A Study on Diffraction of Electromagnetic Wave by Dielectric Wedges

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1. Introduction

Electromagnetic (EM) waves have been known since the work of J. Maxwell, which describes how electric and magnetic fields are generated by charges, currents, and changes in the fields. Among the myriad of electromagnetic problems, electromagnetic wave scattering is always an attractive and challenging topic. It is an important basis for understanding the conditions of radio wave propagation. In wireless communication, EM wave scattering is mostly caused by high buildings. It would be extremely difficult to create a direct solution to the scattering problem of large buildings. However, the problem can be solved more easily by considering the surface of the buildings as a set of wedges for the high-frequency domain. By analyzing the scattering problem of wedges, the effect of the buildings can be evaluated.

Several available solutions such as exact solution [1] and numerical method [2] can be utilized for scattering problems by a limited number of simple shapes and small objects. However, these methods are not ideal for large objects due to unrealistic execution time and memory requirements. Thus, one needs the approximation approaches that can provide both acceptably fast and highly accurate solutions for large objects.

In the high-frequency domain, some rather classical approximation methods may be able to analyze the scattering problems by large conducting objects, such as geometrical optics (GO), geometrical theory of diffraction (GTD) and physical optics (PO) solutions. Here, GO is a ray-based method [3], in which the GO scattering fields can be obtained by using the usual reflection and transmission principles of rays at the interfaces of objects. However, the GO method does not provide information about the diffraction effect and the field behavior in the shadowed areas of the scattering objects. An alternative model of diffraction called GTD [4], [5] was proposed to describe the diffraction behavior in shadowed regions that GO ignores. However, GTD and its uniform extensions [6] may be only applicable to conducting objects and the external field of penetrable objects. This motivates us to look for a more potential solution as PO approximation.

The PO approximation is a rather simple but very powerful solution for antenna applications and electromagnetic problems [7], [8]. This method is developed based on the surface equivalence theorem, in which the scattering field by an object is considered as the radiation from the equivalent induced PO currents on the illuminated surfaces. These PO currents can be easily constructed from the information of the incident wave only. When the scattering objects are made of penetrable materials, the problem becomes more complicated due to the appearance of scattering phenomena inside the object.

Several approximations based on the PO method have been proposed to solve complicated scattering problems of penetrable objects, in which equivalent electric and magnetic currents were proposed to replace the PO currents. These equivalent currents can be obtained from the incident, reflected and transmitted GO rays. From the above basis, a uniform asymptotic PO solution has been derived for the diffraction by dielectric wedges [9]. A hidden rays of diffraction (HRD) solution has also been proposed for the diffracted field of dielectric wedges [10], in which additional non-physical diffracted rays have been introduced to overcome the error of PO. Although the above PO-based approximation methods are available for estimating the high-frequency scattering problem, their reliability has not been clearly verified yet. This motivates us to find out the accuracy of the PO approximation, and to develop a reliable extended solution for the scattering by dielectric objects.

In this research, PO method has been re-investigated in detail for perfectly electrically conducting (PEC) wedges to find a unified diffraction description that can be used for any incident angle [11]. Then, an extended PO (EPO) solution has been presented for scattering by a dielectric wedge [12], in which the scattering field can be formulated as the radiation from equivalent induced electric and magnetic currents on the wedge surfaces. These equivalent currents are obtained from the electric and magnetic fields of the GO incident, reflected and transmitted waves. The scattering fields can then be found by integrating the above equivalent currents. The uniform asymptotic solutions of the edge diffracted field can then be expressed in terms of cotangent functions, which have a one-to-one correspondence with the shadow boundaries of the GO waves. The numerical results have been performed to compare our EPO results with those by other reference methods such as HRD and FDTD (Finite-Difference Time-Domain) simulation. A concept of lateral waves has been proposed to enhance the accuracy of our current solutions.

