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An Oligopoly Model for Market Performance Analysis With an Application to Electricity Market

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Abstract

This paper introduces an oligopoly model for spot prices in non-storable commodity markets under uncertainty. The derived equilibrium spot price captures the effect of market concentration and suppliers' risk attitude on a mark-up. We also propose a simple empirical method to assess mark-ups with the price formulae, and directly apply it to California electricity market as an example. The observed mark-ups are found to depend on both market power and risk premium. The result suggests that the proposed approach can be an effective means of analysis on market performance, i.e. measuring mark-ups and finding the constituent factors.

Keywords: Market Power, Risk Premium, Mark-up, Demand Uncertainty, Strategic Behavior.

JEL Classification: D81, L11, L13, L94

1 Introduction

The main purpose of this study is to investigate mark-ups in non-storable commodity markets under uncertainty. Thus it is necessary and important to find the factors that generate mark-ups. We decompose observed mark-ups into two factors: exercise of market power and risk premium. To this end, we develop a framework that connects each real market with a modeled market. We first construct a non-cooperative game, which expresses the strategic behavior of each market participant under uncertainty, and then explicitly derive Nash equilibrium spot prices. In fact, the constructed model can be widely applied to non-storable commodity markets. With regard to measuring mark-ups, there exist a number of previous studies that focus on electricity markets. Likewise, we apply our model to an electricity market in this paper. To do so, we demonstrate a procedure to relate the real and modeled markets, and then analyze the effects on mark-ups in the California Power Exchange (PX) market from 1998 to 2000.

During the last decade, many electricity markets have been restructured. Thus, the monitoring of the wholesale electricity market has been important to prevent excessive exercise of market power. The instability of electricity prices, such as spikes and strategic price setting by market participants, has been closely observed, especially in the aftermath of the California power crisis in 2000. Regarding the price fluctuations, as discussed by Stoft[33], electricity as a commodity has special features: non-storability, the requirement for the balancing rule, and inelastic demand. Because of these features and uncertainty, prices are volatile and tend to be high during peak periods; thus, spikes can occur. With respect to price setting, bidding behaviors by each market participant cannot be easily observed. There is also a chance that each firm will bid strategically, especially during peak periods, and mark-ups (that is, price-cost margins) may be introduced.

The direct calculation of mark-ups is a representative way to assess market performance. The Lerner Index (LI) is similar to a mark-up, and represents the proportional price-cost margin. However, preliminary tasks must be conducted to determine these values. Both spot price and marginal cost data are required to calculate a mark-up directly. Although it is relatively easy to acquire spot price data, there are two difficulties in numerically deriving marginal cost. The first difficulty arises in calculating cost. To do so, we need various data including fuel costs, generation capacity of each plant, and information on whether or not each power unit is in operation. Although some data are easily acquired from public authorities, other data are not disclosed. In addition, some fuel prices are volatile, changing daily, and thus it can be difficult to track fluctuations in fuel costs. Therefore, it can be expensive to collect all the necessary data for marginal cost estimation. As a cheaper alternative, proxy variables can be used to estimate marginal costs; however, this can result in less accuracy and may cause measurement errors. The second difficulty arises in determining the increase in total cost for an additional unit of output. Therefore, the incremental cost for each production level is required. In the case of a power-generating firm, tremendous effort is needed to obtain the marginal cost curve for each plant, such as fossil fuel, hydroelectric, and nuclear power plants. Even if we calculate the marginal cost curve, the estimate is based on a strong assumption and is subject to measurement bias. In addition to mark-ups and LI, the Herfindahl Hirshman Index (HHI) can be used to assess the conditions of a market. HHI, which shows the degrees of concentration in the market, is one of the representative measures of market power. Under a Cournot oligopolistic model, LI is an increasing function of HHI.

Focusing on real-world situations in electricity markets, the exercise of market power is monitored in various ways. For example, in the Pennsylvania, New Jersey and Maryland (PJM) market, PJM Interconnection, a regional transmission organization (RTO), has assigned Monitoring Analytics to monitor the market on a quarterly basis. Monitoring Analytics observes the market structure, and investigates the exercised power within the market in terms of supply and demand, concentration ratio (HHI), and mark-ups. Their results are shown in “State of the Market Report for PJM”[1].

There is an abundance of previous research on pricing in wholesale electricity markets. With respect to the determination of wholesale electricity prices and mark-ups, we can divide the studies into two approaches: empirical approaches and structural model approaches.

An empirical approach uses data to measure mark-ups or similar indices, such as LI or HHI, and then evaluates market performance. Wolfram[40] measures mark-ups in the British spot electricity market. She calculates a competitive market-wide supply curve and directly estimates mark-ups. In contrast, numerous other studies (mentioned below) calculate mark-ups using a structural model. Wolfram[40] compares her estimations with the results of previous studies, and points out that previous studies tend to overestimate the marginal cost. Borenstein et al.[6] focus on the California wholesale electricity market. To measure market power, they calculate the marginal cost for each plant and construct a competitive marginal cost curve. They then compare the hourly price of the real market with the estimated competitive price from June 1998 to October 2000. They find that significant market power was exercised in the California wholesale market during the summers of 1998, 1999, and 2000. Joskow and Kahn[22] also compare prices in the California market during the summer months of 2000 with those of the previous two years to measure market power. To build short-run marginal costs as competitive benchmark prices, they employ the last units that the market clears. They find that there was a large gap between their benchmark prices and observed prices, and conclude that suppliers exercised market power during the study period.

Wolak[37] builds a model of a competitive electricity market where each generator strategically bids supply curves of forward contracts and spot sales so as to maximize expected profit. This model is applied to the National Electricity market in Australia in 1997, and the result shows that financial contracts could mitigate market power during an earlier date of operation in the wholesale electricity market. Wolak[38] applies the model to the California market from 1998 to 2000, and finds that the market power exercised in June 2000 could be attributed to an expected profit-maximizing response by the electricity suppliers.

Sweeting[35] measures the degrees of market power in the England and Wales wholesale electricity market from 1995 to 2000 by comparing realized spot prices with the estimated competitive benchmark prices. This research shows that considerable market power was exercised by power generators during the study period. Furthermore, he finds that concentration ratios and fuel prices fell significantly during the period when market power was exercised. Mansur[26] also constructs an industry-wide competitive supply curve for the PJM market during the summers of 1998 and 1999 to measure mark-ups. The results show that two main power generating firms exercised market power by reducing their output. The study also finds that vertical integration mitigated market power in the upstream market, and limited the welfare loss. Mansur[27] takes into consideration production constraints for each year to extend the estimation method used

[1]See the website of PJM Interconnection: http://www.pjm.com/

in Mansur[26]. This model is applied to the PJM market during the summers of 1998 and 1999. The estimated mark-ups are less than those identified using the previous competitive benchmark method. Mansur[27] also calculate welfare loss, and finds that the two representative firms in the PJM market exercised market power during the study period. Although the methods adopted in these studies directly estimate mark-ups or welfare loss in one way or another, they encounter the difficulties mentioned above. The costs incurred to collect all the data necessary and estimate the marginal cost curve are significant, and tremendous effort is required to calculate marginal costs from total costs.

The second line of research is the structural model approach, where evaluating market performance is based on a theoretical model. This approach expresses the behavior of firms explicitly as well as measuring market power. The structural model approach is further classified into three sub-categories according to the model used: the Cournot approach, the supply function equilibrium (SFE) approach, and the auction approach.

The first of these, the Cournot approach, is widely used. This approach assumes that each firm in an oligopolistic market selects its quantity of supply so as to maximize its profit, and that the equilibrium prices are higher than the competitive prices, which is how a firm exercises market power. Borenstein and Bushnell[5] use a Cournot model to simulate prices. They derive marginal cost curves from historical data on plant costs and capacity, and find that market power existed in high demand hours and that the extent of market power depends on the level of available hydroelectric production and the elasticity of demand. The Cournot approach is also used as a benchmark. For example, Wolfram[40] shows that this approach tends to overestimate any mark-ups. Bushnell et al.[7] use a modified Cournot model to examine the effect of vertical integration in US spot electricity markets during the summer of 1999 and find that vertical arrangements prevent rising spot prices and reduce production inefficiency.

The SFE approach was designed by Klemperer and Meyer[23]. The main features of the SFE approach are as follows. First, this approach assumes that the demand curve is stochastic, and thus this approach treats uncertainty. Second, the model assumes that each player offers a supply curve strategically so as to maximize expected profit. Both of these features can be found in the wholesale electricity market. In addition to these features, it includes multiple Nash equilibrium prices, which can range from the Cournot level to the Bertrand or competitive prices. This approach sometimes requires the selection of an appropriate price from the various options.

Green and Newbery[14] is the first to apply an SFE model to the electricity market in England and Wales from 1988 to 1989. Assuming symmetric duopoly firms with identical linear marginal cost functions, they calibrate the supply function and find a high mark-up on marginal cost and substantial welfare loss. Green[12] extends the symmetric duopoly model used in Green and Newbery[14] to an asymmetric model, applying this model to the electricity spot market in England and Wales and examining the case where the dominant power generator reduces its capacity by regulatory policy. They find that a reduction in capacity may reduce the market power of the dominant firm and therefore decrease welfare loss. Rudkevich et al.[31] further develop the SFE model. They relax assumptions regarding the differentiability and convexity conditions of marginal costs, and apply the model to actual 1995 data from the Pennsylvania electric system. Their results show that mark-ups exist even in markets with a relatively large number of firms.

In Wolfram[40], as mentioned above, prices calculated from an SFE model are compared with the benchmark prices. The results show that SFE prices tend to be overestimated. Baldick et al.[3] develop a model with piecewise linear and non-decreasing marginal cost/supply functions under capacity constraints. The authors apply this model to the England and Wales wholesale electricity market. They calibrate supply curves and compare prices before and after the divestitures that were implemented in 1996 and 1999. The results show that their SFE model is more suitable than the previous linear one. Baldick et al.[3] also show that the divestitures of plants by dominant firms drastically reduced prices within their numerical examples. Holmberg[17] applies the SFE approach to a real-time market with constant marginal costs and asymmetric capacity constraints. It is found that, compared with symmetric SFE, asymmetric SFE tends to overestimate mark-ups if the size of the market is small. In contrast, if the size of the market is large, mark-ups are underestimated.

The SFE approach has also been applied to analyze the relationship between spot and long-term contract markets, which are essentially the same as forward markets. Newbery[30] and Green[13] construct analytical models to evaluate contracts. Newbery[30] builds a model to consider the relationships among contracts, variable numbers of competitors, and capacity constraints. The following points are identified. If new plants have the same level of variable costs and incumbents face capacity constraints, an entry will occur. If new plants have lower levels of variable costs, then incumbents will make a capacity-increasing investment to deter entry. Green[13] analyzes the effects of the risk attitudes of market participants. He shows that if buyers are risk averse, then power generators have an incentive to sell in the contract market. In this case, the contact prices, that is, the forward prices, tend to exceed the expected spot price.

In addition, Anderson and Philpott[1] derive the necessary conditions for optimal supply functions in circumstances where both electricity demand and other generators' bidding behavior are unforeseeable. Anderson and Xu[2] use the model to investigate supply function strategies in a duopolistic wholesale electricity market, and show the existence of a symmetric Nash equilibrium.

The third approach is the auction approach. This approach focuses mainly on the rules of auctions and the bidding behavior under such rules, and also addresses the electricity market design problem. von der Fehr and Harboard[36] propose an analytical model for spot electricity markets, which is characterized as a first-price, sealed-bid, multi-unit auction under uncertainty. They apply the model to the England and Wales electricity market. von der Fehr and Harboard[36] examine the bidding behavior of National Power and PowerGen in the British market from 1990 to 1991, and find that both power generators offered higher prices during high demand periods and lower prices during low demand periods. Fabra et al.[11] develop two types of multi-unit auction models that reflect various key features of decentralized electricity markets. One is a uniform auction and the other is a discriminatory auction. They characterize the equilibrium-bidding behavior of power generators under each type of auction and compare the equilibrium outcomes in terms of prices. Fabra et al.[11] find that uniform auctions yield higher average prices than those of discriminatory auctions under certain demand. In contrast, under uncertain demand, expected payments to power generators are the same under both types of auctions, at least in a symmetric scenario.

