博士論文要旨 (2022 年度)

Field Equivalence of Physical Optics and Equivalent Current Methods for Smooth Conducting Cylindrical Objects

中央大学大学院理工学研究科
電気・情報系専攻
TA Quang Ngoc

1. Introduction

Electromagnetic (EM) wave scattering is a physical effect, in which the energy carried by an incident EM wave is deflected to other directions when the EM wave impinges on an object or scatterer. Many topics for studying and applications of electromagnetic wave scattering have been conducted over a century [1, 2]. Undeniably, EM scattering phenomena are behind many wireless systems and radio devices people are using today. Accordingly, the studying and understanding of electromagnetic interactions and scattering are fundamental to develop applications that exploit EM waves.

Over a century, variety of methods have been proposed to describe and solve EM scattering problems efficiently. While one may obtain the exact scattering solutions [2, 3], these are restricted in canonical shape objects such as wedge, rectangular and circular shapes. Thus the exact solutions of canonical objects may often be used for calibration of measurement facilities and validation of other methods.

When the geometry of scatterers become complex and inhomogeneous media, one may use the numerical methods such as method of moment (MoM), finite-difference time-domain (FDTD) methods. These numerical methods can give us reasonable results of scattering field from the complex scatterers. Though, they may have problem when applying for the electrically large objects and high frequency analyses. Hence, it is important to develop approximate and physical approaches that supplement these numerical methods by providing fast and acceptably accurate results for objects that are much larger than the wavelength.

The high frequency methods prove their efficient in analyzing electrically large objects in high frequency analysis [4, 5]. These methods mainly split into two branches: the first group is ray based method which involves geometrical optics (GO) and geometrical theory of diffraction (GTD); the second group is current based method which involves physical optics (PO) and physical theory of diffraction (PTD).

GO is a classical ray-based technique that treats the EM wave as light. The rays are conceived as geometrical lines originating from source and propagating through media. However, there is a drawback as GO completely ignores the diffraction effect in the calculation. Therefore, GTD is an alternative model of diffraction, which is used to describe and calculate the diffraction pattern in the shadowed region where GO ignores [4].

Comparing to the ray-based techniques, the current based methods allow the calculation of scattering field everywhere, including the diffraction regions. PO is rather classical, but very powerful and still being widely used in antenna and EM scattering problems [5]. The conventional PO approximation would be constructed by the radiation integral of the induced surface current excited by the incident wave on the surface of perfect electric conducting bodies.

So far, PO approximation has proved itself very powerful method for solving scattering problems by PEC objects. However, PO approximation only considers to the illuminated portion of the object and completely ignores the information of the shadowed portion. Thus it becomes unreliable when applied to solve the EM problems by penetrable objects. For years, some developments of PO approximation has been proposed to enrich the efficient of traditional PO approximation. PTD is an extension of PO approximation that originated in earlier work by Ufimtsev in 1957 [6]. The iterative physical optics (IPO) [7] is a refinement of first-order PO surface currents to cooperate high-order multi-bounce and multiple diffraction effects on electrically large geometries. A modified theory of physical optics method is also used to solve the scattering problem without extra correction terms as in the current based methods [8]. Some of improved PO approximations have been mentioned. However, they are still not clear for the deficient aspect of classical PO method as it only considers to the illuminated structure and ignores the knowledge of the shadowed one. Thus an extension of classical PO approximation that can eliminate this deficiency is important.

In the previous investigation of the electromagnetic scattering by conducting wedge and rectangular cylinders [9], the scattering problem has been solved by deriving the equivalent electric and magnetic currents, which are assumed on both illuminated and shadowed surfaces of the object. The final scattering field of this method coincides with one by traditional PO approximation. Comparing to the PO approximation, this GO-based equivalent current method allows us to formulate the scattering far-field by derived equivalent electric and magnetic currents on both illuminated and shadowed surfaces of the object.

