Analysis of Tourism Demand for Two Major Prefectures in Japan's Kansai Region using the Bayesian VAR

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1. Introduction

The Vector Autoregression / Autoregressive (VAR) model has been frequently applied to analyse economic change since the pioneering work of Sims (1980). In particular, the Structural Vector Autoregression (SVAR)[1] methodology is broadly utilized for macro-economic research. For example, Sims (1992) puts stress on the role of short-term interest rate as an essential factor for monetary policy with recursive identification frameworks of the SVAR. Blanchard and Watson (1986), Gali (1992), Gordon and Leeper (1994), and Lastrapes and Selgin (1995) apply a non-recursive approach to impose contemporaneous restrictions for identification. On the other hand, Bernanke and Mihov (1998) adopt the block-recursive approach to identify the shocks to monetary policy.

There can be another type of the VAR method. The Bayesian Vector Autoregression (BVAR) model has been regarded as the Bayesian-flavored VAR specification for empirical analysis, which connects the priors with information incorporated in the date set. The BVAR decreases a risk of over-parameterization by imposing special restrictions on the parameters in the VAR process (the so-called "shrinking parameters") through their prior probability distribution functions. The prior probability distribution function encloses the prior which includes the mean and variance of the distribution. The prior probability distribution function presents the range of uncertainty of the prior mean, and it revised by sample information if underlying distribution significantly differs from the prior. Moreover, the posterior distribution function of parameters in the BVAR model can be given from the combination of the prior distribution function and the distribution of the sample data. One critical problem is whether the prior distribution function and the

1) SVAR is sometimes called "identified VAR."

posterior are in the same distribution family. In this research, we assume that they are in the same distribution family and utilize one of the stylized conjugate priors — Minnesota (Litterman) prior. Taking the issues described above into consideration, the empirical analysis using the BVAR with Minnesota (Litterman) prior to investigate the influence of economic variables on tourism demand for two major cities in Japan's western region before the COVID-19 pandemic, coronavirus crisis, is conducted in this study.

The reminder of this paper is organized as follows. Section 2 highlights the characteristics of the Bayesian Vector Autoregression analysis. Section 3 is for data set. Section 4 describes the empirical study utilizing the BVAR, and Section 5 presents the concluding remarks.

2. Bayesian Vector Autoregression Analysis

As explained in previous section, the Vector Autoregression (VAR) model has been broadly used as a tool for empirical multivariate economic analysis since the 1980s. However, it is said that the VAR model generates the so-called "overparameterization problem" when it is applied in large models with many parameters. We have at least two kinds of methods to cope with this problem - Structural Vector Autoregression (SVAR: way of application of theoretical constraints) model and Bayesian Vector Autoregression (BVAR: way of application of Bayesian theory) model.[2)]

Generally, the VAR model for estimation is inclined to contain many parameters, and some of them might be significant only by coincidence. In such cases, they do not yield important information if estimated. The BVAR deals with this kind of topic by defining the "prior" for parameters.

Bayesian statistical flamework connects the distribution properties of the prior distribution, likelihood, and posterior distribution based on the assumption that the parameters can be regarded as random variables.[3)] The "prior" expresses the external distributional information derived from the "belief" on the parameters. The "likelihood" means the information of the sample probability distribution function. By the so-called Bayes' theorem, the "prior distribution" is related to the data likelihood results in the "posterior distribution."

If we denote the parameters of interest by $\theta = (\beta, \Sigma)$, the prior distribution by $\pi(\theta)$, the data by y, and the likelihood by $l(y \mid \theta)$, then we can express $\pi(\theta \mid y)$, "posterior distribution", as

$$\pi(\theta \mid y) = \frac{\pi(\theta) l(y|\theta)}{\int \pi(\theta) l(y|\theta) d\theta} \quad (1)$$

2) See Lütkepohl (2006) for details.

3) See Sims and Zha (1998), Lütkepohl (2006), Canova (2007), Giannone, Lenza, and Primiceri (2015), and Jarociński and Marcet (2019) for details.

where the denominator describes a normalizing constant without randomness. In this case, the "posterior" is proportional to the product of $l(y|\theta)$ and the $\pi(\theta)$:

$$\pi(\theta|y) \propto \pi(\theta) l(y|\theta). \tag{2}$$

The theoretical connection between this specification and the BVAR approach requires the following basic VAR(p) model:

$$y_t = a_0 + \sum_{j=1}^{p} a_j y_{t-j} + \epsilon_t \tag{3}$$

where y_t (for $t=1,\cdots,T$) is an $m\times 1$ vector with observations on m different series. ε_t is an $m\times 1$ vector of errors where $\varepsilon_t \sim N(0, \Sigma_\varepsilon)$, *i.i.d.*.

For simplicity, this equation can be written as

$$Y = XA + E \tag{4}$$

or

$$y_t = (I_m \otimes X)\theta + e \tag{5}$$

where Y and E are $T\times m$ matrices, $X = (x_1,\cdots,x_t)'$ *is a* $T\times(mp+1)$ matrix for $x_t = (1, y'_{t-1}{}',\cdots, y'_{t-q})$, I_m is the m-dimension identity matrix, $\theta = vex(x)$, and $e\sim N(0, \Sigma_\varepsilon \otimes I_T)$.

