The Tragedy of the Commons : The Dynamic Adjustment under Unregulated Population Growth

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Abstract

The tragedy of the commons is often represented in terms of Nash equilibrium of a static game of complete information. Such elucidation is misleading since it does not capture the dynamics of an underlying process that eventually invites the tragedy as the number of the users increases without limit. Using a bucolic case as an example, this paper examines Hardin's thesis from the standpoint of a dynamic process by elucidating the inherent logic of entry that is inevitable. The key to this process is the devaluation cost that an added number imposes on the average value of cattle. When a finite number of herdsmen use the commons, the stock carried by each herdsman reaches the state of equilibrium when an additional benefit from an added number equals an additional cost comprised of the private marginal cost and the devaluation cost that this number induces on this stock. But, this equilibrium is constantly disrupted by the entry of another herdsman, for at the point of entry there is no need for this entrant to consider the devaluation cost induced by his own action. As the entrant increases his stock and imposes the devaluation cost on all existing stocks, the incumbent herdsmen are forced to cut down their stocks. The adjustment ceases when an additional benefit is balanced again with an additional cost. This process of entry and adjustment continues until the stock of all herdsmen combined approaches its maximum size allowable under a given private marginal cost, while each herdsman's share of this total falls to zero. Thus, the logic of entry and an accompanying adjustment account for the eventual arrival of the day of reckoning unless population growth is checked. The process itself has little to do with a simultaneous move game which has Nash equilibrium, but a temporary equilibrium reached with a finite number of herdsmen is identical to this equilibrium. The problem is that this number increases without limit and that an entrant at any stage finds it profitable to enter the commons. The analysis confirms Hardin's insight that it is overpopulation that eventually leads to the tragedy. The paper also places the issue and the literature in a historical perspective.

Key Words

tragedy of the commons, unregulated population growth, Cournot− Nash equilibrium, devaluation cost, logic of entry, adjustment process

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Contents

I. Introduction

The tragedy of the commons is a class of problems in which the commons may face an inevitable ruin if it is left to unregulated open access to all individuals, who, acting upon their self interest, may seek the private good at the expense of the social good. The class includes a wide range of problems including exploitation of natural resources, pollution of many sorts, habitat destruction, littering, excessive use of public goods, exploitation of outer space, and arms race, only to name a few. The cause of the problem had been recognized since the time of the Ancient Greece, or, possibly, since time immemorial [e.g., Thucydides, *History of the Peloponnesian War*, Book I, Sec. 141 ; Aristotle, *Politics*, Book II, Chapter III, 1261 b ; see also St. Thomas Aquinas, *Summa Theologica*][1] but it took Thomas Malthus, in *An Essay on the Principle of Population*, to scrutinize the implications of the growing population on the tragedy of checks and balances that Nature plays against the fate of mankind. Recently, ecologist Garrett Hardin (1968), partly drawing on William Forster Loyd's (1833) observation on the abuse of the commons, published a controversial paper in *Science* and reawakened us to the fateful ruin of the commons of open free access under the unchecked population growth. In his view, the problem is not technical but moral in nature, for it is our ego−centric behavior that puts us in an unavoidable dilemma between our own gains and the excessive use of the commons that remains accessible to everybody. Hardin suggests that some sort of mutually−agreed−upon coercion may be necessary as a way of keeping the tragedy from setting in, arguing against making an appeal to conscience, for fear that such an appeal may not only arouse deep feeling of guilt and anxieties in man's psyche but also kindle a selective process through which conscience may disappear altogether from the human race. Convinced that the overpopulation is the ultimate cause of the tragedy of the commons, Hardin stresses the importance of controlling it so that the more precious freedoms may be preserved. Whatever values we may hold, be they the traditional values of individualism, freedom, autonomy, and pursuit of happiness or the values of self−sacrificing altruism and the welfare state, the question is whether such values are consistent with the fact that common resources are limited or that the Earth's biosystem is founded on a delicate balance. If the inevitable consequence of human behavior is excessive use of the commons, or, more generally, the tipping of the balance of the Earth's biosystem, the first moral task is how to contain the cause of the tragedy.