When our GO-based equivalent current method is firstly proposed for the wedge and rectangular conducting cylinder, a question arises as whether this method can also be applied to the curved and smooth objects. This is a motivation of our present study. In this GO-based equivalent current method will be used to evaluate the scattering far-field from a circular conducting cylinder and half circular conducting strip. The final scattering far-field by our GO-based equivalent current method is found to coincide with the PO approximation. In addition, the PO radiation integrals are solved directly by using an uniform asymptotic evaluation for both H and E polarizations. This allows us to describe the reflected and diffracted fields corresponding to the saddle point and endpoint contributions, respectively. The accuracy of PO approximation is checked with those by the exact solution. Additionally, simple formulae are also proposed to estimate the main scattering lobes for H and E polarizations.

2. Field Equivalence Theorem

In this chapter, theory of surface equivalence theorem will be discussed. From this theorem, one may get the relation between the PO approximation and our GO-based equivalent current method when dealing with scattering problems by conducting objects.

According to the surface equivalence theorem, the fields outside an imaginary closed surface are obtained by placing, over the closed surface, suitable electric and magnetic currents that satisfy the boundary conditions. Assuming that the scattering fields $(\boldsymbol{E}^{\mathrm{s}}, \boldsymbol{H}^{\mathrm{s}})$ are excited by incident fields $(\boldsymbol{E}^{\mathrm{i}}, \boldsymbol{H}^{\mathrm{i}})$ impinges on a conducting object. The total fields outside the conducting obstacle $(\boldsymbol{E}, \boldsymbol{H})$ are the summation of the incident and scattering fields. If the scattering fields $(\boldsymbol{E}^{\mathrm{s}}, \boldsymbol{H}^{\mathrm{s}})$ are considered as the fields radiated from the secondary sources on the obstacle, the scattering fields outside the object can be provided from the surface current sources $\boldsymbol{J}_{\mathrm{s}}, \boldsymbol{M}_{\mathrm{s}}$ on a virtual closed surface, which may be found as

$$\begin{aligned} \boldsymbol{J}_{\mathrm{s}} &= \hat{\boldsymbol{n}} \times \boldsymbol{H}^{\mathrm{s}}, && (1) \\ \boldsymbol{M}_{\mathrm{s}} &= \boldsymbol{E}^{\mathrm{s}} \times \hat{\boldsymbol{n}}. && (2) \end{aligned}$$

In the case of the conducting objects, one may choose the closed surface coinciding with the conducting parts of the physical structure of the objects. Then the scattering fields $(\boldsymbol{E}^{\mathrm{s}}, \boldsymbol{H}^{\mathrm{s}})$ can be obtained from the local feature of the scatterer's surface on the high frequency analysis and the field inside the conducting objects will be zero. Since the magnetic current $\boldsymbol{M}_{\mathrm{s}}$ is set to be zero on the perfect electric conducting body and the physical equivalent current is set to be zero in the shadowed region of the scatterer, one needs only to consider the electric current $\boldsymbol{J}_{\mathrm{s}}$ on the illuminated surface of the objects. If the perfect electric conducting object has an infinite, flat and smooth surface, the tangential components of the scattered magnetic field are in phase and approximately equal in amplitude of the tangential components of the incident field. Therefore, the PO equivalent current may be represented as

$$\boldsymbol{J}^{\mathrm{PO}} = \begin{cases} 2\hat{\boldsymbol{n}} \times \boldsymbol{H}^{\mathrm{i}}, & \text{on the illuminated surface,} \\ 0, & \text{on the shadowed surface.} \end{cases} \qquad (3)$$

If the shadowed structure occupies a large part of the object, it is totally ignored by the PO approximation. Thus it is important to find another method that can include the information of the shadowed structure.

In Sect. 2.2, the general case of scattering from a conducting object is described by GO-based equivalent current method. By choosing the close surface right on the scattering body, the scattering fields $(\boldsymbol{E}^{\mathrm{s}}, \boldsymbol{H}^{\mathrm{s}})$ can be obtained from the local feature of the scatterer's surface on the high frequency analysis, and the field inside the conducting object will be zero. If the scatterer geometry is assumed to be large compared with the wavelength, the scattering fields $(\boldsymbol{E}^{\mathrm{s}}, \boldsymbol{H}^{\mathrm{s}})$ may be given by the reflected GO fields $(\boldsymbol{E}^{\mathrm{r}}, \boldsymbol{H}^{\mathrm{r}})$ on the illuminated surface. On the shadowed surface of the conducting object, the total GO fields would be almost null. Therefore the scattering fields must behave to cancel the original incident fields. Then assuming that $\boldsymbol{E}^{\mathrm{s}} = -\boldsymbol{E}^{\mathrm{i}}$ and $\boldsymbol{H}^{\mathrm{s}} = -\boldsymbol{H}^{\mathrm{i}}$, the equivalent currents are represented in the lit and shadowed portions as