Although these articles are mainly analytical, some studies have examined bidding behavior empirically. Wolfram[39] creates a model of a multi-unit auction, and applies it empirically to the England and Wales wholesale electricity market. The results show that generating firms tend to submit large mark-ups if demand is high and the firms have high marginal costs. In addition, compared with small-scale power generators, large-scale firms tend to submit higher prices and obtain mark-ups above their marginal cost.

Sweeting[35], who we mentioned in our discussion of the empirical approach, uses data on generators' bids and costs sourced from two major generators (National Power and PowerGen) in the England and Wales wholesale electricity market. The generators' bidding behaviors are observed during the second half of the 1990s. It is found that the companies could have increased their short-run profits by reducing their prices, which implies that their bidding behaviors were consistent with tacit collusion or as an attempt to raise spot prices in the future. Hortacsu and Puller[18] develop an equilibrium model with a multi-unit auction and examine the bidding behavior of power generators in the Texas electricity spot market. They collect firm-level data on bids, and compare marginal costs with the benchmarks from their model, observing that smaller power generating firms tend to bid excessively steep supply schedules. In contrast, larger generators tend to offer gradual schedules that are close to the benchmark of profit maximization.

This completes our brief outline of the three structural model approaches. To highlight the advantages of our model, we now identify the weaknesses of these approaches in terms of measuring mark-ups. However, as mentioned above, the auction approach focuses more on the issue of auction design rather than on investigating market power. Therefore, we only present the weaknesses of the Cournot and SFE approaches, which can be classified into two types.

The first weakness is with the features of these models themselves. First, the Cournot model expresses a specific situation where each supplier maximizes profit under a certain demand, and moreover, the Cournot model does not contain inelastic demand cases. Second, except in some simple cases, it is difficult to derive equilibria analytically in the SFE approach. In addition, the demand uncertainty is mainly used to constrain the solution. Therefore, strategies and market price are not affected by the demand probability distribution. Third, the SFE model basically assumes that each market participant is risk-neutral. In other words, these models do not take into account the risk attitudes of market participants.

The second category of weakness emerges when we apply these models to measure market performance. As with to the direct calculation of a mark-up, it is necessary to estimate both marginal cost and electricity demand curves. Aside from the two difficulties described above, data collection and the estimation procedure for electricity demand curves are also laborious. In fact, studies that use the Cournot approach, such as Borenstein and Bushnell[5], tend to avoid estimating demand curves. They conduct sensitivity analyses with multiple scenarios, and use parameters from this analysis rather than estimating demand curves. However, more detailed data are required whenever we examine bidding behaviors as well as market power in an electricity market. Some articles use the multi-unit auction approach, such as Hortacsu and Puller[18], who collect detailed bidding data. However, it is not always easy to access such data.

Therefore, to assess market performance in non-storable commodity, especially electricity, markets, we develop a model that overcomes the weaknesses of previous models. Our model has following advantages. Regarding the first type of weakness, and in contrast with the Cournot approach, our model treats uncertainty, which implies that the concept of time is built into our model. In addition, our model can express inelastic demand as well as elastic demand. Compared with the SFE approach, we can derive the Nash equilibria analytically, and show that the demand distribution affects the equilibrium spot price explicitly. We also introduce different expressions of strategy for each firm, which reflects risk attitude. With respect to the second category of weakness, we need only two-dimensional time series data for daily price and daily trading volume (MWh) to measure the degrees of mark-up. At the same time, we can also determine the risk attitudes of power generators and the degrees of concentration without any additional data. Moreover, we can calculate the hourly mark-up, and can compare market power between periods of high and low demand.

To construct the model, we consider two key points. The first is that electricity has non-storable properties and generators have to meet the balancing rule in the market, and the second is that each power generator faces uncertain and inelastic demand. Thus, our model has the following features. First, to determine a strategy under uncertainty, we use an α-quantile of profit distribution as the objective function. Second, we assume that each generator can offer its supply curve strategically, which is greater than the marginal cost curve. Our model will be explained in detail in Section 2.

There are numerous studies that deal with spot electricity price processes, and especially with spikes. As price behavior is the subject of these studies, it easy to apply them to the measurement of market risk. However, an underlying market structure that generates electricity prices and spikes remains outside their scope.

The present research follows two lines of study. The first is a data analysis-based approach that estimates the price process. Johanson and Barz[21] compare eight types of stochastic processes using simulations with electricity price data. Davison et al.[10], Huisman and Mahieu[19], and Mount et al.[28] use a stochastic regime switching model. Hadsell et al.[15] estimate the volatility of spot prices in NYMEX using a TARCH model. In contrast, Hadsell and Shawky[16] apply a GARCH model to simulate the NYISO real-time market. The other line of study is the transformation model, in which an inelastic demand is mapped to a price by an assumed supply function. Barlow[4], and Kanamura and Ohashi[24] use various functions as supply curves to estimate spot price processes. In particular, Kanamura and Ohashi[24] construct a structural model that generate spikes more successfully than do other preceding models.

These studies clearly focus on spikes in the industry, as does Spear[34]. This is a noteworthy study that study proposes a framework of a general equilibrium with a non-cooperative game, and aims to show the relationship between market power and spikes and to explain why capacity did not increase in the California wholesale electricity market. To address these issues, Spear[34] analyzes short-run electricity price determinations and long-run strategies to choose capacity under uncertainty, obtaining the following three properties. First, the degree of the mark-up depends on the number of firms in both peak and off-peak periods. Second, even in a perfectly competitive market, a spike may occur if demand is high enough. Third, each power generator has an incentive to decrease capacity in the long run.

Our model can be applied to estimate price processes and spikes, as well as to measure mark-ups. Therefore, we can use our model for both financial risk management and assessment of market performance. Furthermore, our model has a general versatility. Although we focus on the spot electricity market in this article, our model can easily be applied to other non-storable commodity markets such as emission trading markets, agricultural markets, semiconductor markets, and freight shipping markets. In addition, the model can also be used to analyze commodities with limited storage capacities like natural gas, and refinancing in short-term money markets.

In this article, we first construct a non-cooperative game model that focuses on the strategic behavior of each power generator under uncertainty. We derive the Nash equilibrium spot price formulae, and show that a mark-up consists of two factors: the effect of market concentration and that of risk attitude. The price formula is then applied in an empirical analysis, and the observed mark-ups are decomposed into the two factors. The article is organized as follows. In Sections 2 and 3, we outline our model, derive the Nash equilibrium formulae, and present relevant implications. In Section 4, we construct numerical examples to assess any mark-ups in the California PX market from 1998 to 2000. Finally, we conclude with closing remarks in Section 5.

2 Model

In this section, we describe the assumptions and notation used throughout this article. While our model really does possess a kind of generality, we will explain it in an electricity market setting for the application described in Section 4. We assume an oligopolistic spot electricity market with a finite number of power generating firms and many perfectly competitive retailers. In the market, the power generating firms are suppliers and face spot electricity demand by the retailers, who distribute power to their customers.

For simplicity, our model is one period. We call the beginning of the period *time zero* and the end of the period *time one*. At time zero, spot electricity demand is uncertain, and each power producer has information regarding only distribution. Each supplier then strategically selects a supply function and submits it. After that, the market maker aggregates individual supply functions to construct a market supply function. At time one, the demand curve will be realized, and the spot price is where supply equals demand. It is worth noting that we can easily extend our approach to a multi-period model using the method described in Ishii[20].

Let $(\Omega, \mathcal{F}, \mathrm{P})$ be a probability space. A non-negative continuous random variable Z represents the basic uncertainty of total electricity demand at time one. At time zero, only the probability distribution of Z is known, and the value is unforeseeable for all market participants. The demand curve is given by a set $\{(x,y) \,|\, x + cy = Z,\, 0 \leq x \leq Z\}$, where c is a non-negative constant. In our notation, x denotes quantity and y price. In addition, we assume that the demand curve is linear and its intercept is $\dfrac{Z}{c}$. Thus, the demand curve only moves with parallel shifts. $\dfrac{Z}{c}$ can be interpreted as the maximum willingness to pay.

The number of power generating firms is denoted by $n \geq 2$. Suppose that a and b are positive constants. We set

$$f(x) = ax + b \qquad \text{for } x \in [0, \infty),$$

and

$$C(x) = \int_0^x f(u)\, du = \frac{a}{2}x^2 + bx \qquad \text{for } x \in [0, \infty), \tag{1}$$

where f is the marginal cost function of each power generating firm, and C is the cost function. Without loss of generality, the fixed cost is assumed to be 0 for every firm. Therefore, it follows that the n power generating firms are homogeneous.

We define $g : [0, \infty)^2 \to [0, \infty)$ by

$$g(x, s_j) = ax + b + s_j \qquad \text{for } (x, s_j) \in [0, \infty)^2.$$

The economic interpretation of the function g is as follows. For each $j = 1, 2, \ldots, n$, $s_j \in [0, \infty)$ is a strategy of power producer j. With the distribution of Z and information regarding the other power producers, power producer j selects a strategy s_j and bids its supply function $g(x, s_j)$ to the market at time zero. The supply function is parallel to the marginal cost function. Thus, strategy s_j represents a price increment from the marginal cost. This is illustrated in Figure 1. A higher value of s_j means a higher mark-up by firm j. In contrast, $s_j = 0$ means that firm j offers the marginal cost curve f as a supply curve, and firm j has no mark-up.

Insert **Figure 1: Examples of marginal cost function and supply function** around here.

In addition, for any $s_j \in [0, \infty)$, $g^{-1}(y, s_j)$ is defined as follows:

$$g^{-1}(y, s_j) = \begin{cases} 0 & \text{for } y \in [0\ ,\ b + s_j] \\ \dfrac{y - b - s_j}{a} & \text{for } y \in (b + s_j\ ,\ \infty) \end{cases} . \tag{2}$$

We point out here that the output is zero if, and only if, a spot price y is less than or equal to $b + s_j$.

Hereafter, s denotes a strategy vector whose components are s_1, s_2, $\ldots$, s_n, and $s_{(k)}$ is the kth smallest of $s_1, s_2, \ldots, s_n$, that is,

$$s_{(1)} \le s_{(2)} \le \ldots \le s_{(n)}.$$

In addition, let $r(s, k)$ be the rank of s_k among $s_1, s_2, \ldots, s_n$, so that

$$r(s, k) = \begin{cases} 1 & \text{if } s_k = s_{(1)}, \\ 2 & \text{if } s_k = s_{(2)}, \\ \vdots & \\ n & \text{if } s_k = s_{(n)}, \end{cases}$$

where we put $r(s, i) < r(s, j)$ for $s_i = s_j$ and $i < j$. Thus $\{r(s, 1), r(s, 2), \ldots, r(s, n)\} = \{1, 2, \ldots, n\}$. That is, $r(s, \cdot)$ is a one-to-one correspondence, or a permutation of $\{1, 2, \ldots, n\}$, for each $s \in [0, \infty)^n$. For convenience, $r(s, k)$ will be shortened to $r(k)$.

For any $y \ge 0$ and any strategy vector $s = (s_1, s_2, \ldots, s_n) \in [0, \infty)^n$, we set

$$\begin{aligned} G(y, s) &:= \sum_{i=1}^{n} g^{-1}(y, s_i) \\ &= \begin{cases} 0 & \text{for } y \in [0\ ,\ b + s_{(1)}] \\ \dfrac{k(y - b) - \sum_{i=1}^{k} s_{(i)}}{a} & \text{for } y \in (b + s_{(k)}\ ,\ b + s_{(k+1)}], \\ & \qquad k = 1, 2, \ldots, n-1 \\ \dfrac{n(y - b) - \sum_{i=1}^{n} s_i}{a} & \text{for } y \in (b + s_{(n)}\ ,\ \infty) \end{cases} . \end{aligned} \tag{3}$$

The market-wide supply curve is denoted by $G(y, s)$ when the strategy vector is s.