Hardin's paper has two important immediate predecessors : K. William Kapp (1950) and Scott Gordon (1954). More immediately, Scott Gordon (1954) presented a pioneering work on the economic theory on the utilization of common−property natural resources (such as fishing, hunting and trapping, petroleum production, and use of common pasture), attempting to grasp the extraction of natural resources specifically as an economic problem, and discussed how the over−extraction or over−utilization of such resources may arise from open access. He wrote :

> Perhaps the most interesting similar case is the use of common pasture in the medieval manorial economy. Where the ownership of animals was private but the resource on which they fed was common (and limited), it was necessary to regulate the use of common pasture in order to prevent each man from competing and conflicting with his neighbors in an effort to utilize more of the pasture for his own animals. Thus, the manor developed its elaborate rules regulating the use of the common pasture, or "stinting" the common : limitations on the number of animals, hours of pasturing, etc., designed to prevent the abuses of excessive individualistic competition.
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> There appears, then, to be some truth in the conservative dictum that everybody's property is nobody's property. Wealth that is free for all is valued by none because he who is foolhardy enough to wait for its proper time of use will only find that it has been taken by another. The blade of grass that the manorial cowherd leaves behind is valueless to him, for tomorrow it may be eaten by another's animal ; the oil left under the earth is valueless to the driller, for another may legally take it ; the fish in the sea are valueless to the fisherman, because there is no assurance that they will be there for him tomorrow if they are left behind today. A factor of production that is valued at nothing in the business calculations of its users will yield nothing in income. Common−property natural resources are free goods for the individual and scarce goods for society. Under unregulated private exploitation, they can yield no rent ; that can be accomplished only by methods which make them private property or public (government) property, in either case subject to a unified directing power.
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> H. Scott Gordon (1954, p. 135).

> An analysis of bionomic equilibrium of the fishing industry may, then, be approached in terms of two problems. The first is to explain the nature of the equilibrium of the industry as it occurs in the state of uncontrolled or unmanaged exploitation of a common−property resource. The second is to indicate the nature of a socially optimum manner of exploitation, which is, presumably what governmental management policy aims to achieve or promote.
>
> H. Scott Gordon (1954, p. 136).

Clearly, the issues are too multi−faceted to draw simple general conclusions on the fate of the commons, local or global, even if they are managed. Stevenson (1991) tested how the protection of common property resources fared in comparison with private property on Swiss alpine grazing lands. His study was mixed in its message, pointing to the difficulty of such tasks. In defining the problem of the tragedy of the commons, Stevenson (1991) argues that a distinction should be made between the common property whose use is limited to a group of members and is subject to certain rules on the one hand and the common property that is held with open access, on the other, whether this access is limited or unlimited. The former is close to the category of private property, whose use is limited to only one person and is left to the discretion of this person. It is the common property of the latter kind that faces the possible problem of excessive use. On this point, Stevenson argues that even the common property with limited user open access falls prey to overuse as long as the extraction is unlimited (p. 58), but whether this is the case as a general proposition should be analyzed carefully. We will come back to this point later in our analysis. We keep in mind that historically many of the commons have been preserved by various forms of rules and norms of the community and management, and that there is no clear evidence that the commons, particularly local commons, was always doomed to overuse [see Dasgupta (1982, 1996, 1997) for a detailed exposition of the multidimensionality of the problem of the commons].

On a much more general level, Hardin's work was preceded by K. William Kapp's work of great importance (1950). He pioneered in the economics of social costs and ecological economics and alerted the economics profession that any economic institution that ignores the disruption of the environment and the social costs of production is fundamentally flawed and that any economic analysis that ignores the true cost of the economic activities in terms of the natural resources that are depleted, polluted, and made non-renewable is misleading. In the preface to the enlarged version of this work, Kapp states that he is reassured of his conviction :

> These long-neglected problems and hard facts have been cited by critics and dissenters from the mainstream of economic thought for more than a century. They are finally attracting world-wide attention, after having been allowed to reach the point of an environmental and ecological crisis in many countries. I am convinced today, as I was when this book was written, that the disruption of man's environment and the social costs resulting from productive activities are among the most fundamental and long-term issues mankind has ever faced. Their implications may well turn out to be as far-reaching as some of the problems raised by nineteenth-century social reformers and socialist critics who concentrated their attention on exploitation, poverty, and economic instability. Ultimately the disruption of man's environment may reveal itself as our most crucial problem, exceeded in overall significance only by the urgent necessity of guaranteeing human survival in an age of nuclear weapons. Just as the new techniques of warfare call for new international institutions, the phenomena of environmental disruption and social costs as well as the urgency of devising effective means of control call for a radical change of the present institutionalized system of investment and decision making by private firms and public agencies with regard to the choice of investment and production patterns.
>
> Kapp (1971, preface)