2. PO for PEC Wedge

A scattering problem of a perfectly electrically conducting (PEC) wedge will be solved by applying the PO

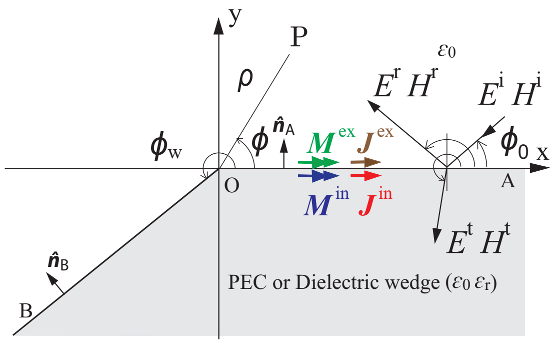


Figure 1: Scattering by wedge.

approximation method. Then, a unified expression of the scattering field is represented for two-dimensional cases.

The PO approximation is constructed based on the surface equivalence theorem. When an object is illuminated by an incident electromagnetic wave ($\boldsymbol{E}^{\rm i}$, $\boldsymbol{H}^{\rm i}$), the scattered field ($\boldsymbol{E}^{\rm s}$, $\boldsymbol{H}^{\rm s}$) may be calculated by the radiation fields from induced electric and magnetic currents $\boldsymbol{J}$ and $\boldsymbol{M}$ on the surface of the object. For the two-dimensional configuration ($\frac{\partial}{\partial z} \equiv 0$), the scattering field ($\boldsymbol{E}^{\rm s}$, $\boldsymbol{H}^{\rm s}$) is then given by integrating these currents on the boundary C of a scattering body with the two-dimensional Green's function G as [1], [5]

$$\boldsymbol{E}^{\rm s} = -\int_{\rm C} \left[j\omega\mu \boldsymbol{J} G(\boldsymbol{r}, \boldsymbol{r}') + \boldsymbol{M} \times \nabla' G(\boldsymbol{r}, \boldsymbol{r}') \right] dl', \quad (1)$$

$$\boldsymbol{H}^{\rm s} = -\int_{\rm C} \left[j\omega\varepsilon \boldsymbol{M} G(\boldsymbol{r}, \boldsymbol{r}') - \boldsymbol{J} \times \nabla' G(\boldsymbol{r}, \boldsymbol{r}') \right] dl'. \quad (2)$$

where ω, ε and μ are the angular frequency, permittivity and permeability, respectively. ∇' indicates differentiation with respect to the prime source coordinates.

When the illuminated object is a PEC object, the induced magnetic current $\boldsymbol{M}$ is zero, and the electric current $\boldsymbol{J}$ can be approximated by the PO current $\boldsymbol{J}^{\rm PO}$ on the illuminated surface of the scattered object as [5]

$$\boldsymbol{J}^{\rm PO} = \begin{cases} 2\hat{\boldsymbol{n}} \times \boldsymbol{H}^{i} & \text{on illuminated surface,} \quad (3a) \\ 0 & \text{on shadowed surface,} \quad (3b) \end{cases}$$

where $\hat{\boldsymbol{n}}$ denotes a unit normal vector on the boundary surface to the area containing the observation point. Based on the formulations of the PO approximation, one then considers a two-dimensional wedge of the wedge angle $\phi_{\rm w}$, which is illuminated by an incident plane wave with the incident angle ϕ_0 as in Fig. 1.

When the wedge is a PEC wedge, the external scattering field can be derived from the PO current in the outside region, while the internal field is null. When surface OA of the wedge is illuminated by an incident plane wave, the corresponding PO current $\boldsymbol{J}_{\rm A}^{\rm PO}$ is excited on surface OA as

$$\boldsymbol{J}_{A}^{\rm PO} = 2\hat{\boldsymbol{n}}_{A} \times \boldsymbol{H}^{\rm i}|_{y=0}. \quad (4)$$

Then the z-component of the scattering fields can be obtained by integrating the current $\boldsymbol{J}_{A}^{\rm PO}$ with two dimensional Green's function G as in Eqs.(1) and (2). The obtained radiation integrals can then be solved by using the saddle point technique on the assumption for a large k $(= \omega\sqrt{\varepsilon_0\mu_0})$. The uniform asymptotic solution of the diffracted field can then be expressed in terms of two cotangent functions and transition functions. The singularity behavior of two cotangent functions has a one-to-one correspondence with one of the shadow boundaries of the GO incident and reflected waves.