By applying equation (5), the likelihood function is written as the following form:

$$l(\theta, \Sigma_\epsilon) \propto \left|\Sigma_\epsilon \otimes I_T\right|^{-0.5} exp\{-0.5(y-(I_m\otimes X)\theta)'(\Sigma_\epsilon\otimes I_T)^{-1}(y-(I_m\otimes X)\theta)\}. \tag{6}$$

Assuming that Σ_ε is known and a multivariate normal prior for θ to explain the derivation of the posterior moments is

$$\Pi(\theta) \propto \left|V_0\right|^{-0.5} exp\{-0.5(\theta-\theta_0)'V_0^{-1}(\theta-\theta_0)\} \tag{7}$$

where θ_0 is the mean of prior and V_0 is the covariance of the prior. The posterior density is expressed by the following form as a multivariate normal probability density function if we combine equation (6) and (7):

$$\Pi(\theta|y) = exp\Big[-0.5\{(V_0^{-0.5}(\theta-\theta_0))'(V_0^{-0.5}(\theta-\theta_0))\} + \{(\Sigma_\epsilon{}^{-0.5}\otimes I_T)y-(\Sigma_\epsilon{}^{-0.5}\otimes X)\theta\}'\{(\Sigma_\epsilon{}^{-0.5}\otimes I_T)y-(\Sigma_\epsilon{}^{-0.5}\otimes X)\theta\}\Big]. \tag{8}$$

Defining ω and W as

$$\omega \equiv \begin{bmatrix} V_0^{-0.5}\theta_0 \\ (\Sigma_\epsilon^{-0.5}\otimes I_T)y \end{bmatrix} \text{ and } W \equiv \begin{bmatrix} V_0^{-0.5} \\ \Sigma_\epsilon^{-0.5}\otimes X \end{bmatrix}, \tag{9}$$

we get the exponential part in equation (8) that is described as

$$\begin{aligned}\Pi(\theta|y) &\propto exp\{-0.5(\omega-W\theta)'(\omega-W\theta)\} \\ &\propto exp\{-0.5(\theta-\overline{\theta})'W'W(\theta-\overline{\theta})+(\omega-W\overline{\theta})'(\omega-W\overline{\theta})\}\end{aligned} \tag{10}$$

where the mean value of the posterior, $\overline{\theta}$, has the specification:

$$\overline{\theta} = (W'W)^{-1}W'\omega = \left[V_0^{-1} + (\Sigma_\epsilon^{-1} \otimes X'X)\right]^{-1}\left[V_0^{-1}\theta_0 + (\Sigma_\epsilon^{-1} \otimes X)'y\right]. \tag{11}$$

If the Σ_ε is known, the $\overline{\theta}$ in equation (10) does not include randomness. In this context, the posterior distribution may be simply described by the following form:

$$\Pi(\theta \mid y) \propto exp\left\{-0.5(\theta - \overline{\theta})'W'W(\theta - \overline{\theta})\right\} = exp\left\{-0.5(\theta - \overline{\theta})'\overline{V}^{-1}(\theta - \overline{\theta})\right\} \tag{12}$$

where the covariance of posterior, $\overline{V}$, is

$$\overline{V} = \left[V_0^{-1} + (\Sigma_\epsilon^{-1} \otimes X'X)\right]^{-1}. \tag{13}$$

In the flamework of Bayesian econometrics, the prior distribution function of the parameter for estimation is constituted based on the "belief" of the researcher to reflect the prior information. One of the critical problems is whether the prior distribution function and the posterior are in the same distribution family. If they are in the same distribution family, then we can conduct the simple analytical estimation process of the Bayesian VAR with some conjugate priors. If not, we should conduct some kinds of simulation-based inference like the Markov Chain Monte Charlo (MCMC) method, the Gibbs Sampling, and so on.[4] In the case of this study dealing with tourism behavior, the research might be in the former one, and we could select some applicable priors, for instance, the so-called "Minnesota (Litterman) prior," "Normal-Wishart Prior," "Sims-Zha normal-Wishart prior," and "Sims-Zha normal-flat prior." In this study, the "Minnesota (Litterman) prior" which assumes a normal prior on θ with fixed Σ_ε in the VAR process is applied. This prior was initially developed by, for instance, Litterman (1986) and Doan, Litterman, and Sims (1984). "Minnesota (Litterman) prior" treats Σ_ε as fixed or known factor. Therefore, Σ_ε should be replaced by the estimated $\widehat{\Sigma_\varepsilon}$. In general, there are three options for estimating $\widehat{\Sigma_\varepsilon}$ − (1) Univariate AR estimate, (2) Diagonal VAR estimate, (3) Full VAR estimate. In this sense, we should specify a prior for θ because we should estimate $\widehat{\Sigma_\varepsilon}$. Usually, the Minnesota (Litterman) prior assumes that $\theta \sim N(\theta_0, V_0)$. The hyper-parameter $\mu_1 = 0$ derives $\theta_0 = 0$ (a zero-mean model). But the prior covariance should not be zero, $V_0 \neq 0$. The $\theta_0 = 0$ case could lessen the risk of over-fitting.

The explanatory variables in any equation of the VAR model are divided into own lags of dependent variable, lags of the other dependent variables, and any exogenous variables, including constant term. The factors of V_0 for exogenous variables are set to infinity. It means that no information of the exogenous variables is contained in the prior. The remainder of V_0 is a diagonal matrix that includes diagonal elements v^l_{ij} for $l = 1, \cdots, p$:

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4) See Chan, Koop, Poirier, and Tobias (2019) for details of the Markov Chain Monte Charlo (MCMC) method, and the Gibbs Sampling.

$$\upsilon_{ij}^{l} = \begin{cases} \left(\dfrac{\lambda_1}{l^{\lambda_3}}\right)^2 & \text{for}\,(i=j) \\ \left(\dfrac{\lambda_1\lambda_2\sigma_i}{l^{\lambda_3}\sigma_j}\right)^2 & \text{for}\,(i\neq j) \end{cases} \tag{14}$$

where σ_i and σ_j are the square roots of the corresponding diagoral elements of Σ_ε, respectively. This way of setting prior simplifies selection of all the elements of V_0 down to the choice of three scalars, the hyper-parameters, λ_1, λ_2, and λ_3. The λ_1 is overall tightness while the λ_2 is relative cross-variable weight. The λ_3 is for the lag decay and coefficients are increasingly shrunk toward zero as lag length increases. These hyper-parameter scalar values may lead to smaller (or larger) variances of coefficients − tightening (or loosening) the prior. The setting of the values for these scalars depends on the empirical estimation[5] in order that researchers can make a trial with different values.