We shall keep these works by Gordon and Kapp in mind as we proceed to discuss the logic of the tragedy of the commons that Hardin laid out in his essay. Hardin's logic is spelled out clearly as follows :

> The tragedy of the commons develops in this way. Picture a pasture open to all. It is to be expected that each herdsman will try to keep as many cattle as possible on the commons. Such an arrangement may work reasonably satisfactorily for centuries because tribal wars, poaching, and disease keep the numbers of both man and beast well below the carrying capacity of the land. Finally, however, comes the day of reckoning, that is, the day when the long-desired goal of social stability becomes a reality. At this point, the inherent logic of the commons remorselessly generates tragedy.

> As a rational being, each herdsman seeks to maximize his gain. Explicitly or implicitly, more or less consciously, he asks, "What is the utility to me of adding one more animal to my herd?" This utility has one negative and one positive component.
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> 1. The positive component is a function of the increment of one animal. Since the herdsman receives all the proceeds from the sale of the additional animal, the positive utility is nearly +1.
> 2. The negative component is a function of the additional overgrazing created by one more animal. Since, however, the effects of overgrazing are shared by all the herdsmen, the negative utility for any particular decision-making herdsman is only a fraction of −1.
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> Adding together the component partial utilities, the rational herdsman concludes that the only sensible course for him to pursue is to add another animal to his herd. And another .... But this is the conclusion reached by each and every rational herdsman sharing a commons. Therein is the tragedy. Each man is locked into a system that compels him to increase his herd without limit−in a world that is limited. Ruin is the destination toward which all men rush, each pursuing his own best interest in a society that believes in the freedom of the commons. Freedom in a commons brings ruin to all.
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> (Hardin, *Science* Vol. 162, 1968, p. 1244)

His logic consists in: (1) In adding one more cow, the herdsman weighs the gain of the proceeds from the sale of the animal against the cost of overgrazing which is shared by all of the herdsmen. Since the former outweighs the latter, the herdsman is compelled to keep adding more to his herd. (2) This being the case, the herdsman has no choice but to raise as many cattle as possible in order to maximize his own gain (utility). Strictly speaking, the second does not follow from the first. Herdsmen will not be able to maximize their gains by keeping as many cattle as possible even if the first utility exceeds the second invariably. This is a fallacy in Hardin's argument, but we know that Hardin's overriding concern is the problem of unregulated population growth. Therefore, we need to keep the two questions separate: The first is, "How is the total stock of cattle on the commons determined when there are a fixed number of herdsmen," and the second is, "How does this stock change when a new herdsman enters, and what is the limit of this stock as new herdsmen keep entering the commons one by one under unregulated population growth?" If the number of the herdsmen is fixed, the herdsmen's decisions might be conceived as being interdependent in the sense that each herdsman decides on the size of his stock by guessing how many cattle his competitors will raise. We may model the equilibrium of such interdependence by a simultaneous move game much after the Cournot−Nash game of n players [see, e.g., Gibbons (1992) and Osborne (2004) for an exposition of this equilibrium]. But, such modeling is misconceived, for it does not address the dynamic process in which the stock of each herdsman or the combined total stock of all herdsmen changes with entry of new herdsmen under unchecked population growth. When herdsmen utilize the commons, they are not rewriting a simultaneous move game every time a new herdsman enters. What we need is the logic of entry that explains why a new entrant finds it profitable to enter the commons when there are already a number of herdsmen utilizing the commons.