When surface OB is illuminated, one has a contribution from additional PO current $\boldsymbol{J}_{\rm B}^{\rm PO}$ on surface OB. The corresponding diffracted field also can then be obtained in terms of two cotangent functions. One notes that the diffracted fields are currently represented separately by different equations depending on the illumination. When the incident direction changes, only one of the two equations has to be selected to describe the corresponding diffracted field behavior. This can sometimes be bothersome for the calculation. Thus, by decomposing the diffraction coefficient due to the currents $\boldsymbol{J}_{\rm A}^{\rm PO}$ and $\boldsymbol{J}_{\rm B}^{\rm PO}$, and by carefully rearranging the terms, one then finds a unified expression that includes four cotangent functions and is valid for all incident and observation angles $0 < (\phi, \phi_0) < \phi_{\rm w}$. When the direction of incident wave ϕ_0 changes, two of four cotangent functions correspond to the non-physical rays and cancel out each other to show exact behavior of the diffracted fields [11].

3. Extended PO for Dielectric Wedge

An extended PO (EPO) approximation for the scattering problem of dielectric wedges is proposed based on equivalent electric and magnetic currents.

As show in Fig. 1, when the incident wave illuminates surface OA of the dielectric wedge of the dielectric constant $\varepsilon_{\rm r}$, it excites not only the reflected wave ($\boldsymbol{E}^{\rm r}$, $\boldsymbol{H}^{\rm r}$) in the outer region, but also the transmitted wave ($\boldsymbol{E}^{\rm t}$, $\boldsymbol{H}^{\rm t}$) inside the wedge. These GO reflected and transmitted waves can be normally derived from the formulation of the incident wave by using Snell's law. Then, the external equivalent currents on surface OA can be given by [12]

$$\boldsymbol{J}^{\rm ex} = \hat{\boldsymbol{n}}_{\rm A} \times (\boldsymbol{H}^{\rm i} + \boldsymbol{H}^{\rm r}), \quad (5)$$

$$\boldsymbol{M}^{\rm ex} = (\boldsymbol{E}^{\rm i} + \boldsymbol{E}^{\rm r}) \times \hat{\boldsymbol{n}}_{\rm A}. \quad (6)$$

Inside the wedge, the corresponding currents on surface OA are obtained as

$$\boldsymbol{J}^{\rm in} = -\hat{\boldsymbol{n}}_{\rm A} \times \boldsymbol{H}^{\rm t} \quad (7)$$

$$\boldsymbol{M}^{\rm in} = \boldsymbol{E}^{\rm t} \times (-\hat{\boldsymbol{n}}_{\rm A}). \quad (8)$$

The scattering fields can then be represented by integrations in Eqs.(1) and (2). These integrals can be evaluated by using the saddle point technique with the same manner as the PEC wedge case in Chapter 2. Then, uniform asymptotic solutions of the diffracted fields can be obtained for each integral from the corresponding GO incident, reflected and transmitted waves. Then combining the above contributions, unified expressions of the scattering fields can be obtained.

The formulation of the external diffracted field also includes four cotangent functions with additional reflection coefficients Γ_A and Γ_B on surfaces OA and OB, respectively. As the dielectric constant ε_r tends to infinity (the dielectric wedge becomes a PEC wedge), these reflection coefficients become a unit. Accordingly, the resulting diffracted field becomes exactly the same as the one formulated by the PO formulation for the PEC wedge in Chapter 2.

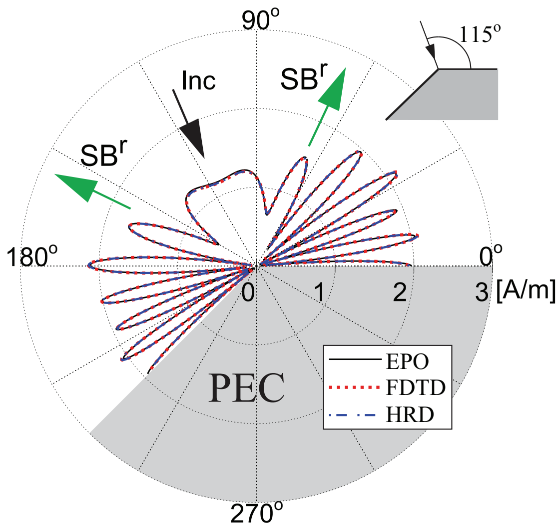
Inside the dielectric wedge, the diffracted field can be found from the electric and magnetic currents, which are excited by the transmitted waves. Then, the uniform expression of the internal diffracted field can then be given by two cotangent functions with corresponding transmission coefficients T_A and T_B from surfaces OA and OB.