After the selection of prior, the posterior for θ takes the following specification:

$$\theta \sim N(\overline{\theta}, \overline{V}) \tag{15}$$

where

$$\overline{V} = \left[V_0^{-1} + (\widehat{\Sigma_\epsilon^{-1}} \otimes X'X)\right]^{-1} \tag{16}$$

and

$$\overline{\theta} = \overline{V}\left[V_0^{-1}\theta_0 + (\widehat{\Sigma_\epsilon^{-1}} \otimes X)'y\right]. \tag{17}$$

3. The Data

This section describes the data set for the empirical analysis utilizing the BVAR methodology to investigate the influence of economic variables on tourism demand for two major prefectures in Japan's western region − Kyoto and Osaka −. Our estimations are conducted with monthly data spanning the period: January 2013 to March 2020. The endpoint of our sample period (March 2020) was decided in order the we may apply our analysis to the period before the COVID-19 pandemic: coronavirus crisis.[6] Our data set is constructed by the following variables.[7]

5) Litterman (1986) proposes the other type of explanation for this problem.

6) The first COVID-19 state of emergency declaration for Osaka prefecture started in April 2020.

7) The data on "approximate total number of overnight guests" can be retrieved from the website of the Japan Tourism Agency, Ministry of Land, Infrastructure, Transport, and Tourism (https://www.mlit.go.jp/kankocho/siryou/toukei/shukuhakutoukei.html). The "heavy fuel oil for industry, type A (for heavy loaded lorry), regional basis (Kinki area)" is obtained from the Agency for Natural Resources and Energy's website (https://www.enecho.meti.go.jp/statistics/petroleum_and_lpgas/

V: approximate total number of overnight guests; prefectural data (Kyoto, Osaka), monthly, data listed in Table 9 in result of the survey "Overnight Travel Statistics," issued by the Japan Tourism Agency, Ministry of Land, Infrastructure, Transport, and Tourism.

C: heavy fuel oil for industry, type A (for heavy loaded lorry), regional basis (Kinki region), monthly, unit: Yen / Litter, excluding consumption tax, in result of the "Petroleum Products Price Survey" issued by the Agency for Natural Resources and Energy - Ministry of Economy, Trade and Industry.

I: indices of industrial production, prefectural data (Kyoto, Osaka), monthly, original index, manufacturing (Item Number: 20000000 for Kyoto and 21000011 for Osaka), base year: 2015, issued by each prefectural government office and the Ministry of Economy, Trade, and Industry.

P: consumer price index, city-level (municipality) data (city of Kyoto and city of Osaka), monthly, original index, all items, base year: 2020, issued by the Ministry of Internal Affairs and Communications.

Our empirical analysis focuses on the two major prefectures in Japan's western region, concretely, prefectures of Kyoto and Osaka. The variables "*V*" and "*I*" are the prefectural-level data that were observed by each local governmental office. "*P*" in our estimation is the city-level (municipality) data as the proxy variable for the prefectural one on consumer price index. The reason of this treatment is the fact that prefectural data on "*P*" is not available. Similarly, because prefectural data on the heavy fuel oil for industry is not available, regional basis data for Kinki area[8)] ("*C*") is adopted. "*V*" and "*C*" work as the proxy variables for tourism demand and transportation cost of the tourism, respectively. "*I*" and "*P*" are the proxy variables for vitality of regional economy and regional price level, respectively. In this study, Logarithmic transformation (natural logarithm) is performed on all the variables listed above.

4. Empirical Result

This section is constructed to investigate the tourism demand for Japan's two major prefectures in the Kansai region, namely, Kyoto and Osaka. The empirical estimations are conducted by utilizing BVAR (Bayesian vector autoregression) model based on the Minnesota (Litterman) prior with the variables explained in the former section. The

pl007/results.html). The "consumer price index" is available from the "e-stat" website (https://www.e-stat.go.jp/stat-search/files?page=1&toukei=00200573). The data on "Indices of Industrial Production (prefectural data)" can be retrieved from the website of the Ministry of Economy, Trade, and Industry (https://www.meti.go.jp/statistics/tyo/iip/chiiki/index.html).

8) It includes prefectures of Kyoto, Osaka, Nara, Shiga, Hyogo, Wakayama, and Fukui.

following specifications are applied. ("*ln*" means the natural logarithm)

$$
\begin{aligned}
lnV_t = {} & \alpha_{1,1} lnV_{t-1} + \alpha_{1,2} lnV_{t-2} + \cdots + \alpha_{1,12} lnV_{t-12} \\
& + \alpha_{2,1} lnC_{t-1} + \alpha_{2,2} lnC_{t-2} + \cdots + \alpha_{2,12} lnC_{t-12} \\
& + \alpha_{3,1} lnI_{t-1} + \alpha_{3,2} lnI_{t-2} + \cdots + \alpha_{3,12} lnI_{t-12} \\
& + \alpha_{4,1} lnP_{t-1} + \alpha_{4,2} lnP_{t-2} + \cdots + \alpha_{4,12} lnP_{t-12} \\
& + c_1 + \varepsilon_{1t}
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
lnC_t = {} & \alpha_{1,1} lnV_{t-1} + \alpha_{1,2} lnV_{t-2} + \cdots + \alpha_{1,12} lnV_{t-12} \\
& + \alpha_{2,1} lnC_{t-1} + \alpha_{2,2} lnC_{t-2} + \cdots + \alpha_{2,12} lnC_{t-12} \\
& + \alpha_{3,1} lnI_{t-1} + \alpha_{3,2} lnI_{t-2} + \cdots + \alpha_{3,12} lnI_{t-12} \\
& + \alpha_{4,1} lnP_{t-1} + \alpha_{4,2} lnP_{t-2} + \cdots + \alpha_{4,12} lnP_{t-12} \\
& + c_2 + \varepsilon_{2t}
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
lnI_t = {} & \alpha_{1,1} lnV_{t-1} + \alpha_{1,2} lnV_{t-2} + \cdots + \alpha_{1,12} lnV_{t-12} \\
& + \alpha_{2,1} lnC_{t-1} + \alpha_{2,2} lnC_{t-2} + \cdots + \alpha_{2,12} lnC_{t-12} \\
& + \alpha_{3,1} lnI_{t-1} + \alpha_{3,2} lnI_{t-2} + \cdots + \alpha_{3,12} lnI_{t-12} \\
& + \alpha_{4,1} lnP_{t-1} + \alpha_{4,2} lnP_{t-2} + \cdots + \alpha_{4,12} lnP_{t-12} \\
& + c_3 + \varepsilon_{3t}
\end{aligned} \tag{20}
$$