$$\boldsymbol{J}_{\mathrm{s}} = \hat{\boldsymbol{n}} \times \boldsymbol{H}^{\mathrm{s}} \simeq \begin{cases} \hat{\boldsymbol{n}} \times \boldsymbol{H}^{\mathrm{r}} & \text{on illuminated } S, \\ \hat{\boldsymbol{n}} \times (-\boldsymbol{H}^{\mathrm{i}}) & \text{on shadowed } S, \end{cases} \qquad (4)$$

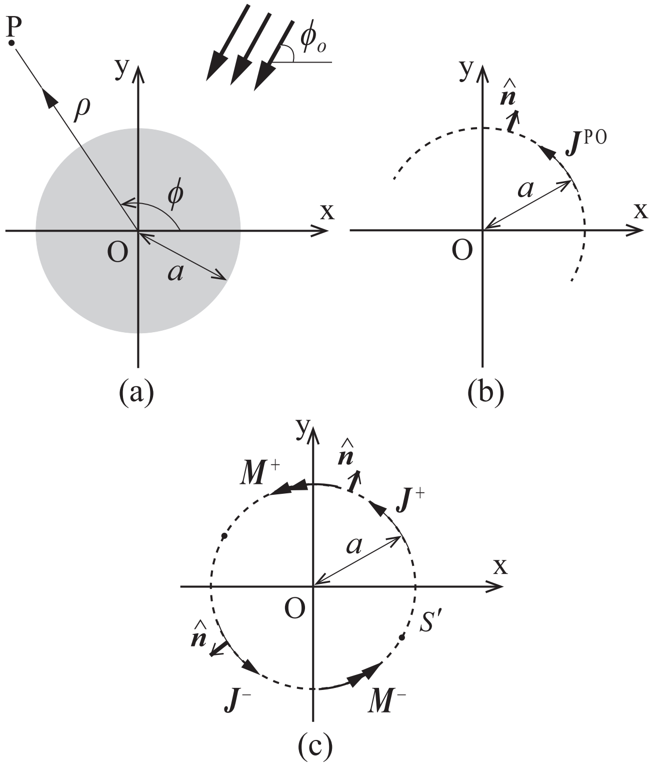
$$\boldsymbol{M}_{\mathrm{s}} = \boldsymbol{E}^{\mathrm{s}} \times \hat{\boldsymbol{n}} \simeq \begin{cases} \boldsymbol{E}^{\mathrm{r}} \times \hat{\boldsymbol{n}} & \text{on illuminated } S, \\ (-\boldsymbol{E}^{\mathrm{i}}) \times \hat{\boldsymbol{n}} & \text{on shadowed } S. \end{cases} \qquad (5)$$

The scattered electric and magnetic far-fields due to the PO current in Eq. (3) and surface equivalent currents in Eqs. (4), (5) are derived by integrating these currents over the surface using the free-space Green's function.

3. Scattering by a Circular Conducting Cylinder

In this chapter, the electromagnetic scattering from a circular conducting cylinder will be analyzed by PO approximation and our GO-based equivalent current method. Figure 1(a) shows the geometry of an EM plane wave impinges on a circular conducting cylinder.