To see how the logic of entry works, we approach the dynamic process inductively, starting with the case of one herdsman monopolizing the commons. The monopolist reaches his equilibrium stock by equating the marginal value and the marginal cost. If this condition is rewritten in terms of the additional benefit from adding an extra animal, which is given by the average value of his stock, and the additional cost, which is composed of the usual marginal cost and the devaluation cost, we see that the monopolist's optimal stock is reached when the average value of his stock equal the marginal cost plus the devaluation cost on his stock. It is this condition that informs why a new entrant finds it profitable to enter the commons. At the point of entry, an entrant can get the value equaling the average value of the stock owned by the monopolist while he has to pay only the marginal cost in entering. He does not have to be pulled back by the devaluation cost on his stock since he is yet to have this stock. It is this difference between the average value of the stock and the marginal cost that drives the entry of a new herdsman. As the entrant increases his stock, the devaluation cost gains its weight, so that new equilibrium is reached with two herdsmen when the average value equals the marginal cost plus the devaluation cost for each party. The monopolist has reduces its quantity, and the entrant expanded his stock, with both owning a stock of the same size. This equilibrium can be approximated by the Cournot–Nash equilibrium of duopoly, but the dynamic process of adjustment is entirely different. The equilibrium of two herdsmen is disturbed by the entry of another entrant by the same logic toward another equilibrium, this time with three herdsmen, which is approximated by the Cournot–Nash equilibrium of oligopoly of three players. By allowing the number of herdsman to increase and tracing this dynamic adjustment, we should be able to see that the ultimate equilibrium is reached when there is no longer any difference between the average value and the marginal cost, which can occur only if the stock owned by any herdsman is so small that the devaluation cost no longer exists. If the marginal cost equals the average cost, there will be no gain from the use of the commons that is overexploited.

Thus, the tragedy of the commons is best represented by the dynamic process based on the logic of entry, not by the Cournot–Nash equilibrium of a simultaneous move game of n players. The temporary equilibrium at any stage of this dynamic process may well be approximated by a sequence of monopoly, duopoly, and oligopoly of increasing numbers of players until the equilibrium (analogous to perfectly competitive equilibrium) is reached with an indefinitely large number of herdsmen utilizing the commons. But, the process does not jump from one Nash equilibrium to another since there is no resetting of the condition under which a simultaneous move game is played. The adjustment is gradual as the incumbents' stocks fall while the entrant's stock increases before new equilibrium is reached. This gradual and continual process from one equilibrium to another is more in line with Hardin's insight on how the tragedy arrives in the end. The value of cattle and the marginal cost of adding a cow are determined in the market. But, any decision to raise more cattle causes devaluation on the market value of the existing stock as this value falls with additional cattle, small as this fall may be. Hence, in each herdsman's calculation of the benefit and the cost of adding an extra cow, the additional benefit from this cow is compared with the additional cost of doing so, which includes not only the private marginal cost of adding a cow but also the devaluation cost consequent upon such addition.

Explicating this cost–benefit calculation and following through the process in which equilibrium shifts with an increasing number of herdsmen, this paper elucidates the inherent logic of the tragedy of the commons that is inevitable under unchecked population growth. Our analysis assumes that the function that represents the average value of cattle is shared by all potential users of the commons and that the cost of raising cattle is identical for all such users. This allows us to represent the temporary equilibrium for a finite number of herdsmen as the symmetric Nash equilibrium. To repeat, this representation of equilibrium as a symmetric Nash equilibrium is devoid of any dynamics inherent in the tragedy, but is applied here only to locate equilibrium. But, the best response function of each incumbent herdsman, reinterpreted properly, indicates how he will change his optimal stock when a new herdsman starts using the commons. The novelty of our analysis lies in the explication of the logic of entry that shows why an entrant finds it profitable to enter the commons, and in the demonstration that the equilibrium that a given number of herdsmen reaches is identical to the Nash equilibrium of a simultaneous move game of these herdsmen although such a game is never played on the field.

2. Analysis

Our analysis starts with the case of one herdsman monopolizing the use of the commons. It may seem tedious, but this analysis is necessary to elucidate our point. Upon entering the commons, the first user increases his stock of cattle one by one, by weighing the additional benefit against the additional cost of adding a number. The former equals the average market value of the stock of cattle raised, while the latter is comprised of the private marginal cost and the devaluation cost on the value of the stock (which is initially zero because a stock is yet to be owned). When the devaluation cost becomes large enough, the first user as a monopolist reaches his equilibrium, at which the additional benefit is balanced with the additional cost. We then show why a new entrant finds it profitable to enter the commons. With his entry, the entrant disrupts the monopolist's equilibrium. He keeps increasing his stock as long as the average market value exceeds the additional cost including the devaluation cost on the stock he owns. But, as the entrant's stock increases, the first user's stock is devalued as the market value falls. This adjustment process, with the entrant increasing his stock and the incumbent reducing his stock, is completed when the additional benefit and the additional cost are balanced for both herdsmen, and this equilibrium is identical to the Cournot–Nash equilibrium of a duopoly game that would obtain if the two herdsmen were thought to play the game hypothetically. Following this logic of entry and adjustment, we show how the herdsmen's stocks of cattle move over time as the number of herdsmen increases through entry, passing through the temporary equilibrium that is identified with the Cournot–Nash equilibrium of a simultaneous move game of n herdsmen.