$$
\begin{aligned}
lnP_t = {} & \alpha_{1,1} lnV_{t-1} + \alpha_{1,2} lnV_{t-2} + \cdots + \alpha_{1,12} lnV_{t-12} \\
& + \alpha_{2,1} lnC_{t-1} + \alpha_{2,2} lnC_{t-2} + \cdots + \alpha_{2,12} lnC_{t-12} \\
& + \alpha_{3,1} lnI_{t-1} + \alpha_{3,2} lnI_{t-2} + \cdots + \alpha_{3,12} lnI_{t-12} \\
& + \alpha_{4,1} lnP_{t-1} + \alpha_{4,2} lnP_{t-2} + \cdots + \alpha_{4,12} lnP_{t-12} \\
& + c_4 + \varepsilon_{4t}
\end{aligned} \tag{21}
$$

Considering the fact that we use the monthly data set, the lag length for each estimation is decided as 12. With respect to the BVAR process with Minnesota (Litterman) prior, some conditions explained in section 2 should be settled before estimation. First, initial residual covariance matrix is obtained by the full-VAR estimation in this study. Second, the hyper-parameters, μ_1, λ_1, λ_2, and λ_3 have to be determined as scalar values. The μ_1, the AR(1) coefficient, is set as 0 in our estimation describing the zero-mean model. The λ_1 is the overall tightness on the variance of the first lag, and this parameter expresses the relative importance of sample and prior information. In our estimation, it is set as $\lambda_1 = 0.1$. The λ_2 represents relative tightness of the variance of other variables, and we set it as $\lambda_2 = 0.99$. The λ_3 (> 0) shows the relative tightness of the variance of lags, and it is set as $\lambda_3 = 1$.

As is widely known, VAR-type analysis provides impulse response for grasping the marginal effects of the assumed shock to one of the variables on the current and future levels of other endogenous variables.[9] With that in mind, the analysis by utilizing

9) See Koop (2013) for details.

Figure 1 Cumulative Impulse Responses of Kyoto by utilizing BVAR model

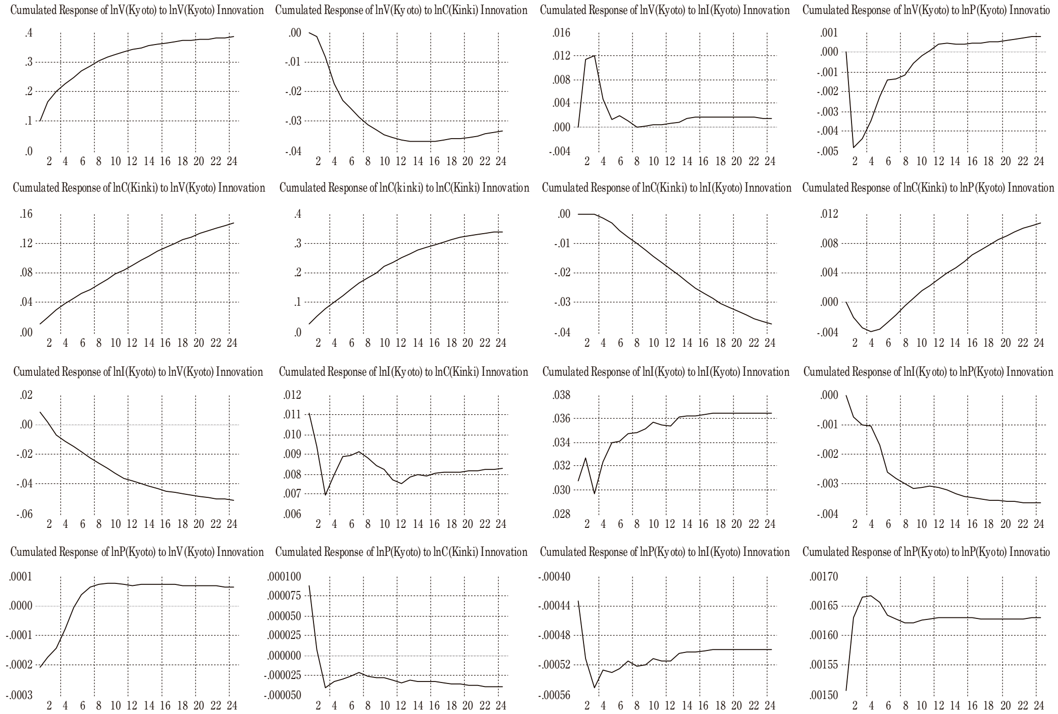
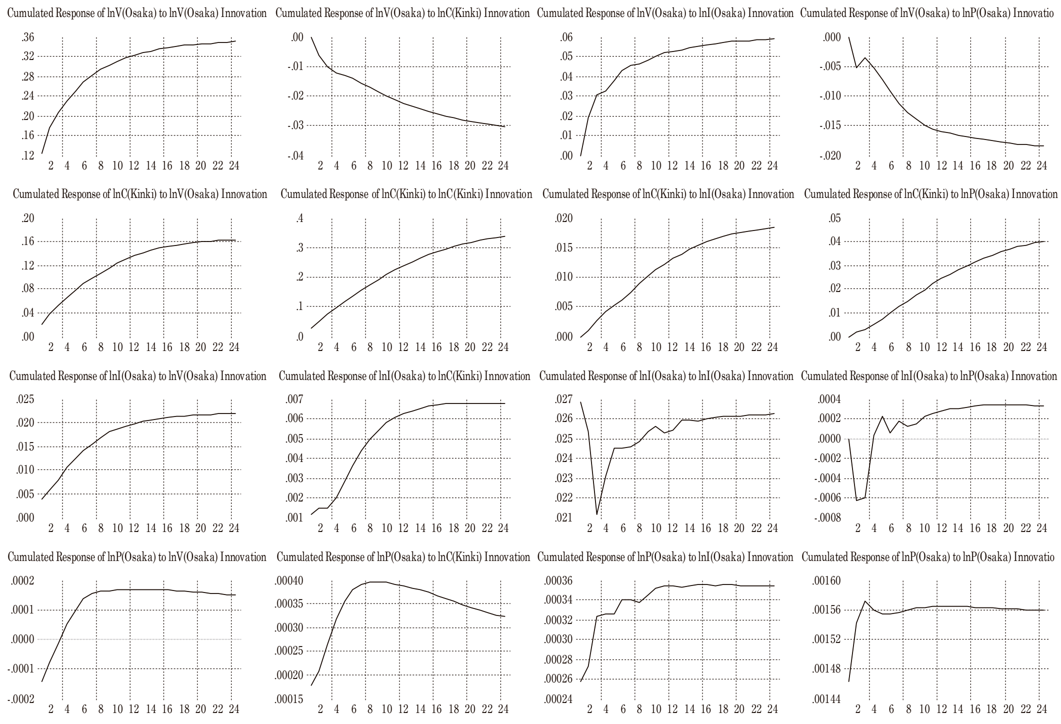