We define $I_k(s)(k = 0, 1, 2, \dots, n)$ as follows:

$$I_0(s) := \Big[0, c(b + s_{(1)})\Big)\ ,\ I_1(s) := \left[c(b + s_{(1)}), \frac{1}{a}(s_{(2)} - s_{(1)}) + c(b + s_{(2)})\right)\ ,$$

$$I_k(s) := \left[\frac{1}{a}\sum_{i=1}^{k-1}(s_{(k)} - s_{(i)}) + c(b + s_{(k)}), \frac{1}{a}\sum_{i=1}^{k}(s_{(k+1)} - s_{(i)}) + c(b + s_{(k+1)})\right) \quad \text{for} \quad k = 2, 3, \dots, n-1,$$

$$I_n(s) := \left[\frac{1}{a}\sum_{i=1}^{n-1}(s_{(n)} - s_{(i)}) + c(b + s_{(n)}), \infty.\right)$$

To determine the spot electricity price at time one, we consider a system of equations in two unknowns x and y:

$$\begin{cases} x + cy = z \\ x = G(y, s) \end{cases}$$

for $z > 0$. As $G(y, s)$ is strictly increasing on $[b + s_{(1)}\ ,\ \infty)$, it is apparent that the above system has a unique solution in $[0, \infty)^2$ if, and only if, $z \geq c(b + s_{(1)})$. Let $(\varphi_x(z, s), \varphi_y(z, s))$ represent the solution. It can easily be shown that

$$\varphi_x(z, s) = \frac{1}{k + ac}\left(kz - c\sum_{i=1}^{k}(b + s_{(i)})\right), \tag{4}$$

$$\varphi_y(z, s) = \frac{1}{k + ac}\left(az + \sum_{i=1}^{k}(b + s_{(i)})\right), \tag{5}$$

for $z \in I_k(s)$ and $k = 1, 2, \dots, n$. In addition, we define $\varphi_x(z, s) = \varphi_y(z, s) = 0$ for $z \in I_0(s)$, which does not lead to loss of generality. If event $\{Z \in I_0(s)\}$ occurs, the demand curve $\{(x, y) \mid x + cy = Z,\ 0 \leq x \leq Z\}$ and the market wide supply curve $G(y, s)$ do not intersect. Thus, the spot electricity price cannot be determined. We will refer to this situation as *untradable*.

At time one, the market maker solves the system to set a quoted electricity spot price that balances total supply with total demand. Then $\varphi_x(z, s)$ denotes the amount of electricity traded, and $\varphi_y(z, s)$ denotes the spot electricity price. Figure 2 shows an example of spot prices. As mentioned above, a supply curve $G(y, s)$ is determined at time zero. Z is unforeseeable at time zero, and will be realized at time one. In the case that $Z = z_1$, the realized demand curve is l_1, and the spot price is determined as $\varphi_y(z_1, s)$. In addition, $Z = z_2$ leads to the realized demand curve l_2, and the spot price is $\varphi_y(z_2, s)$.

Insert **Figure 2: Examples of demand curves and spot prices** around here.

When the spot price at time one is $\varphi_y(z, s)$, $g^{-1}(\varphi_y(z, s), s_j)$ expresses the supply of power producer j. Substituting (5) into (2) gives us $g^{-1}(\varphi_y(z, s), s_j)$ as follows.

If $r(j) = l$ i.e., $s_j = s_{(l)}$ for $l = 1, 2, \ldots, n$,

$$
g^{-1}(\varphi_y(z, s), s_j) = \begin{cases} 0 & \text{for } z \in \displaystyle\bigcup_{i=0}^{l-1} I_i(s) \\ \dfrac{az - (k + ac)(b + s_j) + \displaystyle\sum_{i=1}^{k}(b + s_{(i)})}{a(k + ac)} & \text{for } z \in I_k(s) \text{ and} \\ & \quad k = l, l+1, \ldots, n \end{cases} . \tag{6}
$$

Moreover, we define $F_j : (0, \infty) \times [0, \infty)^n \to \mathbf{R}$ by

$$
F_j(z, s) := \varphi_y(z, s) \cdot g^{-1}(\varphi_y(z, s), s_j) - C(g^{-1}(\varphi_y(z, s), s_j)).
$$

$F_j(z, s)$ represents the profit of power generating firm j at time one. Using (1), (5), and (6), $F_j(z, s)$ can be expressed explicitly.

When $r(j) = l$, namely $s_j = s_{(l)}$ for $l = 1, 2, \ldots, n$,

$$
\begin{aligned}
&F_j(z, s) \\
&= \begin{cases} 0 \quad \text{for } z \in \displaystyle\bigcup_{i=0}^{l-1} I_i(s) \\ \dfrac{1}{2a(k + ac)^2} \left\{ -(k - 1 + ac)(k + 1 + ac) \left(s_j - \dfrac{a(z - bc) + \displaystyle\sum_{i \in N(j,k)} s_{(i)}}{(k - 1 + ac)(k + 1 + ac)} \right)^2 \right. \\ \qquad\qquad \left. + \dfrac{(k + ac)^2 \left(a(z - bc) + \displaystyle\sum_{i \in N(j,k)} s_{(i)} \right)^2}{(k - 1 + ac)(k + 1 + ac)} \right\} \\ \qquad\qquad \text{for } z \in I_k(s) \text{ and } k = l, l+1, \ldots, n, \end{cases}
\end{aligned} \tag{7}
$$

where $N(j, k) = \{1, 2, \ldots, k\} \backslash \{r(j)\}$.

3 Nash Equilibrium

Let $\alpha \in (0, 1)$ be arbitrary, and consider the following non-cooperative game. For each $j = 1, 2, \ldots, n$,

$$
\begin{cases} \text{the strategy set for player } j \text{ is } [0, \infty), \\ \text{the payoff to player } j \text{ is } \inf \{ u \in \mathbf{R} \mid \mathrm{P}(F_j(Z, s) \leq u) \geq \alpha \}. \end{cases} \tag{8}
$$

In the above non-cooperative game, the players are n power producers. At time zero, each producer strategically chooses a bid supply curve to maximize the α-quantile of its profit distribution given the probability distribution of Z, which is the random shock on the demand curve. Note that in this model we use a quantile function as the objective function. Applications of quantile functions to economics are investigated by Manski [25], Chambers[8, 9], and Rostek[32].

We present a rough illustration of the relationship between this quantile function and existing objective functions. When $\alpha = 0.5$, the objective function of power producer j is the median of the profit distribution. In this case, we encounter a problem similar to maximizing the expected profit. If $\alpha < 0.5$, it can be interpreted that each power producer is concerned with downside risk of the profit shortfall. This is especially true if $\alpha = 0.01$, as the producers manage 1% value at risk (VaR) of the profit. With respect to the risk measures, we provide a more detailed explanation. There are many ways to measure risk. Chambers[9] characterizes the quantile functions as risk measures by some meaningful axioms.

There are many instances where the choice of α depends on the risk attitude. Following Manski[25], Chambers[9], and Rostek[32], we assume that players choose the α-quantile of the profit distribution as the risk measure, that is, they intend to control it, where α implicitly reflects the risk attitudes. In the quantile maximization model, an α-maximizer is weakly more averse toward downside risk than an α'-maximizer if, and only if, $\alpha < \alpha'$ holds (see Rostek[32]). Needless to say, to compare between these two values is quite straightforward. Thus, α can serve as an explicit comparative measure of risk attitude.

There are also some other advantages in using the α-quantile of future profits as the objective function. In Lemma 1 below, we show a simple form of the α-quantile that is a function of only z_α and s. Nash equilibria depend on demand distributions; however, the derivation of them does not. Therefore, compared with other objective functions such as expected profit, it is easier to solve the game to some extent. In particular, an expected objective function requires concavity to characterize risk attitudes, which involves more complex procedures or calculations.

From an empirical perspective, as Rostek[32] points out, the quantile maximization model has following advantages compared with the expected objective function model: 1) one does not have to make any parametric assumptions about each player's objective function; 2) to compare agent risk attitudes, there is no need to consider the concavity of objective function from the data; 3) in order to make policy recommendations based on the quantile model, it suffices to estimate a unique parameter α; 4) the quantile model is robust to fat tails and works well with distributions that do not possess finite moment. We use these advantages actively in the numerical example in the next section.

We will now go through a number of steps to rigorously derive the unique Nash equilibrium in an explicit form. Hereafter, let z_α be the α-quantile of Z.

Lemma 1
Assume the conditions given in Section 2. Then

$$\inf \{u \in \mathbf{R} \mid \mathrm{P}(F_j(Z, s) \leq u) \geq \alpha\} = F_j(z_\alpha, s), \tag{9}$$

for $j = 1, 2, \ldots, n$ and $s \in [0, \infty)^n$.

It is easy to show that $F_j(z, s)$ is increasing in z. The proof of this lemma is straightforward.

Lemma 2
Suppose that $z_\alpha > bc$. If there exists an $m = 1, 2, \ldots, n$ such that $F_m(z_\alpha, s) = 0$, then s is not a Nash equilibrium.

Proof. See the Appendix.

Lemma 3
Suppose that $z_\alpha > bc$. Then the following conditions are equivalent:

(a) $s \in [0, \infty)^n$ satisfies $F_k(z_\alpha, s) > 0$ for each $k = 1, 2, \ldots, n$;

(b) $s \in [0, \infty)^n$ satisfies $\sum_{i=1}^{n-1}(s_{(n)} - s_{(i)}) + ac(b + s_{(n)}) < az_\alpha$.

Proof. See the Appendix.

To prove that the game (8) has a unique Nash equilibrium for $z_\alpha > bc$, we introduce the following system of n linear equations in a n-dimensional column vector ξ:

$$A\xi = a(z_\alpha - bc)\mathbf{1}, \tag{10}$$

where A is a symmetric $n \times n$ matrix in which each diagonal element is $(n - 1 + ac)(n + 1 + ac)$ and each off-diagonal element is -1, and $\mathbf{1}$ is a vector whose n components are 1.

Lemma 4
Suppose that $z_\alpha > bc$. The solution set of (10) then contains all the Nash equilibria of the game (8).

Proof. See the Appendix.

Lemma 5
The solution of system (10) is

$$s^* = ({s^*}_1, {s^*}_2, \ldots, {s^*}_n) = \left(\frac{a(z_\alpha - bc)}{(n + ac)^2 - n}, \frac{a(z_\alpha - bc)}{(n + ac)^2 - n}, \ldots, \frac{a(z_\alpha - bc)}{(n + ac)^2 - n} \right). \tag{11}$$

Proof. See the Appendix.

The following theorem is a consequence of the preceding two lemmas.

Theorem 1
In the non-cooperative game (8), we have the following:

(a) if $z_\alpha \le bc$, every strategy vector in $[0, \infty)^n$ is a Nash equilibrium, and all payoff functions are 0;

(b) if $z_\alpha > bc$, s^* is the unique Nash equilibrium.

Substituting s^* for (4) and (5) gives us the following corollary.

Corollary 2
Let $z_\alpha > bc$. If $Z \geq bc + \dfrac{ac(z_\alpha - bc)}{(n+ac)^2 - n}$,

$$\varphi_x(Z, s^*) = \frac{n}{n+ac}\left(Z - bc - \frac{ac(z_\alpha - bc)}{(n+ac)^2 - n}\right), \tag{12}$$

$$\varphi_y(Z, s^*) = \frac{n}{n+ac}\left(\frac{aZ}{n} + b + \frac{a(z_\alpha - bc)}{(n+ac)^2 - n}\right). \tag{13}$$

Otherwise, the spot electricity is untradable.

We make some mild assumptions regarding the demand uncertainty. The deviation of the Nash equilibria does not depend on the distribution. Therefore, the spot price formulae can be said to be distribution-free property.

As explained in Section 2, each power producer offers its supply curve with strategy s_j. Strategy vector $s = (0, 0, \ldots, 0)$ can be interpreted as perfect competition where no mark-up exists in the market. In contrast, the equilibrium strategy ${s^*}_j$ leads to a positive mark-up in the equilibrium. In the case of perfectly inelastic demand, as we will see below, the effects of these factors on the spot prices can be expressed in a simpler form. First, we study the equilibrium properties of the model in the following paragraphs.