2 a. The case of one herdsman

Let the market value of cattle be derived from the market demand for the products from the cattle raised. Assume that this value is a decreasing function of the number of cattle, and that this function is linear as in (1). We may call this function the market valuation curve. Once this valuation is written, the rest follows straightforwardly.

(1) $V(Q) = a - bQ$,

where $V(Q)$ represents the average value of the cattle; Q is the number of the cattle; a and b are constant coefficients. This function defines the total value $TV(Q)$ and the marginal value $MV(Q)$ as functions of Q.

(2) $TV(Q) = V(Q)Q = (a - bQ)Q$,
(3) $MV(Q) = a - 2bQ$.

Assume that the cost of maintaining the stock of the cattle increases with Q in a linear fashion.

(4) $C(Q) = cQ$

where c is a constant representing the average as well as the marginal cost of this maintenance.

The herdsman maximizes his profit in value terms, $\Pi(Q)$, defined as the difference between $TV(Q)$ and $C(Q)$. This difference is the net value (the value net of the cost), but we call it profit to draw an analogy between this net value and the profit in the theory of firms.

(5) $\Pi(Q) = TV(Q) - C(Q)$
$= V(Q)Q - cQ = (a - c)Q - bQ^2$,

from which, upon differentiation, is obtained the profit maximization condition (which requires that the marginal value equal the marginal cost of maintenance).

(6) $a - 2bQ = c$,

which gives the profit maximizing stock as

(7) $Q = \frac{a-c}{2b}$.

The average value of the cattle when the herdsman keeps this number is given by

(8) $V\left(\frac{a-c}{2b}\right) = \frac{1}{2}(a+c)$.

The herdsman's profit at this number, therefore, equals

(9) $\Pi\left(\frac{a-c}{2b}\right) = (a-c)\left(\frac{a-c}{2b}\right) - b\left(\frac{a-c}{2b}\right)^2 = \frac{(a-c)^2}{4b}$.

All of this is straightforward. To show the logic of our argument, we illustrate this case using a familiar diagram shown in Figure 1. The profit maximization takes place at point B where the marginal value is equal to the marginal cost. Instead of characterizing the equilibrium in such terms, it is more useful for our purpose to characterize it in terms of an additional benefit and an additional cost, defined as follows: If the herdsman adds one more cow at point A, the additional benefit obtained from this cow equals $\frac{1}{2}(a+c)$, which is the market value at A, but doing so incurs the additional cost which arises from two sources: the marginal (private) cost and *an induced devaluation cost* on his existing stock, which, in this case, amounts to $b\left(\frac{a-c}{2b}\right)$ (that is, as the stock is increased by one, it loses its value by b times this stock). Thus, at point A, the additional benefit $\frac{1}{2}(a+c)$ equals the additional cost $c + b\left(\frac{a-c}{2b}\right)$. That is,

(10) $\frac{1}{2}(a+c) = c + b\left(\frac{a-c}{2b}\right)$.