In Sect. 3.2, PO approximation is applied to observe the EM scattering by a circular conducting cylinder for both H and E polarization. The PO current $\boldsymbol{J}^{\mathrm{PO}}$ is approximated from the incident magnetic field as Eq. (3) and flows on the physically illuminated surface of the circular conducting cylinder as shown in Fig. 1(b). Then the scattering far-field from circular conducting cylinder is derived by integrating the PO current using the



(a) (b) (c)

Figure 1: Scattering from a circular conducting cylinder. (a) The circular conducting cylinder illuminated by a plane wave. (b) PO current $\boldsymbol{J}^{\mathrm{PO}}$ by the incident field on the illuminated surface, (c) Equivalent currents $\boldsymbol{J}^{\pm}, \boldsymbol{M}^{\pm}$ derived by GO fields.

free-space Green's function. The obtained PO radiation integrals are then solved asymptotically by saddle point method by deforming the integration contour from original real value into a steepest descent path in the complex plane. Then the integration may be divided into three parts for the evaluation. The first one is the saddle point contribution, which is corresponding to the reflected field. The remaining ones, corresponding to the diffracted field, are evaluated by non-uniform and uniform solutions. The non-uniform solution, which uses the integration-by-part technique, is fail near the forward direction since there are two saddle points simultaneously contributing to the integral when the observation angle gets close to the forward direction. Thus the uniform asymptotic approximation has been applied to solve the problem in all observation angles.

In Sect. 3.3, GO-based equivalent current method is applied to observe the EM scattering by a circular conducting cylinder for both H and E polarizations. Figure. 1(c) shows the equivalent electric and magnetic currents $\boldsymbol{J}^{\pm}, \boldsymbol{M}^{\pm}$. The equivalent currents $\boldsymbol{J}^{+}, \boldsymbol{M}^{+}$ on the illuminated surface are approximated by the GO reflected rays $\boldsymbol{E}^{r}, \boldsymbol{H}^{r}$. Meanwhile the equivalent currents $\boldsymbol{J}^{-}, \boldsymbol{M}^{-}$ on the shadowed surface are obtained from the incident GO fields $-\boldsymbol{E}^{i}, -\boldsymbol{H}^{i}$. Then the total scattering far-field is given by summing up all contributions derived from these equivalent electric and magnetic currents. The final scattering far-field radiated from these equivalent currents is found to be exactly the same as the result obtained by PO approximation in both H and E polarizations.

In Sect. 3.4, the numerical results of evaluating the scattering far-field for both H and E polarizations are presented and discussed to assess the correctness of the proposed solutions. It is observed that PO approximation result is in a good agreement with the exact solution when the circular conducting strip is very large compared to the wavelength.

4. Scattering by a Half Circular Conducting Strip

In this chapter, the field equivalence between PO and equivalent current method is shown for another example of the electromagnetic scattering, a circular conducting strip is illuminated by a plane wave in Figs. 2(a) and (c) for the case of fully illuminated and partially illuminated, respectively.

The scattering fields by a fully and partially illuminated half circular conducting strip will be derived from the PO approximation in Sect. 4.1. In the fully illuminated case, the lit surface of the strip is found the same as one by a circular conducting cylinder. Therefore the scattering field from a fully illuminated half circular conducting strip would be found to be the same as the circular conducting cylinder case. In the partially illuminated case, the PO current is found to be the same as the fully illuminated case, but only a part of the circular strip is lit by the incident wave as shown in Fig. 2(c).

Next in Sect. 4.2, GO-based equivalent current method is applied to observe the scattering far-field by a circular conducting strip in both full and partial illuminated cases. Figure 2(b) shows the equivalent electric and magnetic currents $\boldsymbol{J}^{\pm}, \boldsymbol{M}^{\pm}$ surrounding the half circular strip in the full illuminated case. The equivalent currents $\boldsymbol{J}^{+}, \boldsymbol{M}^{+}$ are found the same as those in the circular conducting cylinder. However, they become different in the shadowed portion since the unit normal vector $\hat{\boldsymbol{n}}$ becomes $-\hat{\boldsymbol{\rho}}$. Therefore, the electric currents $\boldsymbol{J}^{\pm}$ are the same, while the equivalent magnetic currents on the lit and shadowed surface $\boldsymbol{M}^{\pm}$ cancel each other, then the total equivalent current become the same as PO current. For the partial illuminated case, it can be seen from Fig. 2(c) that the shadowed portion occupies a large part of area, however, it is completely ignored by the PO formulation. Thus GO-based equivalent current method is proposed to include the information of the shadowed portion. By GO-based equivalent current method, the electric and magnetic equivalent currents are shown in Fig. 2(d) and divided into two sets: those for lit portion $(\boldsymbol{J}_1^{\pm}, \boldsymbol{M}_1^{\pm})$, and for shadowed portion $(\boldsymbol{J}_2^{\pm}, \boldsymbol{M}_2^{\pm})$. The lit portion currents can be derived