Theorem 3
In the case that $c > 0$ and $z_\alpha > bc$, consider a single electricity supplier facing the stochastic demand curve whose marginal cost function is f. In other words, we assume a monopolistic market with demand uncertainty. This firm is assumed to submit a supply function g to maximize the α-quantile of its future profit distribution at time zero. Then:

(a) at time zero, the monopolistic firm bids $g\left(x, \dfrac{z_\alpha - bc}{c(2+ac)}\right) = ax + b + \dfrac{z_\alpha - bc}{c(2+ac)}$ to the market;

(b) if $Z \geq bc + \dfrac{z_\alpha - bc}{2+ac}$, the traded amount of electricity and the spot electricity price are rspectively

$$\frac{1}{1+ac}\left(Z - bc - \frac{z_\alpha - bc}{2+ac}\right),$$
$$\frac{1}{1+ac}\left(aZ + b + \frac{z_\alpha - bc}{c(2+ac)}\right),$$

otherwise the spot electricity is untradable.

Proof. See the Appendix.

Theorem 3 describes the equilibrium quantity and price in a monopolistic market where the single firm has a powerful influence. The monopolistic market is one extreme, while the other is perfect competition, where many power generating firms exist in the market and no firm can exercise market power. Under the assumption $c > 0$, we can extend the results from the monopoly to perfect competition (that is, for every $n \geq 1$) without difficulty.

As the demand curve is assumed to be linear, it is easy to obtain the deadweight loss in the equilibrium. We show this in the following corollary.

Corollary 4
Suppose that $c > 0$ and $z_\alpha > bc$. In the Nash equilibrium, the deadweight loss of the mark-up depends on Z, such that

(a) $Z < bc \Rightarrow 0$;

(b) $bc \leq Z < bc + \dfrac{ac(z_\alpha - bc)}{(n+ac)^2 - n} \Rightarrow \dfrac{n(Z-bc)^2}{2c(n+ac)}$;

(c) $Z \geq bc + \dfrac{ac(z_\alpha - bc)}{(n+ac)^2 - n} \Rightarrow \dfrac{a^2cn(z_\alpha - bc)^2}{2(n+ac)\{(n+ac)^2-n\}^2}$.

Proof. See the Appendix.

Corollary 5
If there exists a $z_0 > bc$ such that $\mathrm{P}(Z = z_0) = 1$, the unique Nash equilibrium is

$$s^*{}_j = \frac{a(z_0 - bc)}{(n+ac)^2 - n} \quad \text{for } j = 1, 2, \ldots, n.$$

This corollary shows the equilibrium strategies in the spot electricity market without uncertainty, that is, the demand curve facing each firm is $\{(x, y) \,|\, x + cy = z_0,\ 0 \leq x \leq z_0\}$. Since the concentration, which depends on a finite number of the suppliers n, enables them to exercise market power through their strategies and the spot market, there exists a positive mark-up in the equilibrium. Then we call $\dfrac{1}{(n+ac)^2 - n}$ the market power parameter, which affects the sizes of the equilibrium strategy and mark-up even under certainty.

Now, we can rewrite equation (13) as

$$\varphi_y(Z, s^*) = \frac{n}{n+ac}\left(\frac{aZ}{n} + b + \frac{a(Z-bc)}{(n+ac)^2 - n} + \frac{-a(Z - z_\alpha)}{(n+ac)^2 - n}\right). \tag{14}$$

Here, the equilibrium mark-up is proportional to the sum of the third and fourth terms in the parentheses. If the demand curve is predictable, that is, the value of Z is perfectly forecasted for all market participants, and the third term is the equilibrium mark-up without uncertainty as we have just seen above. More specifically, it is the inevitable mark-up in this market, which is generated to a great degree by the market power. While the market power parameter appears in both terms, the sign of fourth term is determined by the difference between Z and z_α and is negative if and only if $Z > z_\alpha$. Then the equilibrium mark-up is expected to be less than in the certain case, when each producer selects a small α such that $\mathrm{P}(Z > z_\alpha)$ is great enough. In other words, the suppliers throw away the chance of higher mark-ups to improve the minimum performance, or protect possible profits in the case that a left-tail event would occur with probability α. The opposite situation holds when α is close to 1. The suppliers are willing to accept small profits in an event with probability α in exchange for pursuing the best performance in a right-tail event with probability $1 - \alpha$. This might increase the equilibrium mark-up beyond the certain case. Therefore, the risk attitude of suppliers is reflected in the fourth term, which is interpreted as the risk premium for the suppliers. In addition to this decomposition of mark-up into the market power parameter and risk premium, we can also obtain further insights in the Nash equilibrium strategy.

It is clear that the equilibrium strategy $s^*{}_j$ depends on z_α, a, b, and c. We see it here as a function of a and c.

Corollary 6
Suppose that $c > 0$ and $z_\alpha > bc$.

(a) The equilibrium strategy is maximized only at $a = \dfrac{\sqrt{n(n-1)}}{c}$.

(b) The more elastic the demand curve is, the smaller the equilibrium strategy is.

Proof. See the Appendix.

Corollary 6 focuses on the mark-up. We can also see the equilibrium payoff $F_j(z_\alpha, s^*)$ as a function of a, and the following corollary is obtained.

Corollary 7
Suppose that $c > 0$ and $z_\alpha > bc$.

(a) For any $n \in \mathbf{N}$, there exists a unique non-negative a that maximizes $F_j(z_\alpha, s^*)$. We denote the maximizer by $a^*(n)$.

(b) $a^*(1) = 0$.

(c) $\dfrac{n-1}{c} < a^*(n) < \dfrac{n}{c}$ for $n \geq 2$.

Proof. See the Appendix.

We now present our findings from the corollaries.

First, we focus on the relationship between n and a, looking at the results in part (a) of Corollary 6 and Corollary 7. The first corollary provides the slope of the marginal cost function, which maximizes the equilibrium strategy, and the objective function in Corollary 7 is the α-quantile of future profits. In both cases, the maximizer a increases with the number of firms n. That is, in a market where a large number of firms exist, each firm simultaneously chooses a low elastic marginal cost curve with a high slope. Each power generating firm selects less efficient plants such as a small fossil plant rather than a large plant, and its capacity tends to be limited. In contrast, a monopolistic firm selects a flat and perfectly elastic marginal cost curve $a = 0$ to maximize its payoff. This implies that in a monopolistic market with $n = 1$, the firm will employ the most efficient plant, and no capacity constraints exist. In summary, these results suggest that the monopolistic firm has an investment incentive to flatten the marginal cost curve, whereas firms in a more competitive market do not have such incentives. This observation shows the advantage of the single firm operation. Regarding this point, Bushnell et al.[7] compare three electricity markets in 1999: California, New England and PJM. Based on their summary of market conditions, which is described in Table 1, it is found that the higher HHI tends to have the higher proportion of nuclear operation. This suggests that the equipment with the shallowest slope of a is chosen in a market where a smaller number of firms exists.

Second, we see a relationship between c and a. The maximizer a is a decreasing function of c in each corollary. A lower value of c makes the slope a steeper. One can deduce that each firm simultaneously selects less efficient plants with capacity constraints when the elasticity of demand is low. Thus, in a low elasticity demand market, each firm has less incentive to invest in more efficient plants with large capacity.

Third, we see a relationship between c and the equilibrium strategy. In our setting, the slope of the demand curve is $1/c$. All other things being equal, the elasticity of demand becomes high if

c increases. Therefore, part (b) of Corollary 6 shows that a higher elasticity of demand decreases the equilibrium strategy, which leads to a lower equilibrium mark-up. This is consistent with Borenstein and Bushnell[5]. Their numerical examples show that the lower elasticity of demand results in the higher mark-up.

We can apply the equilibrium spot price formula to assess the impact of suppliers in a real market. The numerical examples in the following section show that this approach is effective for estimating the average behavior of all firms within a spot electricity market. As we mentioned in Section 1, electricity as a commodity has special features, that is, "non-storability," "balancing rule," and "inelastic demand." Throughout the following section, we assume that the demand is perfectly inelastic. For the estimation, we introduce the following corollary.

Corollary 8
Demand is assumed to be inelastic, that is $c = 0$. The unique Nash equilibrium is

$$s^* = ({s^*}_1, {s^*}_2, \ldots, {s^*}_n) = \left(\frac{az_\alpha}{n(n-1)}, \frac{az_\alpha}{n(n-1)}, \ldots, \frac{az_\alpha}{n(n-1)} \right), \tag{15}$$

and the equilibrium spot price is

$$\varphi_y(Z, s^*) = \frac{aZ}{n} + b + \frac{az_\alpha}{n(n-1)}. \tag{16}$$

In the right-hand side of Equation (16), the sum of the first and second terms expresses the market-wide marginal cost curve, and the third term $\frac{az_\alpha}{n(n-1)}$ is the equilibrium mark-up. As mentioned above, this can be decomposed into two factors: the market power parameter, which is determined by concentration, and the risk premium, which corresponds to the effect of risk attitude. By substituting various pairs of the α-quantile for the future demand distribution and the number of suppliers n into this equilibrium spot price formula, we can obtain a set of modeled electricity markets. Hence, equation (16) provides a set into which a real market can be connected.

In the following section, we use a statistical procedure for this connection. The mark-ups of spot electricity prices are then described by the two factors, and are examined within the modeled market.

4 Numerical Example

Under the assumption that the electricity demand is inelastic, the equilibrium spot price formula (16) is applied to investigate the existence, degree, and two constituents of mark-ups in the California PX market from May 30, 1998 to December 31, 2000. In this analysis, we use only two time-series datasets, prices and trading volumes.

The data are hourly prices and trading volumes sourced from the California PX market. We specifically use data from 4am and 6pm because our intention is to identify differences in the strategic behavior of the average power generator between peak and off-peak periods. In short, we treat generated electricity at 4am and 6pm as different products. The data covers the period from April 1, 1998 to December 31, 2000 [2]. To explain the analytical method used, we denote

[2] Data are downloaded from the University of California Energy Institute web site: http://www.ucei.berkeley.edu/datamine/datamine.htm

each trading volume and price at time t by $x(t)$ and $y(t)$ without distinguishing between 4am and 6pm. Note that the trading volume is equal to the quantity of demand under the inelastic demand assumption.

Similar to Kanamura and Ohashi[24], suppose that the electricity demand $X(t)$ is the sum of a deterministic term $h(t)$ and a stochastic term $\zeta(t)$, that is,

$$X(t) = h(t) + \zeta(t).$$

For simplicity, ζ is assumed to follow a first-order auto-regressive process, hereafter denoted by AR(1):

$$\zeta(t+1) = \gamma_0 + \gamma_1\zeta(t) + \varepsilon(t+1),$$

where $\varepsilon(t+1) \sim \mathrm{N}(0, {\gamma_2}^2)$.

For each of the peak and off-peak periods, the deterministic trend $h(t)$ is first estimated by the 10-day moving average of trading volumes. Therefore, the estimated deterministic trend $\hat{h}(t)$ is given by

$$\hat{h}(t) = \frac{1}{10}\sum_{k=0}^{9} x(t-k).$$

We then use the differences $\{x(t)-\hat{h}(t)\}$ to estimate the parameters of the AR(1). In this regard, we sequentially estimate the parameters in every t-day period with the latest 50 differences, that is, using a 50-day moving estimation. Thus, we have a series of estimated parameters $\{(\hat{\gamma}_0(t), \hat{\gamma}_1(t), \hat{\gamma}_2(t))\}$. For each t-day period, the α-quantile of $X(t+1)$ can be determined by

$$\hat{h}(t) + \hat{\gamma}_0(t) + \hat{\gamma}_1(t)(x(t) - \hat{h}(t)) + \Phi^{-1}(\alpha)\hat{\gamma}_2(t),$$

where Φ is the distribution function of a standard normal random variable.

Let

$$\begin{aligned} \hat{H}(t+1) &= \hat{h}(t) + \hat{\gamma}_0(t) + \hat{\gamma}_1(t)(x(t) - \hat{h}(t)), \\ \hat{\sigma}(t+1) &= \hat{\gamma}_2(t). \end{aligned}$$

We now explain the procedure for calibrating the average risk attitude α and the number of firms n. Based on the equilibrium spot price (16), we consider a multiple linear regression model where the response variable is the spot price, $y(t)$, and the explanatory variables are the trading volume, $x(t)$, and the α-quantile of one-day ahead electricity demand, $\hat{H}(t)+\Phi^{-1}(\alpha)\hat{\sigma}(t)$. Apparently, $x(t)$ and $\hat{H}(t)+\Phi^{-1}(\alpha)\hat{\sigma}(t)$ correspond to Z and z_α, respectively, on the right hand side of (16).