4. Numerical results and discussion

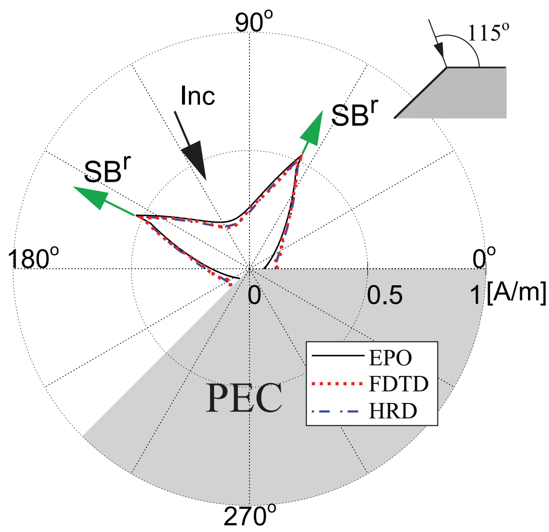
Accuracies of the EPO solutions have been evaluated by comparing the numerical results with those obtained from reference methods such as HRD solution and FDTD simulation. The numerical FDTD calculation is rather simple, but it consumes a lot of time and memory, and one needs a special attention to obtain the results for infinitely large structures. The following comparison results are made for the TM (transverse magnetic) polarization case, in which the magnetic field of the incident wave is perpendicular to the plane of incidence.

Figure 2 shows the total and diffracted fields of the PEC wedge, in which the EPO has the same result as the conventional PO solution. For the total field, three solutions match well in all directions as in Fig. 2(a). Then, subtracting the GO incident and reflected rays, the comparison of diffracted fields is made in Fig. 2(b). The diffracted field is small compared with the GO rays and distributes mainly in the vicinity of the geometrical shadow boundaries SB^r. It can be seen that the HRD and FDTD results are almost identical, and have some differences from the EPO result. These differences become more apparent as the observation point approaches the wedge surface. This is due to the fact that the diffracted field of the PO approximation doesn't satisfy the boundary and edge conditions. On the other hand, the non-physical parts of the HRD solution contribute to satisfying the boundary condition, and the modified angular period of the cotangent functions relate to the edge condition.

When the PEC wedge is replaced by a dielectric wedge, one has more contributions due to transmitted waves inside the wedge, and the corresponding total and diffracted fields are shown in Fig. 3. As can be seen from Fig. 3(a), the total fields also have a good agreement, in which the exterior field pattern is quite similar to the PEC case, while the incident wave mostly transmits into the dielectric region to yield a main scattering lobe in the forward direction. For the diffracted field, the results of EPO match well with the FDTD simulation in the outer region, while those of HRD show some differences. This is in contrast to the PEC wedge case, in which HRD has better agreement with the FDTD simulation than EPO. This change in accuracy of EPO and HRD may be related to the edge condition.