Figure 2 Cumulative Impulse Responses of Osaka by utilizing BVAR model



impulse response on one standard deviation shock with Cholesky decomposition and degrees of freedom correction is conducted.

Figure 1 indicates the estimated cumulative impulse responses of Kyoto. (The estimated coefficients of the specification described by the equations (18), (19), (20), and (21) for Kyoto are reported in the Appendix 1) Our cumulative impulse responses derived by the Bayesian vector autoregression estimation based on the Minnesota (Litterman) prior capture the marginal effects for 24 months after the one-time shock considering one of the variables on the present and future values of other endogenous variables. The most important result to investigate the influence of economic variables on tourism demand is displayed in the first row. Considering the shock to "C" (transportation cost of the tourism) described in the second column of the first row, it can be seen that the response of "V" (tourism demand) is consistent with the usual assumption, that is, a rise in transportation cost is followed by a decline in tourism demand in the long term. The response of tourism demand toward the shock to "I" (vitality of regional economy) indicated in the third column of the first row is positive just after the shock but it soon takes a turn to negative. It might show that the positive effect of vitality of regional economy on tourism demand is not always persistent. On the other hand, the shock to "P" (regional price level) displayed in the fourth column is followed by negative response of tourism initially. However, the cumulative response turns into positive shortly. In line with this result, the impact of regional price level on tourism is limited in Kyoto. As a whole, these impulse responses of Kyoto to the shocks on economic variables are virtually consistent with the conventional belief.

Figure 2 displays the cumulative impulse responses of Osaka (The estimated coefficients of the specification described by the equations (18), (19), (20), and (21) for Osaka are reported in the Appendix 2) The responses do not always indicate the same patterns as Kyoto. With respect to the first row, the cumulative impulse response derived by a shock to transportation cost coincides with the usual assumption. In short, a positive shock to transportation cost is followed by a negative response of tourism demand. The impulse response toward a shock to vitality of regional economy is within expectations since it gets sustained positive response of tourism. Now let us turn our eyes to the response to price level. A shock to regional price level brings the understandable result. Namely, a rise in price level derives the negative cumulative response of tourism demand. Overall, the estimated impulse responses of tourism demand in Osaka derived by several economic shocks seem to be very realistic.

5. Concluding Remarks

The Bayesian Vector Autoregression (BVAR) model has been regarded as the Bayesian-flavored VAR specification for empirical analysis, which connects priors with information incorporated in the data set. The BVAR can decrease the risk of over-parameterization by the imposition of special restrictions on the parameters in the VAR process through their prior probability distribution functions. In this research, we utilize one of the conjugate priors, Minnesota (Litterman) prior, in the BVAR estimation assuming that the prior and the posterior distribution functions are in the same distribution family. Taking these factors into account, the empirical analysis to investigate the influence of economic variables on tourism demand for Kyoto and Osaka as the two major prefectures in Japan's western region is conducted in this study. The endpoint of our sample period (March 2020) was decided in order that we may apply our analysis to the period before the COVID-19 pandemic: coronavirus crisis.

With respect to the estimated cumulative impulse response of Kyoto, the response of tourism demand toward the shock to "C" (transportation cost) is consistent with the usual assumption. Namely, a rise in transportation cost is followed by a decline in tourism demand in the long term. The response of tourism demand toward the shock to "I" (vitality of regional economy) indicated in the third column of the first row is positive just after the shock but it soon takes a turn to negative. It might show that the positive effect of vitality of regional economy on tourism demand is not always persistent. On the other hand, the shock to "P" (regional price level) displayed in the fourth column is followed by negative response of tourism initially. However, the cumulative response turns into positive shortly. In line with this result, the impact of regional price level on tourism is limited in Kyoto. As a whole, these impulse responses of Kyoto to the shocks on economic variables are virtually consistent with the conventional belief.

As for Osaka, the responses do not always indicate the same patterns as Kyoto. With respect to the first row, the cumulative impulse response derived by a shock in transportation cost coincides with the usual assumption. Namely, a positive shock in transportation cost is followed by a negative response of tourism demand. The impulse response toward a shock to vitality of regional economy is within expectations since it gets sustained positive response of tourism. In addition, a shock to regional price level brings the understandable result. In other words, a rise in price level derives the negative cumulative response of tourism demand. Concerning these results, the estimated impulse responses of tourism demand in Osaka to the several economic shocks seem to be very realistic.

On the whole, the impulse responses of Kyoto and Osaka do not always show the same patterns. To be specific, the impulse responses of Osaka seem to be realistic, or the ones of Kyoto imply a kind of vulnerability of regional economy.