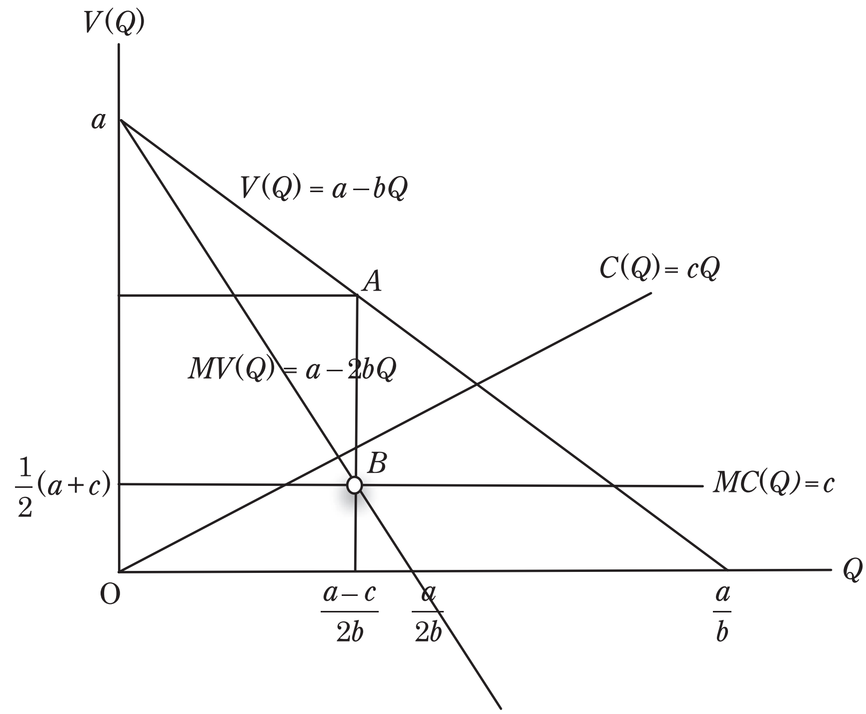


Figure 1

This is obviously equivalent to condition (6) above, which is expressed in terms of the marginal benefit $a-2bQ$ and the marginal cost c The monopolizing herdsman cannot pass any part of the devaluation cost to any third party. Notice that the devaluation cost is practically nonexistent initially when his stock is small, which means that the value of cattle (an additional benefit) is higher than an additional cost (which is composed of only the marginal cost). As the monopolist increases his stock one by one, the additional benefit (the market value of his cattle) remains greater than the additional cost even if the devaluation cost is taken into account. That is, for any Q less than the equilibrium value, it holds that

(11) $c+bQ<a-bQ$.

This is an alternative expression of the condition $c<a-2bQ$ (the marginal revenue being greater than the marginal cost), but it makes the devaluation cost explicit. His equilibrium is reached when his stock becomes large enough to establish the identity :

(12) $c+bQ=a-bQ$.

This monopolist case indicates that if a new competitor joined him and raised his stock by quantity k, the monopolist demand curve becomes truncated with its intercept changed to $a-bk$. This implies that the market value of his cattle cannot be any higher than $a-bk$. With this hypothetical competitor, he would face the market valuation curve starting from this intercept.

(13) $V(Q)=(a-bk)-bQ$.

Hence, his profit would be

(14) $\Pi(Q) = V(Q)Q - cQ = (a - bk - c)Q - bQ^2$.

This allows us to write the monopolist's stock as a function of the stock of a hypothetical competitor, which is analogous to his best response function in the context of a game.

(15) $Q = \frac{a - bk - c}{2b}$.

In the light of this function, the monopolist case can be viewed as a special case in which $k = 0$. This may appear trivial, but the function (15) makes it possible to see how the monopolist as an incumbent adjusts his stock when a newly entered herdsman starts raising his stock gradually as well as to obtain the new equilibrium with two herdsmen (at this equilibrium, it holds $Q = k = q^*$, so that the new equilibrium stock equals $q^* = \frac{a - c}{3b}$ for each herdsman). Note that the condition (15) can be rewritten as

(15)′ $c + bQ = (a - bk) - bQ$,

which shows that the additional benefit on the right side is balanced with the additional cost (the marginal cost c plus the devaluation cost bQ) on the left side.

Obviously, the number of cattle that the monopolist raises can be interpreted as the social optimum since the monopolist extracts the maximum profit from the commons, which would be what the social planner would do if the commons were entrusted to his care. The social optimum is achieved in both cases because the cost of the induced devaluation on the existing stock is included in the cost–benefit analysis.

For the sake of comparison with subsequent cases with multiple herdsmen, let the results of this monopoly case be summarized.

The summary : The case of monopoly

The profit maximizing stock : $Q = \frac{a - c}{2b}$

The average value of the cattle at the profit maximizing stock : $V\left(\frac{a - c}{2b}\right) = \frac{1}{2}(a - c)$

The maximum profit : $\Pi\left(\frac{a - c}{2b}\right) = \frac{(a - c)^2}{4b}$

The equilibrium condition :

$$\frac{1}{2}(a + c) = c + b\left(\frac{a - c}{2b}\right)$$

The additional benefit equals the additional cost (the marginal cost plus the devaluation cost). This is identical to $a - 2bQ = c$ (i.e., the marginal value equals the marginal cost)

As long as the additiona cost is less than the additional benefit (i.e., $c + bQ < a - bQ$), the stock increases.