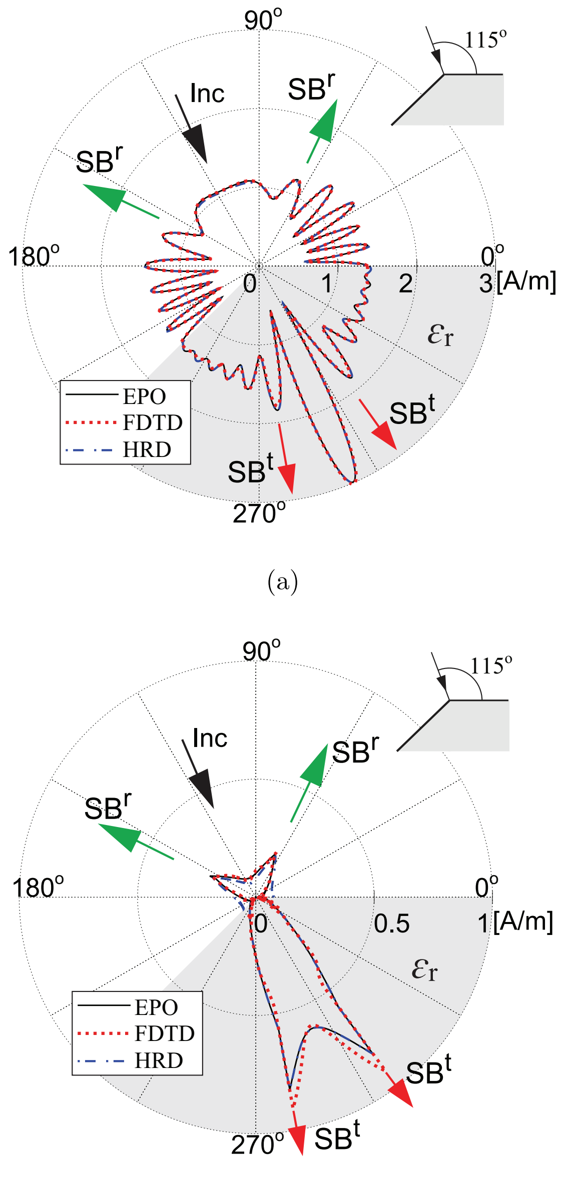
(a)



(b)

Figure 2: Total and diffracted TM fields of PEC wedge: $\phi_w = 225°$, $\phi_0 = 115°$ and $\rho = 3\lambda$. (a) Total field. (b) Diffracted field.

For the interior region, the diffracted fields by the EPO and HRD solutions exhibit almost the same behavior and have some differences from the FDTD simulation. One observes that, unlike the PEC wedge, the diffracted fields of both EPO and HRD solutions don't satisfy the boundary condition for the dielectric case. This is due to the fact that the diffracted field excited at the wedge tip propagates in the exterior and interior regions with different wave numbers. This difference should be compensated by the lateral waves excited in the denser medium as in Fig. 4. The excitation of these lateral waves depends on the surface field. The lateral wave behavior may be found from the contribution of the branch point in the integrals of the Green's function for the two-media problem.



(a)

(b)

Figure 3: Total and diffracted TM fields of dielectric wedge: $\phi_w = 225°$, $\phi_0 = 115°$, $\varepsilon_r = 6$ and $\rho = 3\lambda$. (a) Total field. (b) Diffracted field.

5. Conclusion

From the PO approximation, unified expressions of the diffracted field have been derived for the PEC wedge. Then, the EPO solution has then been obtained for the scattering problem of the dielectric wedge. The accuracy of EPO was evaluated by comparing the numerical results with other reference methods. EPO has a better comparison with FDTD than HRD in the outer region of the dielectric wedge. For the internal field, EPO and HRD were found to exhibit almost the same results. Accordingly, EPO may be suitable for the scattering problem of the dielectric wedge without the non-physical additional terms of HRD, and requires significantly less computational resources than FDTD. From the difference with FDTD, the lateral wave is needed to enhance the accuracy of EPO and HRD inside the

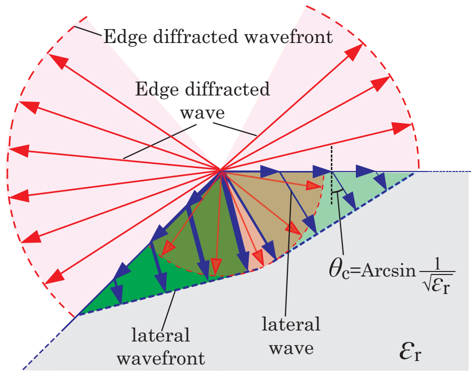


Figure 4: Possible lateral wave inside wedge.

wedge. The accuracy change of the EPO and HRD solutions between PEC and dielectric cases also reminds an additional consideration of the edge condition. These aspects are motivations for our next investigations in the future.

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