Appendix 1: estimated coefficients by the Bayesian VAR Analysis with the Minnesota (Litterman) prior for Kyoto

Bayesian VAR Estimates, Sample (adjusted) : 2014M01 2020M03
Included observations: 75 after adjustments
Prior type: Minnesota (Litterman), Initial residual covariance: Full VAR
Constant included in covariance calculation
Hyper-parameters: $\mu_1 = 0$, $\lambda_1 = 0.1$, $\lambda_2 = 0.99$, $\lambda_3 = 1$, Standard errors in ( )

	lnV (Kyoto)	*lnC* (Kinki)	*lnI* (Kyoto)	*lnP* (Kyoto)
lnV (Kyoto) (-1)	0.597146	0.002296	-0.063351	0.000890
	(0.05377)	(0.01554)	(0.01763)	(0.00084)
lnV (Kyoto) (-2)	0.047592	0.000749	-0.032608	-0.000129
	(0.04208)	(0.01213)	(0.01376)	(0.00065)
lnV (Kyoto) (-3)	0.034645	-0.006893	-0.006878	0.000136
	(0.03015)	(0.00870)	(0.00987)	(0.00047)
lnV (Kyoto) (-4)	0.023339	-0.008018	0.002837	0.000218
	(0.02337)	(0.00675)	(0.00766)	(0.00036)
lnV (Kyoto) (-5)	0.018374	0.000572	-0.011411	-0.000113
	(0.01910)	(0.00552)	(0.00626)	(0.00030)
lnV (Kyoto) (-6)	0.005688	0.002854	-0.005762	-0.000123
	(0.01618)	(0.00468)	(0.00531)	(0.00025)
lnV (Kyoto) (-7)	0.004408	0.001287	-0.001377	-3.24E-05
	(0.01397)	(0.00404)	(0.00458)	(0.00022)
lnV (Kyoto) (-8)	-0.000915	0.001886	-0.004745	-8.18E-05
	(0.01228)	(0.00355)	(0.00403)	(0.00019)
lnV (Kyoto) (-9)	0.000561	0.001940	-0.006224	-7.36E-05
	(0.01097)	(0.00317)	(0.00360)	(0.00017)
lnV (Kyoto) (-10)	0.000433	0.001151	-0.002987	-3.40E-05
	(0.00992)	(0.00287)	(0.00326)	(0.00015)
lnV (Kyoto) (-11)	0.003163	0.000192	-0.000181	-5.13E-06
	(0.00902)	(0.00261)	(0.00296)	(0.00014)
lnV (Kyoto) (-12)	0.005306	-1.78E-05	-0.000464	1.36E-05
	(0.00828)	(0.00240)	(0.00272)	(0.00013)
lnC (Kinki) (-1)	-0.173019	0.927768	-0.082092	-0.002592
	(0.15202)	(0.04438)	(0.05005)	(0.00238)
lnC (Kinki) (-2)	0.023942	0.030193	0.023984	0.001245
	(0.14536)	(0.04270)	(0.04803)	(0.00227)
lnC (Kinki) (-3)	0.059163	-0.008868	0.025314	0.001301
	(0.10110)	(0.02974)	(0.03344)	(0.00158)
lnC (Kinki) (-4)	0.044885	-0.004894	0.018398	0.000335
	(0.07745)	(0.02281)	(0.02564)	(0.00121)
lnC (Kinki) (-5)	0.028233	0.002969	-0.001005	-7.67E-05
	(0.06279)	(0.01850)	(0.02079)	(0.00098)
lnC (Kinki) (-6)	0.003141	0.006165	-0.009827	-0.000141
	(0.05280)	(0.01556)	(0.01749)	(0.00082)
lnC (Kinki) (-7)	-0.001081	0.002340	-0.009050	-0.000113
	(0.04555)	(0.01343)	(0.01509)	(0.00071)
lnC (Kinki) (-8)	0.003006	-0.002671	-0.005469	-0.000131
	(0.04005)	(0.01181)	(0.01327)	(0.00062)
lnC (Kinki) (-9)	0.002216	-0.006307	-6.03E-05	-5.34E-05
	(0.03570)	(0.01053)	(0.01183)	(0.00056)
lnC (Kinki) (-10)	-0.001846	-0.009080	0.003626	-2.83E-05
	(0.03217)	(0.00949)	(0.01066)	(0.00050)