With the first comer extracting the maximum profit from the commons, a new herdsman finds it profitable to enter the commons. The logic of this entry needs to be elucidated. A new entrant compares the additional benefit from his first cow (the prevailing market value of cattle) with the marginal cost of having this cow, and finds that the former, $\frac{1}{2}(a+c)$, is greater than the latter, c. At the time of entry, no stock is possessed by the entrant ; therefore, there is no need to be held back by the devaluation cost induced by his own action. Knowing that $\frac{1}{2}(a+c) > c$, he enters the commons. Note that condition (10) is met at the equilibrium of the monopolist since he owns a stock sufficiently large to cause enough devaluation on his own stock, but it is precisely this condition that gives a potential entrant enough incentives to enter the commons, for the benefit from his first number exceeds the additional cost which, for the time being, consists only of the marginal cost. This shows that as long as there is another potential herdsman waiting to enter the commons, he will not hesitate.

3. The case of two herdsmen

The monopolist's decision shows how he would react to the entry of his potential competitors. Once a new herdsman enters, this user's number increases as long as the market value per head is greater than the marginal cost plus the devaluation cost. The entrant's stock induces devaluation on the stock of the incumbent due to a fall in the market value, which pushes up the additional cost above the additional benefit. The adjustment will cease when the new equilibrium is reached, at which the market value per head is equal to the sum of the marginal cost and the devaluation cost on the stock of each herdsman. That is, when $Q = k = q^*$ in (15), it holds that $\frac{a-c}{3b} = \frac{a - \frac{a-c}{3b} - c}{2b}$. We call this equilibrium the *adjustment equilibrium*. Again, this equilibrium is identical to the Cournot−Nash equilibrium of a hypothetical game in which the incumbent and the entrant play a simultaneous move game, for only at this equilibrium the cost −benefit balance with devaluation taken into account is restored at the margin. We repeat that this is not the way the equilibrium is reached when a new herdsman enters the commons. The incumbent is already using the commons, and a new entrant disturbs his original equilibrium, but the two herdsmen's stocks, one increasing and the other decreasing, approach the equilibrium $\frac{a-c}{3b}$. At this stock the average value of cattle $a - b\left(2\frac{a-c}{3b}\right) = \frac{a+2c}{3b}$ is equal to the marginal cost c plus the devaluation cost $b\,\frac{a-c}{3b}$ for each herdsman ; i.e., $\frac{a+2c}{3b} = c + b\,\frac{a-c}{3b}$.

To draw a parallelism between the adjustment equilibrium and the Cournot−Nash equilibrium of a simultaneous move duopoly game, let it be assumed that all players are symmetric. This assumption allows us to obtain the Cournot−Nash equilibrium as a symmetric Nash equilibrium. We also show that at the strategic level, the two player's strategies are strategically substitutable (meaning that an increase in the strategy of one player causes a decrease in the strategy of the other player),

For the sake of this formal identity, let the numbers of cattle raised by the two herdsmen be denoted by q_1 and q_2, and their strategy spaces by $[0,\infty)$. The herdsmen are facing the market valuation curve given in (1) : $V(Q) = a - bQ$, where $Q = q_1 + q_2$. But, the valuation function that each herdsman faces with his anticipation on the stock of the other herdsman is represented by a truncated one :

(16) $V^1(q_1,q_2) = (a - bq_2) - bq_1,$

(17) $V^2(q_1,q_2) = (a - bq_1) - bq_2.$

With these functions, the payoff functions of the herdsmen are given by :

(18) $\Pi^1(q_1,q_2) = V^1(q_1,q_2)q_1 - C(q_1) = (a - b(q_1 + q_2))q_1 - cq_1,$

(19) $\Pi^2(q_1,q_2) = V^2(q_1,q_2)q_1 - C(q_1) = (a - b(q_1 + q_2))q_2 - cq_2.$

Maximization of these payoffs gives :