We use the Akaike information criterion (AIC) to select $\Phi^{-1}(\alpha)$, or equivalently α. It is necessary to introduce a theorem to choose the statistical model or α with a minimum AIC value. We set the electricity spot prices $\{y(t)\}$, trading volume $\{x(t)\}$, $\{\hat{H}(t)\}$, and $\{\hat{\sigma}(t)\}$ correspond to $\mathbf{y}$, $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$, respectively, which are used in Lemma 8 of the Appendix. It is now simple to obtain the following theorem.

Theorem 9
If $\{y(t)\}$, $\{x(t)\}$, $\{\hat{H}(t)\}$, and $\{\hat{\sigma}(t)\}$ satisfy at least one condition of Lemma 8, then there exists a unique α minimizing the AIC of the multiple regression model.

If the α-quantile of one-day ahead electricity demand is a linear function of $\Phi^{-1}(\alpha)$, Theorem 9 is valid. Therefore, instead of AR(1), any discrete time stochastic process can be used in this method, where the conditional distribution of $\zeta(t+1)$ given $\zeta(t)$ is a normal distribution.

From Theorem 9, we can obtain the value $\Phi^{-1}(\alpha)$ that minimizes the AIC value. For comparison, we can also fit a simple linear regression model in which the response variable is $y(t)$ and the explanatory variable is $x(t)$, and then compute the AIC value. If the AIC value for the multiple linear regression model is smaller than that obtained with the simple one, then there is evidence to support $\hat{H}(t) + \Phi^{-1}(\alpha)\hat{\sigma}(t)$ being a contributing variable. Furthermore, it tells us the overall risk attitude of suppliers. In the case that the estimated α-quantile is small (large), the average power generator has willingness to pay (receive) a risk premium.

The estimated coefficients imply the number of power generating firms n. As can be seen from (16), the estimated coefficients of $x(t)$ and of $\hat{H}(t) + \Phi^{-1}(\alpha)\hat{\sigma}(t)$ correspond to $\dfrac{a}{n}$ and $\dfrac{a}{n(n-1)}$, respectively. We can then easily calculate the number of firms n with these estimated coefficients. Thus, we can show the impact of the degree of concentration on spot prices.

We present in Table 1 the results of the multiple and simple regression analyses during the entire period (from May 30, 1998 to December 31, 2000) for 4am and 6pm. In Table 2, we show the results of the regressions for 4am and 6pm in each of the following years: 1998 (from May 30, 1998 to December 31, 1998), 1999 (from January 1, 1999 to December 31, 1999), and 2000 (from January 1, 2000 to December 31, 2000).

Insert **Table 1: Results of regression analysis for the entire period** around here

Insert **Table 2: Results of regression analysis for 1998, 1999, and 2000** around here

Regarding the multiple regressions in each table, the first row shows $\Phi^{-1}(\alpha)$, which minimizes AIC, and the second presents the estimated value of α. Note that if the absolute value of the derived α-quantile that minimizes AIC is larger than 6, we set it as -6 (if the sign is negative) or 6 (if positive), and use this to calculate α. The reason this truncation rule is used is that the probability that the absolute value of a standard normal random variable is greater than 3 is less than 0.01, and it is extremely rare event that the value is greater than 6. Furthermore, the goal is the estimation of α and not of $\Phi^{-1}(\alpha)$. Thus, based on the probability distribution, meaningless extremely small or large values of $\Phi^{-1}(\alpha)$ are truncated.

We label the estimated coefficients of $\{x(t)\}$ as the "coefficient of trading volume", and those of $\left\{\hat{H}(t) + \Phi^{-1}(\alpha)\sigma(t)\right\}$ as the "coefficient of α-quantile". "Estimated number of firms", "estimated slope of supply curve", and "average mark-up" are also presented in these tables.

We now compare the results of the multiple regression with the simple regression. As we can see in Table 1, the AIC values of the multiple regression are smaller than those of the simple regression for both 4am and 6pm. Therefore, the α-quantile term should be included. In Table 2, most of the AIC values indicate that the α-quantile terms should be included in the models. However, the AIC values of the simple regression for 6pm in 2000 are smaller than those of the multiple regression.

From the results of the multiple regression in Table 1, most of the parameters are significant at the 1% level. However, the estimated coefficient of trading volume for 6pm is negative.

Furthermore, the scatter plot of the entire period (from May 30, 1998 to December 31, 2000) in Figure 3-a shows a weak negative correlation between prices and trading volumes. In this case, the slope of the supply curve is downward: larger trading volumes result in lower spot prices, which seems inconsistent with the common assumptions in the context of microeconomics.

Insert **Figure 3: Scatter plots for prices and trading volumes** around here

Tables 2-a and 2-b show the results for 1998 and 1999, respectively. Regarding risk attitudes, the estimated values of α are 0 for 4am in 1999, and for 6pm in both years. These results imply that suppliers paid risk premium during these periods. In contrast, the value is 1 for 4am in 1998, which reflects that suppliers required risk premium during that period.

We now focus on the estimated number of suppliers n, which represents the degree of market concentration. For both years, the values of n for 4am are smaller than those for 6pm. Thus, the degree of concentration for off-peak periods is higher than for peak periods. If these periods are compared, the result suggests that a small (large) number of plants was in operation during the off-peak (peak) period. Furthermore, the estimated slopes of the marginal cost curves for 4am in 1998 and 1999 are also smaller than those for 6pm. This implies that the capacity constraints may be tighter during peak periods than during off-peak periods.

The coefficients of the α-quantiles are positive for both 4am and 6pm, and the average mark-ups are all positive in 1998 and 1999. Borenstein et al.[6] find that the average mark-up in 1998 is larger than that in 1999. This result is consistent with our measurement for both 4am and 6pm. In addition, our results imply that demand uncertainty or risk attitude as well as market concentration impact mark-ups. During the off-peak period in 1998, the estimated α is 1, and each supplier with higher concentration required more risk premium to earn high profits. In this case, the mark-up is higher than that during peak-period. In contrast, although the estimated number of firms for 4am is smaller than that for 6pm in 1999, that is, the degree of market concentration is high during the off-peak period, the demand distribution leads to a lower mark-up than during peak-period. Then a high degree of concentration does not always result in a high mark-up. These findings are important from the point of assessing market performance. As repeatedly emphasized, although previous empirical studies could confirm only the existence and/or degrees of mark-ups, our above results do show how mark-ups are constituted. In addition, the empirical procedure is simpler than those of previous studies.

Table 2-c shows the results for 2000. It is widely considered that drastic structural changes occurred in 2000. In this regard, Borenstein et al.[6] find that the monthly average spot price jumped from \$47.22 MWh (May 2000) to \$120.20 MWh in June 2000 because of an extremely hot summer, and high mark-ups have been observed ever since. From the results for 6pm, we see that the estimated coefficient of the α-quantile is negative, which suggests the possibility that a negative mark-up exists. However, this estimated parameter is not significant, and has weak explanation power. Furthermore, as mentioned above, the AIC value of the multiple regression for 6pm is larger than that of the simple regression, which shows that the inclusion of the α-quantile is not necessary useful. These results are presumed to be due to the disruptions in 2000, which imply that our approach is more appropriate in stable markets than unstable ones.

Thus, one might be led to conclude that we are prevented from using linear functions as marginal cost curves during periods of significant change such as the energy crisis in 2000. In this case, an alternative method may be to perform a similar analysis using the equilibrium price formula based on non-linear marginal cost functions.

We obtain following policy implications from the results under high degree of uncertainty.

First, it is possible to monitor electricity markets with our parsimonious but sophisticated way. In such circumstances, an observed mark-up might stem largely from a normal risk premium. Then, it is difficult for previous approaches to judge the exercise of market power appropriately. In contrast, our approach can evaluate not only the existence but also the constituents. We can examine to what extent the mark-ups, if exist, attribute to the exercised market power.

Second, our approach provides useful information for market policies to limit mark-ups. If the observed mark-ups mainly depend on the exercises of market power, competition promoting policies should be introduced. On the other hand, if the mark-ups are largely attributed to risk premiums, we can restrict them by policies inducing a change in risk attitudes of market participants. In other words, mark-ups can be decreased if some policies make market participants more risk averse. This kind of policies might be different from the competion promoting policies. If that is the case, our approach can help policy makers to confirm which kind of policies is appropriate to employ.

5 Conclusions

We now summarize our results and contributions. To assess the degree of mark-ups in electricity markets, we have developed a framework for connecting each real market with a modeled market. We then investigated the mark-ups in the California PX market from May 30, 1998 to December 31, 2000 using the corresponding modeled market. The main contributions of this study can be stated in two parts.

Regarding the first part of our contributions, we have derived equilibrium spot price formulae, where the mark-up depends on the number of suppliers, the α-quantile of the future demand distribution, and the slope of the marginal cost curve. As described in Section 3, we decompose a mark-up into two factors from the formulae: the effect of market concentration and the effect of risk attitude. The former factor can be related to exercising market power, and the latter to risk premium. In addition, some properties in the equilibrium indicate a relationship between the number of generators and the slope of the optimal marginal cost function. Further, we impose some mild assumptions regarding the demand uncertainty. Therefore, the spot price formulae enjoy distribution-free properties. In that sense, our model is robust.

With respect to the second part outlined in Section 4, we have presented a simple empirical implementation to investigate market performance. In this procedure, we only used two time series datasets, spot prices and trading volumes, for measuring and decomposing mark-ups. In this study, we attempted to assess the California PX market. The results demonstrate that both the market concentration and averaged risk attitude of suppliers can affect the degree of mark-up.

Despite the value of our contributions, some issues remain. In our model, it is assumed that power producers are homogeneous. However, it may be necessary to include non-homogeneous power producers. Furthermore, as mentioned in Section 4, we assume that the marginal cost function is linear. We intend to extend the marginal cost function to a non-linear function in future research.

Although there is room to extend our model, it can also be applied to many analyses as mentioned in Section 1. For example, our model can be applied to forecasting electricity spot price behavior and spikes, as well as measuring mark-ups. It can also be used to manage market risk in electricity markets. Furthermore, our model has a general versatility suitable for use in other non-storable commodity markets.

Appendix

This Appendix provides the proofs of all the results presented in the article.

□ **Proof of Lemma 2.** As the n producers are homogeneous, it suffices to prove our assertion for $m = 1$. For any $s' = ({s'}_1, {s'}_2, \ldots, {s'}_n) \in \{s \in [0,\infty)^n \mid F_1(z_\alpha, s) = 0\}$, we put $s^0 = (0, {s'}_2, \ldots, {s'}_n)$. Thus, $0 = \min(0, {s'}_2, \ldots, {s'}_n)$. This implies that $r(1) = 1$. Equation (7) and assumption $z_\alpha > bc$ give us

$$F_1(z_\alpha, s^0) \geq \frac{a(z_\alpha - bc)^2}{2(n+ac)^2} > 0 = F_1(z_\alpha, s').$$

This inequality tells us that producer 1 can increase the payoff by changing their strategy from ${s'}_1$ to 0. Therefore, s' is not a Nash equilibrium.