lnC (Kinki) (-11)	-0.002271	-0.009960	0.006973	-4.54E-05
	(0.02928)	(0.00863)	(0.00970)	(0.00046)
lnC (Kinki) (-12)	-0.003385	-0.009867	0.008032	-6.71E-06
	(0.02686)	(0.00791)	(0.00890)	(0.00042)
lnI (Kyoto) (-1)	0.326317	-0.030891	0.056451	-0.001379
	(0.15492)	(0.04495)	(0.05110)	(0.00243)
lnI (Kyoto) (-2)	-0.202858	0.016257	-0.089052	-0.001177
	(0.11520)	(0.03333)	(0.03803)	(0.00181)
lnI (Kyoto) (-3)	-0.209957	-0.035440	0.109428	0.000842
	(0.08830)	(0.02563)	(0.02931)	(0.00138)
lnI (Kyoto) (-4)	-0.018619	-0.006157	0.010418	-0.000188
	(0.07029)	(0.02047)	(0.02343)	(0.00110)
lnI (Kyoto) (-5)	0.070400	-0.010834	-0.007330	0.000145
	(0.05714)	(0.01666)	(0.01907)	(0.00089)
lnI (Kyoto) (-6)	-0.020833	-0.003276	0.010459	0.000174
	(0.04813)	(0.01404)	(0.01608)	(0.00075)
lnI (Kyoto) (-7)	-0.008759	0.004812	-0.008238	-0.000221
	(0.04155)	(0.01213)	(0.01389)	(0.00065)
lnI (Kyoto) (-8)	0.037554	-0.006888	0.006577	0.000221
	(0.03656)	(0.01067)	(0.01223)	(0.00057)
lnI (Kyoto) (-9)	0.001021	-0.006079	0.013035	0.000237
	(0.03266)	(0.00954)	(0.01093)	(0.00051)
lnI (Kyoto) (-10)	0.002382	0.000416	-0.008601	-5.47E-05
	(0.02949)	(0.00862)	(0.00987)	(0.00046)
lnI (Kyoto) (-11)	0.012004	-0.000782	-0.002401	3.30E-05
	(0.02687)	(0.00785)	(0.00900)	(0.00042)
lnI (Kyoto) (-12)	0.004252	-0.005114	0.014010	0.000224
	(0.02465)	(0.00721)	(0.00826)	(0.00038)
lnP (Kyoto) (-1)	-3.252331	-1.384433	-0.527807	0.081460
	(4.83160)	(1.39660)	(1.58431)	(0.07640)
lnP (Kyoto) (-2)	2.419390	0.518243	-0.421956	0.007641
	(2.91958)	(0.84804)	(0.96234)	(0.04604)
lnP (Kyoto) (-3)	0.292697	0.492369	-0.123665	-0.002327
	(2.03712)	(0.59366)	(0.67360)	(0.03208)
lnP (Kyoto) (-4)	0.356395	0.378201	-0.304550	-0.008316
	(1.55386)	(0.45350)	(0.51453)	(0.02445)
lnP (Kyoto) (-5)	0.308885	0.345895	-0.380019	-0.010740
	(1.25340)	(0.36611)	(0.41536)	(0.01972)
lnP (Kyoto) (-6)	-0.065810	0.154472	-0.025624	-0.004708
	(1.05014)	(0.30693)	(0.34820)	(0.01652)
lnP (Kyoto) (-7)	0.097613	0.029392	-0.041605	-0.002308
	(0.90258)	(0.26388)	(0.29936)	(0.01420)
lnP (Kyoto) (-8)	0.193690	-0.002060	-0.053601	-0.001120
	(0.79089)	(0.23125)	(0.26234)	(0.01244)
lnP (Kyoto) (-9)	-0.007976	-0.075103	0.071478	0.001060
	(0.70368)	(0.20577)	(0.23343)	(0.01107)
lnP (Kyoto) (-10)	-0.031552	-0.041994	0.031380	-0.000206
	(0.63377)	(0.18534)	(0.21026)	(0.00997)
lnP (Kyoto) (-11)	0.021443	-0.013066	0.033423	0.000947
	(0.57649)	(0.16861)	(0.19127)	(0.00907)
lnP (Kyoto) (-12)	-0.085254	0.021975	-0.009478	-0.000451
	(0.52872)	(0.15464)	(0.17543)	(0.00831)
Const.	0.491547	-4.971670	29.73905	12.67748

	(88.0815)	(25.5445)	(28.9806)	(1.39054)
R-squared	0.675759	0.962132	0.382008	0.126336
Adj. R-squared	0.077162	0.892223	-0.758900	-1.486582
Sum sq. resids	1.422155	0.111169	0.240149	0.000162
S.E. equation	0.233877	0.065389	0.096107	0.002496
F-statistic	1.128904	13.76255	0.334828	0.078328
Mean dependent	14.32133	4.178442	4.564679	13.50334
S.D. dependent	0.243458	0.199179	0.072466	0.001583

Appendix 2: estimated coefficients by the Bayesian VAR Analysis with the Minnesota (Litterman) prior for Osaka

Bayesian VAR Estimates, Sample (adjusted) : 2014M01 2020M03
Included observations: 75 after adjustments
Prior type: Minnesota (Litterman), Initial residual covariance: Full VAR
Constant included in covariance calculation
Hyper-parameters: $\mu_1=0$, $\lambda_1=0.1$, $\lambda_2=0.99$, $\lambda_3=1$, Standard errors in ( )

	lnV (Kyoto)	*lnC* (Kinki)	*lnI* (Kyoto)	*lnP* (Kyoto)
lnV (Osaka) (-1)	0.438187	-0.018721	0.012757	0.000470
	(0.06751)	(0.01861)	(0.01524)	(0.00084)
lnV (Osaka) (-2)	0.052475	0.000817	0.017348	-5.24E-05
	(0.04457)	(0.01225)	(0.00992)	(0.00055)
lnV (Osaka) (-3)	0.040312	-0.002508	0.009139	7.01E-05
	(0.03159)	(0.00867)	(0.00697)	(0.00039)
lnV (Osaka) (-4)	0.033193	-0.005425	0.004364	6.13E-05
	(0.02425)	(0.00665)	(0.00533)	(0.00030)
lnV (Osaka) (-5)	0.018575	-0.002473	-0.000467	5.89E-05
	(0.01964)	(0.00539)	(0.00431)	(0.00024)
lnV (Osaka) (-6)	0.002926	0.000629	0.001589	-5.87E-06
	(0.01646)	(0.00451)	(0.00361)	(0.00020)
lnV (Osaka) (-7)	0.004591	-0.000192	0.002842	-9.19E-06
	(0.01412)	(0.00387)	(0.00309)	(0.00017)
lnV (Osaka) (-8)	0.004091	-0.000376	0.001255	-1.67E-07
	(0.01241)	(0.00340)	(0.00272)	(0.00015)
lnV (Osaka) (-9)	0.003630	-0.000361	8.20E-05	-1.17E-07
	(0.01105)	(0.00303)	(0.00242)	(0.00013)
lnV (Osaka) (-10)	0.002624	-0.000222	9.58E-05	1.01E-05
	(0.00996)	(0.00273)	(0.00218)	(0.00012)
lnV (Osaka) (-11)	0.002029	-0.000192	0.000928	-9.44E-07
	(0.00906)	(0.00248)	(0.00198)	(0.00011)
lnV (Osaka) (-12)	0.003542	-0.000478	0.000371	6.96E-06
	(0.00831)	(0.00228)	(0.00182)	(0.00010)
lnC (KINKI) (-1)	-0.241199	0.905541	0.015705	0.000792
	(0.16305)	(0.04549)	(0.03728)	(0.00207)
lnC (Kinki) (-2)	0.152196	0.033445	-0.002713	0.001190
	(0.15315)	(0.04283)	(0.03441)	(0.00191)
lnC (Kinki) (-3)	0.074217	-0.006139	0.013225	0.000217
	(0.10678)	(0.02986)	(0.02390)	(0.00132)
lnC (Kinki) (-4)	0.001752	0.001825	0.013387	-0.000519
	(0.08180)	(0.02288)	(0.01826)	(0.00101)
lnC (Kinki) (-5)	-0.020122	0.006247	0.005664	-0.000506
	(0.06642)	(0.01858)	(0.01479)	(0.00082)

