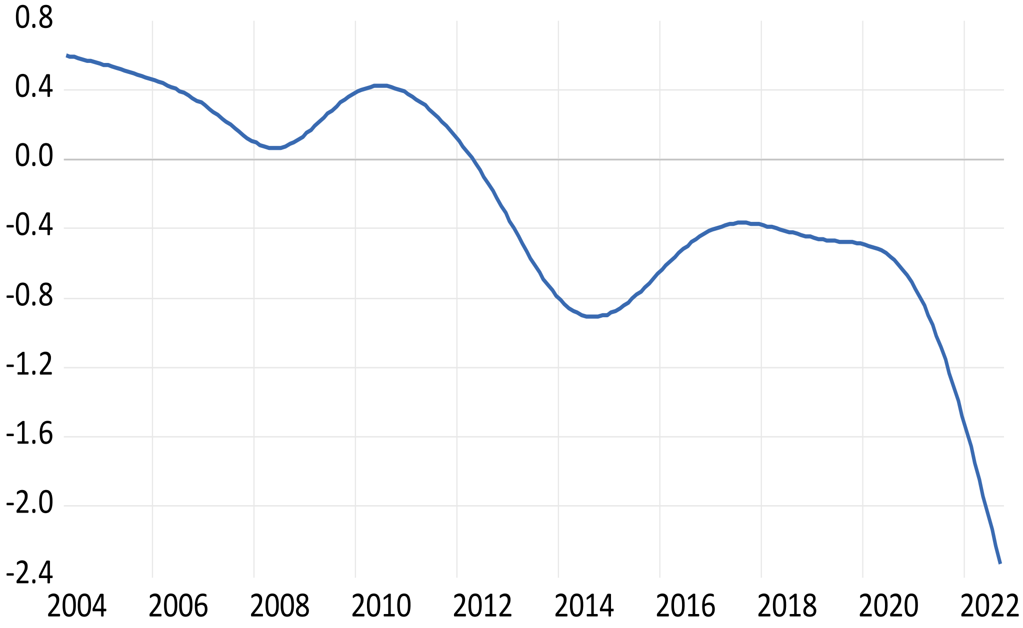
[10] The data on “uncollateralized overnight call rate” were retrieved from “BOJ Time-Series Data Search” in the website of The Bank of Japan (in English) “https://www.stat-search.boj.or.jp/index_en.html”.

Figure 2: Estimated Natural Rate of Interest by utilizing the Hodrick-Prescott Filter (unit: %)



difficulty since it is hard to deal with such a situation by conducting a monetary policy based on the usual framework.

5. Concluding Remarks

Our estimation result brings us to the conclusion that the natural rate of interest in Japan shows a persistent decline in recent 20 years. If the natural rate of interest is declining persistently, usual (or traditional) framework of monetary policy cannot be pursued. In this kind of situation, monetary policy should be conducted to guide real interest rate to negative level by raising expected inflation rate or by decreasing nominal interest rate. One difficult problem is whether the central bank can induce real interest rate to be negative when the natural rate is negative with the non-negative constraint of nominal rate. Therefore, the understanding the future direction of the natural rate and the cause of its decline is the essential factor for considering effective monetary policy.

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中央大学経済研究所
(INSTITUTE OF ECONOMIC RESEARCH, CHUO UNIVERSITY)
代表者　林　光洋　(Director: Mitsuhiro Hayashi)
〒192-0393　東京都八王子市東中野　742-1
(742-1 Higashi-nakano, Hachioji, Tokyo 192-0393　JAPAN)
TEL: 042-674-3271　　+81 42 674 3271
FAX: 042-674-3278　　+81 42 674 3278
E-mail: keizaiken-grp@g.chuo-u.ac.jp
URL: https://www.chuo-u.ac.jp/research/institutes/economic/