□ **Proof of Lemma 3.** From $z_\alpha > bc$, it follows that $s = (0, 0, \ldots, 0)$ belongs to both (a) and (b), i.e., both sets are not empty. Thus, we check whether they are equal. In the case that $s_j = s_{(n)}$, (7) is arranged as

$$F_j(z,s) \\ = \begin{cases} 0 \quad \text{for } \dfrac{a(z-bc) + \sum_{i\neq j} s_i}{n-1+ac} \leq s_j \\ \dfrac{1}{2a(n+ac)^2} \left\{ -(n-1+ac)(n+1+ac)\left(s_j - \dfrac{a(z-bc)+\sum_{i\neq j} s_i}{(n-1+ac)(n+1+ac)}\right)^2 \right. \\ \qquad \left. + \dfrac{(n+ac)^2 \left(a(z-bc)+\sum_{i\neq j} s_i\right)^2}{(n-1+ac)(n+1+ac)} \right\} \\ \qquad \text{for } s_j < \dfrac{a(z-bc)+\sum_{i\neq j} s_i}{n-1+ac} \end{cases} . \tag{17}$$

For each s in (a), there exists an $m = 1, 2, \ldots, n$ such that $s_m = s_{(n)}$. Then, from (17), we see that $F_m(z_\alpha, s) > 0$ if, and only if,

$$s_m < \frac{a(z_\alpha - bc) + \sum_{i\neq m} s_i}{n-1+ac}.$$

Rewriting this inequality, we have

$$\sum_{i=1}^{n-1}(s_{(n)} - s_{(i)}) + ac(b + s_{(n)}) < az_\alpha.$$

To see that (b) implies (a), we use the fact that $F_{r^{-1}(n)}(z_\alpha, s) > 0$ leads to $F_k(z_\alpha, s) > 0$ for all $k = 1, 2, \ldots, n$. For any s satisfying (b), there exists an $m = 1, 2, \ldots, n$, such that $r(m) = n$, i.e., $s_m = s_{(n)}$. Hence, an argument opposite to that described above gives us

$$F_{r^{-1}(n)}(z_\alpha, s) = F_m(z_\alpha, s) > 0.$$

□ **Proof of Lemma 4.** Because of the assumption that the n power producers are homogeneous, it suffices to show that

$$\{\text{Nash equilibrium of the game (8)}\} \subset \left\{ (s_1, s_2, \ldots, s_n) \;\middle|\; s_1 = \frac{a(z_\alpha - bc) + \sum_{i=2}^{n} s_i}{(n - 1 + ac)(n + 1 + ac)} \right\}.$$

For $j = 2, 3, \ldots, n$, let t_j denote a strategy of the power producer j, where we assume that $0 \le t_2 \le t_3 \le \ldots \le t_n$ without loss of generality. Let us arrange s_1, t_2, $\ldots$, t_n in non-decreasing order $t_{(1)} \le t_{(2)} \le \cdots \le t_{(n)}$. If $(s_1, t_2, \ldots, t_n)$ is a Nash equilibrium, then Lemma 2 and Lemma 3 give us

$$\sum_{i=1}^{n-1}(t_{(n)} - t_{(i)}) + ac(b + t_{(n)}) < az_\alpha.$$

This inequality implies that

$$(n - 1 + ac)t_n - \sum_{i=2}^{n-1} t_i - a(z_\alpha - bc) < s_1 < \frac{a(z_\alpha - bc) + \sum_{i=2}^{n} t_i}{n - 1 + ac}.$$

It is apparent that

$$\frac{a(z_\alpha - bc) + \sum_{i=2}^{n} t_i}{(n - 1 + ac)(n + 1 + ac)} < \frac{a(z_\alpha - bc) + \sum_{i=2}^{n} t_i}{n - 1 + ac}.$$

Therefore, if

$$(n - 1 + ac)t_n - \sum_{i=2}^{n-1} nt_i - a(z_\alpha - bc) \le \frac{a(z_\alpha - bc) + \sum_{i=2}^{n} t_i}{(n - 1 + ac)(n + 1 + ac)}$$

holds, the payoff to power producer 1 is maximized at $s_1 = \dfrac{a(z_\alpha - bc) + \sum_{i=2}^{n} t_i}{(n-1+ac)(n+1+ac)}$. Otherwise, $s_1 = (n-1+ac)t_n - \sum_{i=2}^{n-1} t_i - a(z_\alpha - bc)$ maximizes the payoff. Here, we put

$$t = \left((n-1+ac)t_n - \sum_{i=2}^{n-1} t_i - a(z_\alpha - bc), t_2, \ldots, t_n \right).$$

As we have $t_n = \dfrac{a(z_\alpha - bc) + s_1 + \sum_{i=2}^{n-1} t_i}{n-1+ac}$, Lemma 3 implies $F_n(z_\alpha, t) = 0$. Hence, t is not a Nash equilibrium by Lemma 2.

□ **Proof of Lemma 5.** It is easy to show that A is nonsingular, and

$$A^{-1} = \frac{1}{\{(n+ac)^2 - n\}(n+ac)^2} B,$$

where each diagonal element of B is $(n+ac)^2 - n + 1$ and each off-diagonal element is 1. Therefore,

$$A^{-1} a(z_\alpha - bc)\mathbf{1} = \frac{a(z_\alpha - bc)}{(n+ac)^2 - n}\mathbf{1}.$$

□ **Proof of Theorem 3.** Suppose that the monopolistic firm selects a strategy $s_1 \in [0, \infty)$. For $z \geq c(b + s_1)$, the dispatched electricity is

$$\frac{1}{1+ac}(z - c(b + s_1)), \tag{18}$$

and the spot electricity price is

$$\frac{1}{1+ac}(az + b + s_1). \tag{19}$$

Otherwise, the spot electricity is untradable. The profit function at time one is then given by

$$\begin{cases} 0 & \text{for } z < b + s_1, \\ \dfrac{1}{2a(1+ac)} \left\{ -ac(2+ac)\left(s_1 - \dfrac{z-bc}{c(2+ac)}\right)^2 + \dfrac{a(1+ac)^2(z-bc)^2}{c(2+ac)} \right\} & \text{for } z \geq b + s_1 \end{cases}$$

as explained in Section 2. Thus, it is apparent that s_1 maximizing the α-quantile of the profit distribution is

$$s_1^* = \frac{z_\alpha - bc}{c(2+ac)} = \frac{a(z_\alpha - bc)}{(1+ac)^2 - 1}.$$

By substituting s_1^* into (18) and (19), we obtain (b).

□ **Proof of Corollary 4.** If $Z \leq bc$, the spot electricity is untradable in both the perfect competition and the equilibrium. Thus, the total surplus is zero in each case. When the event $bc \le Z < bc + \dfrac{ac(z_\alpha - bc)}{(n+ac)^2 - n}$ occurs, the electricity spot is traded at $\varphi_y(Z,0) = \dfrac{n}{n+ac}\left(\dfrac{aZ}{n} + b\right)$ in the perfect competition. In contrast, spot electricity is untradable in the equilibrium. Therefore, the deadweight loss of market power in the equilibrium is $\dfrac{n(Z-bc)^2}{2c(n+ac)}$, which is equal to the total surplus generated by the perfect competition. For $Z \ge bc + \dfrac{ac(z_\alpha)}{(n+ac)^2 - n}$, the deadweight loss of market power is

$$\frac{1}{2} \cdot \frac{n}{n+ac}\left(1 + \frac{ac}{n}\right)\frac{a(z_\alpha - bc)}{(n+ac)^2 - n} \cdot \frac{n}{n+ac}\frac{ac(z_\alpha - bc)}{(n+ac)^2 - n} = \frac{a^2cn(z_\alpha - bc)^2}{2(n+ac)\{(n+ac)^2 - n\}^2}.$$

□ **Proof of Corollary 6.** To show (a), we have

$$\frac{\partial s^*{}_j}{\partial a} = \frac{(z_\alpha - bc)\{n(n-1) - a^2c^2\}}{\{(n+ac)^2 - n\}^2}.$$

Therefore, $s^*{}_j$ is maximized at the point where the partial derivative is equal to zero, that is, $a = \dfrac{\sqrt{n(n-1)}}{c}$.

The partial derivative of $s^*{}_j$ with respect to c gives us the proof of (b) as follows:

$$\frac{\partial s^*{}_j}{\partial c} = \frac{-ab\{(n+ac)^2 - n\} - a(z_\alpha - bc)\{2a(n+ac)\}}{\{(n+ac)^2 - n\}^2} \le 0.$$

For the proof of Corollary 7, it is convenient to introduce a family of polynomials with degree 4:

$$p_n(t) = t^4 + 2nt^3 + 3(n-1)t^2 - 2n(n-1)^2t - n(n-1)^2(n+1) \quad \text{for } n \in \mathbf{N}. \tag{20}$$

We show a related property of these polynomials.

Lemma 6
For each $n \in \mathbf{N}$, $p_n(t) = 0$ has a unique non-negative root $\lambda(n)$. Moreover, $\lambda(1) = 0$ and

$$n - 1 < \lambda(n) < n \quad \text{for } n \ge 2.$$

□ **Proof of Lemma 6.** If $n = 1$,

$$p_1(t) = t^4 + 2t^3 = t^3(t+2).$$

Therefore, it is apparent that $\lambda(1) = 0$.
For $n \ge 2$, we first prove the existence and uniqueness of the non-negative root. The first derivative of p_n is

$$\frac{d}{dt}p_n(t) = 4t^3 + 6nt^2 + 6(n-1)t - 2n(n-1)^2.$$

It is easy to see that

$$\frac{d}{dt}p_n(0) = -2n(n-1)^2 < 0 \quad \text{and} \quad \lim_{t\to\infty}\frac{d}{dt}p_n(t) = \infty.$$

As the second derivative of p_n satisfies

$$\frac{d^2}{dt^2}p_n(t) = 12t^2 + 12nt + 6(n-1) > 0 \quad \text{for } t \geq 0,$$

$\frac{d}{dt}p_n(t)$ is increasing on $[0,\infty)$. These imply the existence of a unique positive value $t_{n,0}$, which satisfies $\frac{d}{dt}p_n(t_{n,0}) = 0$. The behavior of p_n on $[0,\infty)$ is summarized in the following table.

t	0	$\cdots$	$t_{n,0}$	$\cdots$	∞
$\frac{dp_n}{dt}$	$-2n(n-1)^2$	$-$	0	$+$	∞
p_n	$-n(n-1)^2(n+1)$	$\searrow$	minimum	$\nearrow$	∞

Hence, the existence of a unique positive root is proved. By substituting both $n-1$ and n for $p_n(t)$,

$$p_n(n-1) = -2(n-1)^2 < 0,$$
$$p_n(n) = n(3n^2 + 4n(n-1) + n^2 - 1) > 0.$$

Therefore, we obtain the inequalities.

□ **Proof of Corollary 7.** We substitute the Nash equilibrium (11) into $F_j(z_\alpha, s)$ and obtain:

$$F_j(z_\alpha, s^*) = \frac{a(n-1+ac)(n+1+ac)(z_\alpha - bc)^2}{2\{(n+ac)^2 - n\}^2}.$$

For $n = 1$,

$$F_j(z_\alpha, s^*) = \frac{(z_\alpha - bc)^2}{2c(2+ac)^2},$$

which is a decreasing function of a. For any $n = 2, 3, \ldots$, we find the partial derivative of $F_j(z_\alpha, s^*)$ with respect to a:

$$\begin{aligned}&\frac{\partial}{\partial a}F_j(z_\alpha, s^*)\\ = &\frac{-(z_\alpha - bc)^2\{(ac)^4 + 2n(ac)^3 + 3(n-1)(ac)^2 - 2n(n-1)^2(ac) - n(n-1)^2(n+1)\}}{2\{(n+ac)^2 - n\}^3}.\end{aligned}$$

Thus, Lemma 6 yields the desired results.

Lemma 7
Let $q(x) = \frac{x^2 + b_1x + b_2}{x^2 + b_3x + b_4}$ with ${b_3}^2 - 4b_4 < 0$.

(a) If $b_1 \neq b_3$, there exist a unique maximum point and a unique minimum point for q.

(b) Suppose $b_1 = b_3$. If $b_2 < b_4$, the maximum of q occurs only at $x = -\dfrac{b_1}{2}$; if $b_2 > b_4$, the minimum of q occurs at only $x = -\dfrac{b_1}{2}$.