lnC (Kinki) (-6)	-0.021806	0.006045	-0.002476	-0.000337
	(0.05591)	(0.01564)	(0.01244)	(0.00069)
lnC (Kinki) (-7)	-0.006399	0.000494	-0.004153	-0.000240
	(0.04824)	(0.01349)	(0.01073)	(0.00059)
lnC (Kinki) (-8)	0.004683	-0.004101	-0.003929	-0.000192
	(0.04242)	(0.01186)	(0.00943)	(0.00052)
lnC (Kinki) (-9)	0.004879	-0.005669	-0.003478	-0.000136
	(0.03782)	(0.01058)	(0.00840)	(0.00047)
lnC (Kinki) (-10)	-0.000508	-0.006389	-0.003784	-5.33E-05
	(0.03409)	(0.00954)	(0.00758)	(0.00042)
lnC (Kinki) (-11)	0.000823	-0.006759	-0.002982	-1.10E-05
	(0.03102)	(0.00868)	(0.00690)	(0.00038)
lnC (Kinki) (-12)	0.006249	-0.007555	-0.003063	4.46E-05
	(0.02843)	(0.00795)	(0.00633)	(0.00035)
lnI (Osaka) (-1)	0.750429	0.025257	-0.050430	2.94E-05
	(0.24397)	(0.06769)	(0.05616)	(0.00310)
lnI (Osaka) (-2)	0.133790	0.049724	-0.171939	0.001321
	(0.17494)	(0.04855)	(0.04060)	(0.00224)
lnI (Osaka) (-3)	0.007412	-0.000791	0.033902	-5.24E-05
	(0.13262)	(0.03678)	(0.03036)	(0.00167)
lnI (Osaka) (-4)	0.066037	-0.003037	0.011873	8.06E-06
	(0.10372)	(0.02876)	(0.02358)	(0.00129)
lnI (Osaka) (-5)	0.037014	0.001251	0.009253	0.000216
	(0.08490)	(0.02353)	(0.01922)	(0.00105)
lnI (Osaka) (-6)	-0.032618	0.015752	6.56E-05	-0.000249
	(0.07208)	(0.01997)	(0.01623)	(0.00089)
lnI (Osaka) (-7)	-0.025837	0.011455	-0.002699	-0.000174
	(0.06225)	(0.01725)	(0.01400)	(0.00077)
lnI (Osaka) (-8)	0.004642	-0.000468	0.009189	0.000132
	(0.05482)	(0.01519)	(0.01230)	(0.00067)
lnI (Osaka) (-9)	0.013525	-0.003388	0.004202	0.000117
	(0.04902)	(0.01358)	(0.01098)	(0.00060)
lnI (Osaka) (-10)	0.010943	-0.000593	-0.010494	6.47E-05
	(0.04425)	(0.01226)	(0.00991)	(0.00054)
lnI (Osaka) (-11)	-0.009462	0.004177	-0.001343	-1.52E-05
	(0.04040)	(0.01119)	(0.00903)	(0.00050)
lnI (Osaka) (-12)	-0.000685	0.000151	0.008387	-7.22E-05
	(0.03702)	(0.01026)	(0.00828)	(0.00045)
lnP (Osaka) (-1)	-3.672096	1.162770	-0.455631	0.052692
	(6.13146)	(1.70272)	(1.42261)	(0.07937)
lnP (Osaka) (-2)	3.640140	-0.192451	0.060482	0.011723
	(3.68225)	(1.02140)	(0.83545)	(0.04667)
lnP (Osaka) (-3)	-1.621591	0.348811	0.402642	-0.012297
	(2.58228)	(0.71577)	(0.57832)	(0.03232)
lnP (Osaka) (-4)	-0.977427	0.333453	0.143537	-0.006235
	(1.97440)	(0.54709)	(0.43955)	(0.02457)
lnP (Osaka) (-5)	-0.515222	0.170686	-0.059576	-0.001543
	(1.59281)	(0.44128)	(0.35365)	(0.01977)
lnP (Osaka) (-6)	-0.513182	0.126149	0.078799	-0.002261
	(1.33400)	(0.36955)	(0.29570)	(0.01653)
lnP (Osaka) (-7)	-0.098064	0.038960	-0.053806	0.000849
	(1.14740)	(0.31783)	(0.25404)	(0.01420)
lnP (Osaka) (-8)	-0.147741	0.013096	0.027426	-0.000341

	(1.00605)	(0.27867)	(0.22259)	(0.01245)
lnP (Osaka) (-9)	-0.035600	-0.010950	0.032993	-4.03E-05
	(0.89512)	(0.24794)	(0.19798)	(0.01107)
lnP (Osaka) (-10)	0.065149	-0.009490	0.000286	0.000704
	(0.80650)	(0.22339)	(0.17832)	(0.00997)
lnP (Osaka) (-11)	0.062228	-0.017205	0.010374	-0.000479
	(0.73381)	(0.20325)	(0.16219)	(0.00907)
lnP (Osaka) (-12)	0.163739	-0.028537	-0.001834	-5.04E-05
	(0.67298)	(0.18640)	(0.14872)	(0.00832)
Const.	50.90874	-25.81226	2.022877	12.91044
	(112.291)	(31.1596)	(25.6631)	(1.43277)
R-squared	0.620805	0.957116	0.278235	0.125842
Adj. R-squared	-0.079247	0.877946	-1.054253	-1.487989
Sum sq. resids	1.306113	0.125896	0.134078	0.000123
S.E. equation	0.224132	0.069586	0.071811	0.002171
F-statistic	0.886799	12.08931	0.208809	0.077977
Mean dependent	14.86551	4.178442	4.626779	13.50354
S.D. dependent	0.215746	0.199179	0.050103	0.001376

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