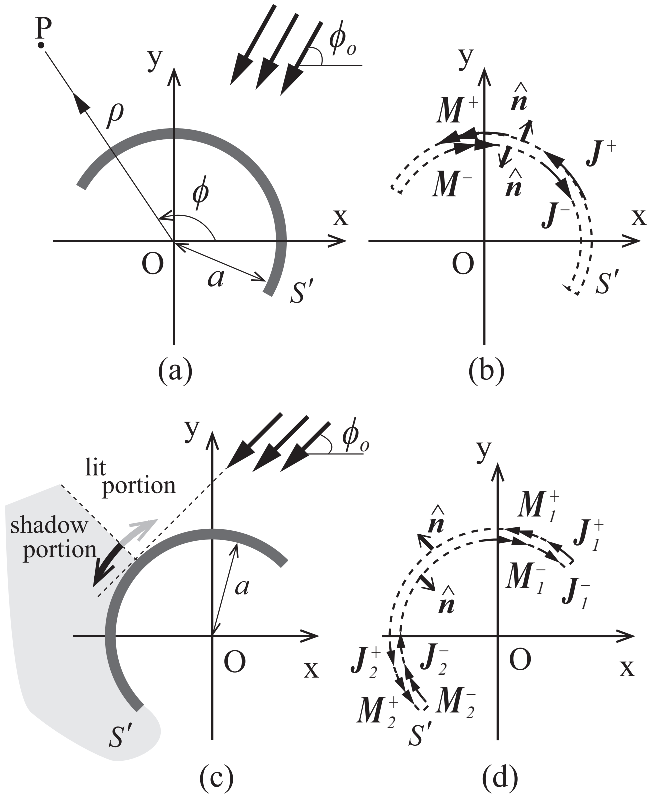


Figure 2: A half circular conducting cylinder illuminated by a plane wave. (a) Original problem. (b) Equivalent currents $\boldsymbol{J}^+, \boldsymbol{M}^+$ on the illuminated surface and $\boldsymbol{J}^-, \boldsymbol{M}^-$ on the shadowed surface. (c) A partial illuminated case. (d) Equivalent currents $\boldsymbol{J}^{\pm}_{1,2}, \boldsymbol{M}^{\pm}_{1,2}$.

the same as those of full illuminated case. The shadowed portion currents are found as $\boldsymbol{J}_2^+ = -\boldsymbol{J}_2^-, \boldsymbol{M}_2^+ = -\boldsymbol{M}_2^-$ due to the opposite direction of the normal vector $\hat{\boldsymbol{n}}$. Consequently, the total currents $\boldsymbol{J}_2^{\pm}, \boldsymbol{M}_2^{\pm}$ become zero in the shadowed portion. Therefore the final result is only the sum of the fields derived from equivalent current in the lit portion $\boldsymbol{J}_1^{\pm}, \boldsymbol{M}_1^{\pm}$. Accordingly, the final results are found exactly the same as PO formulation for both H and E polarizations in Sect. 4.1.

5. Conclusion

The field equivalence between physical optics and GO-based equivalent current methods has been shown for the plane wave scattering by a circular conducting cylinder and a half circular conducting strip for both H and E polarizations. It has been shown that our GO-based equivalent current method can give us the same scattering far-field as the PO approximation, meanwhile it includes the information about the shadowed structure of the circular conducting cylinder and the half circular strip. This is an advantage of our GO-based equivalent current method since it can be extended to formulate the scattering far-field by penetrable objects. The high frequency asymptotic solutions of the PO radiation integral have also been derived for the plane wave scattering far-field by a circular conducting cylinder, which can be interpreted regard as the reflected and diffracted fields. The results have been checked with the exact solution and yield a good agreement when the radius of the circular conducting cylinder is large. In addition, simple formulae are also proposed to estimate the main scattering lobes for both H and E polarizations.

Based on the obtained results, one may extend our formulations to more practical analyses such as smooth penetrable objects.

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