$\square$ **Proof of Lemma 7.** First, we provide a proof for (a). We have

$$\frac{d}{dx}q(x) = \frac{(b_3 - b_1)x^2 + 2(b_4 - b_2)x + (b_1 b_4 - b_2 b_3)}{(x^2 + b_3 x + b_4)^2} \tag{21}$$

and

$$\begin{aligned}
&(b_4 - b_2)^2 - (b_3 - b_1)(b_1 b_4 - b_2 b_3) \\
&= b_4 \left(b_1 - \frac{b_3(b_2 + b_4)}{2b_4} \right)^2 + \frac{(b_2 - b_4)^2(-{b_3}^2 + 4b_4)}{4b_4} > 0,
\end{aligned}$$

where we have used the fact that ${b_3}^2 - 4b_4 < 0$ gives us $b_4 > 0$. Hence, there exist two distinct real numbers x_1 and x_2 such that $\dfrac{d}{dx}q(x_1) = \dfrac{d}{dx}q(x_2) = 0$. The behavior of q is obtained as follows. For $b_1 < b_3$,

x	$\cdots$	x_1	$\cdots$	x_2	$\cdots$
$\dfrac{dq}{dx}$	$+$	0	$-$	0	$+$
q	$\nearrow$	maximum	$\searrow$	minimum	$\nearrow$

and for $b_1 > b_3$,

x	$\cdots$	x_1	$\cdots$	x_2	$\cdots$
$\dfrac{dq}{dx}$	$-$	0	$+$	0	$-$
q	$\searrow$	minimum	$\nearrow$	maximum	$\searrow$

In addition,

$$\lim_{x \to -\infty} q(x) = \lim_{x \to \infty} q(x) = 1.$$

Thus, we have the desired result.

In the case that $b_1 = b_3$, the derivative of q reduces to

$$\frac{d}{dx}q(x) = \frac{(b_4 - b_2)(2x + b_1)}{(x^2 + b_1 x + b_4)^2},$$

which leads to the conclusion.

To show the following lemma, we also introduce some mathematical settings. Given independent column vectors $\mathbf{1} = {}^{\mathrm{t}}(1, 1, \ldots, 1)$, $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3$, and $\mathbf{y}$ in $\mathbf{R}^N$ with $N \geq 5$, let $\mathbf{x}_0$ be the orthogonal projection of $\mathbf{y}$ onto the span of $\{\mathbf{1}, \mathbf{x}_1, \mathbf{x}_2 + \lambda \mathbf{x}_3\}$, where the transpose of matrix M is denoted by ${}^{\mathrm{t}}M$. We set

$$S_e(\lambda) = {}^{\mathrm{t}}(\mathbf{y} - \mathbf{x}_0)(\mathbf{y} - \mathbf{x}_0).$$

In addition,

$$
\begin{aligned}
&\overline{x}_j := \frac{{}^{\mathrm{t}}\mathbf{1}\mathbf{x}_j}{N} \quad \text{for } j=1,2,3, \quad \overline{y} := \frac{{}^{\mathrm{t}}\mathbf{1}\mathbf{y}}{N} \\
&S_{ij} := {}^{\mathrm{t}}(\mathbf{x}_i - \overline{x}_i\mathbf{1})(\mathbf{x}_j - \overline{x}_j\mathbf{1}) \quad \text{for } 1 \le i \le j \le 3, \\
&S_{jy} := {}^{\mathrm{t}}(\mathbf{x}_j - \overline{x}_j\mathbf{1})(\mathbf{y} - \overline{y}\mathbf{1}) \quad \text{for } j=1,2,3, \qquad S_{yy} := {}^{\mathrm{t}}(\mathbf{y} - \overline{y}\mathbf{1})(\mathbf{y} - \overline{y}\mathbf{1}), \\
&A_1 := S_{11}S_{33} - (S_{13})^2, \quad A_2 := S_{11}S_{23} - S_{12}S_{13}, \quad A_3 := S_{11}S_{22} - (S_{12})^2, \\
&B_1 := (S_{1y})^2 S_{33} - 2S_{13}S_{1y}S_{3y} + S_{11}(S_{3y})^2, \\
&B_2 := S_{11}S_{2y}S_{3y} - S_{13}S_{1y}S_{2y} - S_{12}S_{1y}S_{3y} + (S_{1y})^2 S_{23}, \\
&B_3 := S_{11}(S_{2y})^2 - 2S_{12}S_{1y}S_{2y} + S_{22}(S_{1y})^2.
\end{aligned}
$$

The following matrix is positive-definite:

$$
\begin{pmatrix} S_{11} & S_{13} \\ S_{13} & S_{33} \end{pmatrix}.
$$

Then we can see that

$$
A_1 = \det \begin{pmatrix} S_{11} & S_{13} \\ S_{13} & S_{33} \end{pmatrix} > 0
$$

and

$$
B_1 = \begin{pmatrix} S_{3y} & -S_{1y} \end{pmatrix} \begin{pmatrix} S_{11} & S_{13} \\ S_{13} & S_{33} \end{pmatrix} \begin{pmatrix} S_{3y} \\ -S_{1y} \end{pmatrix} > 0.
$$

Lemma 8
If either $\dfrac{A_2}{A_1} \neq \dfrac{B_2}{B_1}$ or $\dfrac{A_3}{A_1} > \dfrac{B_3}{B_1}$ holds, then there exists a unique λ that minimizes $S_e(\lambda)$.

□ **Proof of Lemma 8.** According to the assumption, there exist b_0, b_1 and b_2, such that

$$
\begin{aligned}
&\mathbf{x}_0 = b_0\mathbf{1} + b_1\mathbf{x}_1 + b_2(\mathbf{x}_2 + \lambda\mathbf{x}_3), \\
&{}^{\mathrm{t}}\mathbf{1}\{\mathbf{y} - b_0\mathbf{1} - b_1\mathbf{x}_1 - b_2(\mathbf{x}_2 + \lambda\mathbf{x}_3)\} = 0, \\
&{}^{\mathrm{t}}\mathbf{x}_1\{\mathbf{y} - b_0\mathbf{1} - b_1\mathbf{x}_1 - b_2(\mathbf{x}_2 + \lambda\mathbf{x}_3)\} = 0, \\
&{}^{\mathrm{t}}(\mathbf{x}_2 + \lambda\mathbf{x}_3)\{\mathbf{y} - b_0\mathbf{1} - b_1\mathbf{x}_1 - b_2(\mathbf{x}_2 + \lambda\mathbf{x}_3)\} = 0.
\end{aligned}
$$

Then we have

$$
b_0 = \frac{{}^{\mathrm{t}}\mathbf{1}\mathbf{y} - b_1{}^{\mathrm{t}}\mathbf{1}\mathbf{x}_1 - b_2{}^{\mathrm{t}}\mathbf{1}(\mathbf{x}_2 + \lambda\mathbf{x}_3)}{N} \tag{22}
$$

and

$$
\begin{aligned}
\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = &\frac{1}{S_{11}(S_{22} + 2\lambda S_{23} + \lambda^2 S_{33}) - (S_{12} + \lambda S_{13})^2} \\
&\begin{pmatrix} S_{22} + 2\lambda S_{23} + \lambda^2 S_{33} & -S_{12} - \lambda S_{13} \\ -S_{12} - \lambda S_{13} & S_{11} \end{pmatrix} \begin{pmatrix} S_{1y} \\ S_{2y} + \lambda S_{3y} \end{pmatrix}.
\end{aligned} \tag{23}
$$

Here, the matrix

$$
\begin{pmatrix} S_{11} & S_{12} + \lambda S_{13} \\ S_{12} + \lambda S_{13} & S_{22} + 2\lambda S_{23} + \lambda^2 S_{33} \end{pmatrix}
$$

is positive definite for any $\lambda \in \mathbf{R}$. Therefore, the determinant, which is the denominator in equation (23), is positive. Substituting (22) and (23) into $S_e(\lambda)$ yields

$$S_e(\lambda) = S_{yy} - \frac{B_1\lambda^2 + B_2\lambda + B_3}{A_1\lambda^2 + A_2\lambda + A_3} = S_{yy} - \frac{B_1}{A_1} \frac{\lambda^2 + \dfrac{B_2}{B_1}\lambda + \dfrac{B_3}{B_1}}{\lambda^2 + \dfrac{A_2}{A_1}\lambda + \dfrac{A_3}{A_1}}.$$

Therefore, Lemma 7 provides us with the conclusion.

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TABLE 1: Results of regression analysis for the entire period

Entire period (1998/5/30-2000/12/31)

Multiple regression for 4am

	Estimate	Std. Error	t value	P(>|t|), P(>F)	
α-quantile that minimizes AIC	-6.00000				
α	0.00000				
Intercept	117.50652	15.40971	7.62548	0.00000	***
Coefficient of Trading Volume	0.02290	0.00258	8.89437	0.00000	***
Coefficient of α-quantile	-0.03331	0.00232	-14.34410	0.00000	***
F-statistic	293.90000			0.00000	***
adjusted R-squared	0.37480				
AIC	4001.39900				
Estimated Number of Firms	0.31237				
Estimated slope of marginal cost curve	0.00715				
Averaged Mark-up	-465.26910				

*** 1% significance ** 5% significance * 10% significance

Simple regression for 4am

	Estimate	Std. Error	t value	P(>|t|), P(>F)	
Intercept	265.90000	12.56000	21.16000	0.00000	***
Coefficient of Trading Volume	-0.01281	0.00072	-17.77000	0.00000	***
R-squared	0.24440				
AIC	4092.49300				

*** 1% significance ** 5% significance * 10% significance

Multiple regression for 6pm

	Estimate	Std. Error	t value	P(>|t|), P(>F)	
α-quantile that minimizes AIC	-6.00000				
α	0.00000				
Intercept	217.10000	22.98000	9.44735	0.00000	***
Coefficient of Trading Volume	-0.00062	0.00143	-0.43408	0.66400	
Coefficient of α-quantile	-0.00690	0.00141	-4.91032	0.00000	***
F-statistic	33.18000			0.00000	***
adjusted R-squared	0.06181				
AIC	4843.19300				
Estimated Number of Firms	1.08991				
Estimated slope of marginal cost curve	-0.00068				
Averaged Mark-up	-113.32880				

*** 1% significance ** 5% significance * 10% significance

Simple regression for 6pm

	Estimate	Std. Error	t value	P(>|t|), P(>F)	
Intercept	233.60000	23.00000	10.15800	0.00000	***
Coefficient of Trading Volume	-0.00599	0.00093	-6.42500	0.00000	***
R-squared	0.04058				
AIC	4852.92300				

*** 1% significance ** 5% significance * 10% significance

TABLE 2: Results of regression analysis for 1998, 1999, and 2000
2-a
1998 (1998/5/30-1998/12/31)

Multiple regression for 4am

	Estimate	Std. Error	t value	P(>|t|), P(>F)	
α-quantile that minimizes AIC	6.00000				
α	1.00000				
Intercept	-53.08000	7.52500	-7.05382	0.00000	***
Coefficient of Trading Volume	0.00099	0.00085	1.16922	0.24360	
Coefficient of α-quantile	0.00243	0.00072	3.36515	0.00091	***
F-statistic	43.70000			0.00000	***
adjusted R-squared	0.28430				
AIC	432.75860				
Estimated Number of Firms	1.40781				
Estimated slope of marginal cost curve	0.00140				
Averaged Mark-up	51.47169				

*** 1% significance ** 5% significance * 10% significance

Simple regression for 4am

	Estimate	Std. Error	t value	P(>|t|), P(>F)	
Intercept	-47.35000	7.50400	-6.30900	0.00000	***
Coefficient of Trading Volume	0.00351	0.00041	8.52000	0.00000	***
R-squared	0.25330				
AIC	435.83220				

*** 1% significance ** 5% significance * 10% significance

Multiple regression for 6pm

	Estimate	Std. Error	t value	P(>|t|), P(>F)	
α-quantile that minimizes AIC	-6.00000				
α	0.00000				
Intercept	-103.70000	10.01000	-10.35964	0.00000	***
Coefficient of Trading Volume	0.00356	0.00084	4.21633	0.00004	***
Coefficient of α-quantile	0.00281	0.00112	2.51028	0.01280	**
F-statistic	107.20000			0.00000	***
adjusted R-squared	0.49690				
AIC	662.43810				
Estimated Number of Firms	2.26700				
Estimated slope of marginal cost curve	0.00807				
Averaged Mark-up	50.45958				

*** 1% significance ** 5% significance * 10% significance

Simple regression for 6pm

	Estimate	Std. Error	t value	P(>|t|), P(>F)	
Intercept	-102.90000	10.12000	-10.16000	0.00000	***
Coefficient of Trading Volume	0.00545	0.00038	14.25000	0.00000	***
R-squared	0.48690				
AIC	663.08600				

*** 1% significance ** 5% significance * 10% significance

2-b
1999 (1999/1/1-1999/12/31)

Multiple regression for 4am

	Estimate	Std. Error	t value	P(>|t|), P(>F)	
α-quantile that minimizes AIC	-6.00000				
α	0.00000				
Intercept	-45.24000	4.46800	-10.12534	0.00000	***
Coefficient of Trading Volume	0.00163	0.00048	3.39219	0.00077	***
Coefficient of α-quantile	0.00239	0.00062	3.83577	0.00015	***
F-statistic	98.94000			0.00000	***
adjusted R-squared	0.34990				
AIC	676.20600				
Estimated Number of Firms	1.68412				
Estimated slope of marginal cost curve	0.00275				
Averaged Mark-up	34.00577				

*** 1% significance ** 5% significance * 10% significance

Simple regression for 4am

	Estimate	Std. Error	t value	P(>|t|), P(>F)	
Intercept	-39.15000	4.25300	-9.20300	0.00000	***
Coefficient of Trading Volume	0.00324	0.00024	13.28500	0.00000	***
R-squared	0.32710				
AIC	680.97180				

*** 1% significance ** 5% significance * 10% significance

Multiple regression for 6pm

	Estimate	Std. Error	t value	P(>|t|), P(>F)	
α-quantile that minimizes AIC	-6.00000				
α	0.00000				
Intercept	-63.23000	8.00100	-7.90276	0.00000	***
Coefficient of Trading Volume	0.00254	0.00039	6.51628	0.00000	***
Coefficient of α-quantile	0.00224	0.00070	3.19270	0.00154	***
F-statistic	92.04000			0.00000	***
adjusted R-squared	0.33340				
AIC	1056.13600				
Estimated Number of Firms	2.13482				
Estimated slope of marginal cost curve	0.00543				
Averaged Mark-up	37.33383				

*** 1% significance ** 5% significance * 10% significance

Simple regression for 6pm

	Estimate	Std. Error	t value	P(>|t|), P(>F)	
Intercept	-48.82000	6.68900	-7.29800	0.00000	***
Coefficient of Trading Volume	0.00346	0.00027	13.02300	0.00000	***
R-squared	0.31840				
AIC	1058.60400				

*** 1% significance ** 5% significance * 10% significance

2-c
2000 (2000/1/1-2000/12/31)

Multiple regression for 4am

	Estimate	Std. Error	t value	P(>|t|), P(>F)	
α-quantile that minimizes AIC	-6.00000				
α	0.00000				
Intercept	200.19263	43.70469	4.58058	0.00001	***
Coefficient of Trading Volume	0.04604	0.00534	8.62732	0.00000	***
Coefficient of α-quantile	-0.06525	0.00599	-10.88605	0.00000	***
F-statistic	61.60000			0.00000	***
adjusted R-squared	0.24930				
AIC	1593.88000				
Estimated Number of Firms	0.29436				
Estimated slope of marginal cost curve	0.01355				
Averaged Mark-up	-933.37900				

*** 1% significance ** 5% significance * 10% significance

Simple regression for 4am

	Estimate	Std. Error	t value	P(>|t|), P(>F)	
Intercept	166.31409	50.13834	3.31700	0.00100	***
Coefficient of Trading Volume	-0.00539	0.00285	-1.88700	0.06000	*
R-squared	0.00968				
AIC	1643.06800				

*** 1% significance ** 5% significance * 10% significance

Multiple regression for 6pm

	Estimate	Std. Error	t value	P(>|t|), P(>F)	
α-quantile that minimizes AIC	-6.00000				
α	0.00000				
Intercept	-211.50000	68.18000	-3.10208	0.00207	***
Coefficient of Trading Volume	0.01682	0.00292	5.75830	0.00000	***
Coefficient of α-quantile	-0.00280	0.00188	-1.49149	0.13676	
F-statistic	16.61000			0.00000	***
adjusted R-squared	0.07880				
AIC	1853.41800				
Estimated Number of Firms	-4.99857				
Estimated slope of marginal cost curve	-0.08408				
Averaged Mark-up	-47.53690				

*** 1% significance ** 5% significance * 10% significance

Simple regression for 6pm

	Estimate	Std. Error	t value	P(>|t|), P(>F)	
Intercept	-228.30000	67.36000	-3.39000	0.00078	***
Coefficient of Trading Volume	0.01554	0.00280	5.55800	0.00000	***
R-squared	0.07824				
AIC	1851.88600				

*** 1% significance ** 5% significance * 10% significance

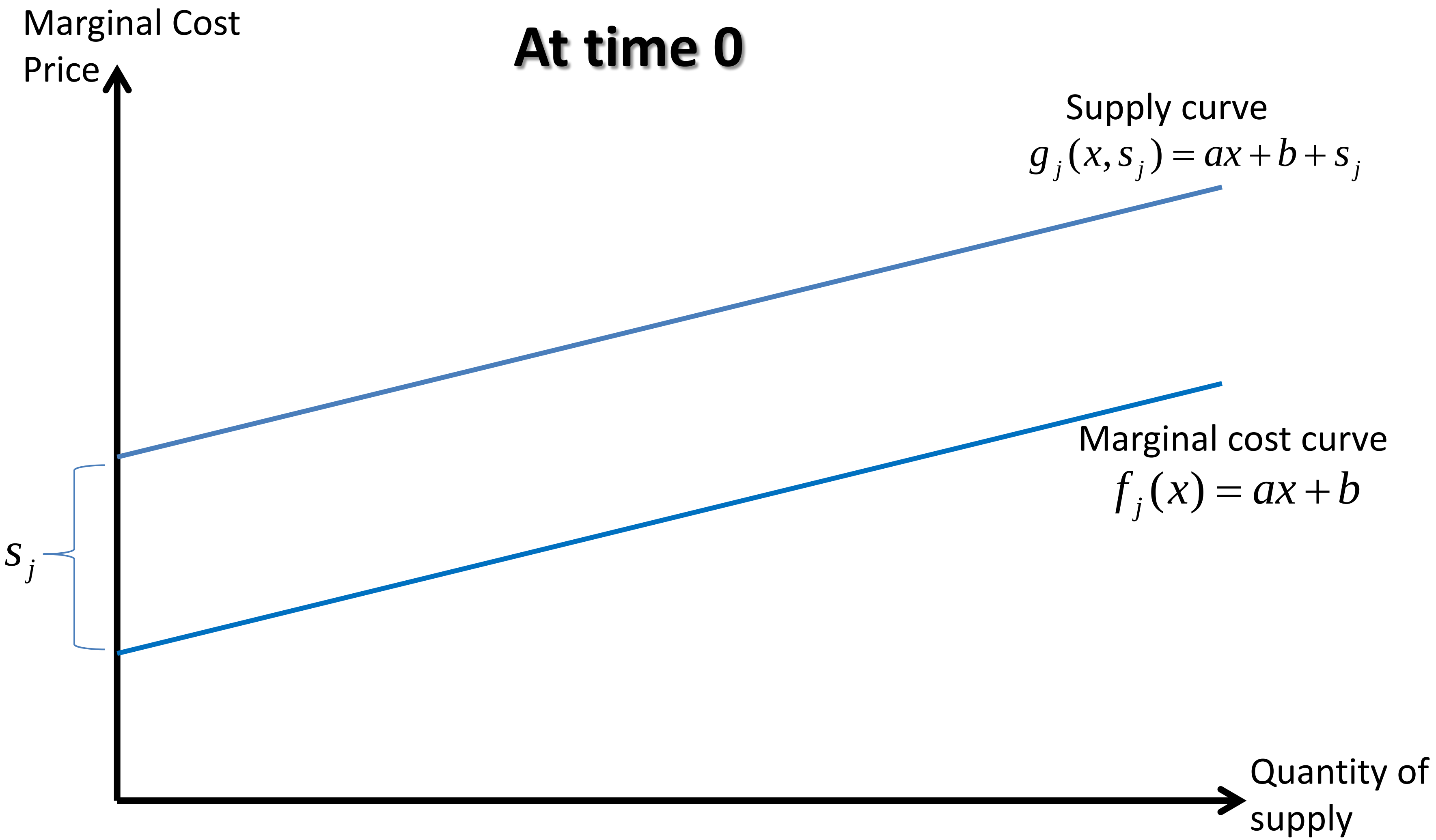

Figure 1: Examples of marginal cost function and supply function

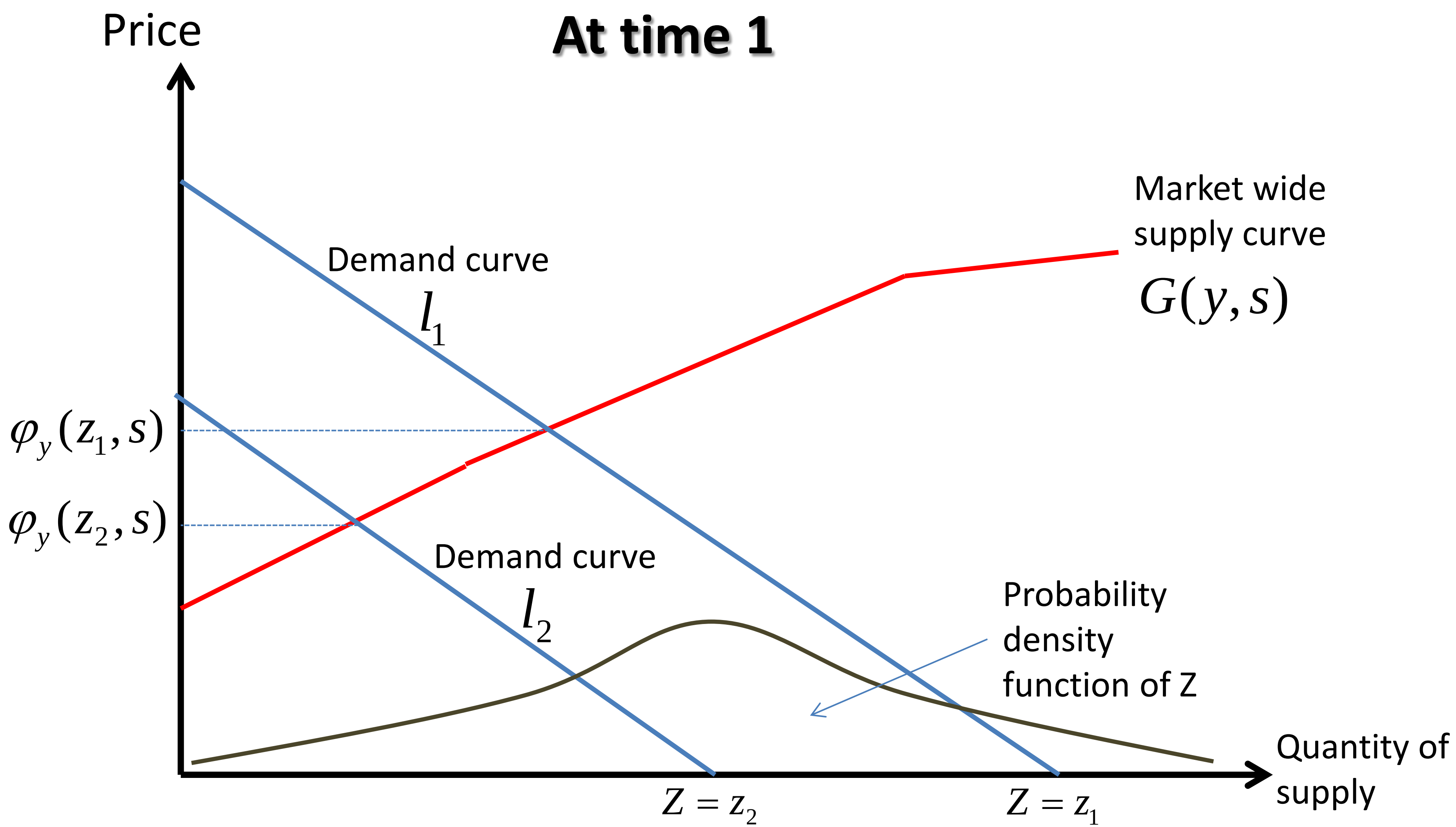

Figure 2: Examples of demand curves and spot prices

Figure 3 Scatter plots for prices and trading volumes