(20) $\dfrac{\partial\ \Pi^1(q_1,q_2)}{\partial q_1} = a - bq_2 - c - 2bq_1 = 0,$

(21) $\dfrac{\partial\ \Pi^2(q_1,q_2)}{\partial q_2} = a - bq_1 - c - 2bq_2 = 0.$

The best response function of each herdsman against the anticipated move by the other is, therefore, given by :

(22) $q_1^*(q_2) = \dfrac{a - bq_2 - c}{2b} = \dfrac{a-c}{2b} - \dfrac{1}{2}q_2,$

(23) $q_2^*(q_1) = \dfrac{a - bq_1 - c}{2b} = \dfrac{a-c}{2b} - \dfrac{1}{2}q_1.$

The Cournot−Nash equilibrium takes place at the intersection of these functions. The best response functions of the two herdsmen, each with the intercept $\frac{a-c}{2b}$ and slope $-\frac{1}{2}$, are drawn in Figure 2. The Cournot−Nash equilibrium is obtained as $\left(q_1 = \dfrac{a-c}{3b},\ q_2 = \dfrac{a-c}{3b}\right)$.

Fig. 3 shows how the best response function is obtained graphically. If herdsman 1 anticipates that herdsman 2's number is q_2, then he knows that the value of his cattle will be no more than $a - bq_2$. Therefore, his effective valuation curve, under his anticipation, is a truncated one starting at point A, and the marginal value schedule starts at point A with the slope $-2b$. This schedule intersects the marginal cost

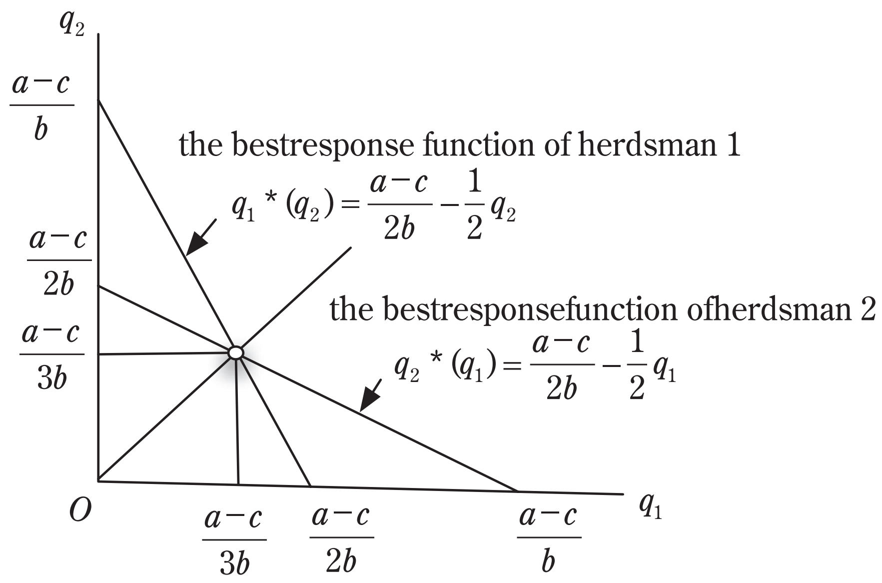


Figure 2

curve at $q_1 = \dfrac{a - bq_2 - c}{2b}$, which gives the best response function $q_1 = \dfrac{a-c}{2b} - \dfrac{1}{2}q_2$. In the bottom graph, the payoff function of herdsman 1 is drawn along with the marginal payoff function: $MV^1(q_1,q_2) = (a - bq_2) - 2bq_1$. Since the former is quadratic, the latter is linear. Both the payoff function and the marginal payoff function shift with a change in the number of cattle of herdsman 2.

In Fig. 4 are drawn again the four functions: the market valuation function, the marginal valuation function, the payoff function, and the marginal payoff function, when the number of cattle of herdsman 2 is given. They are now drawn in reference to the same origin. As seen, the marginal value function shifts downward as the market valuation function of herdsman 1 shifts to the left as the stock of herdsman 2 increases. Likewise, the payoff function of herdsman 1 becomes smaller as the stock of herdsman 2 increases, and this causes the marginal payoff function of herdsman 1 to shift downward. The fact that this function shifts downward can be seen by differentiating the marginal payoff function with respect q_2.

(24) $$\Pi^1_{12} \equiv \frac{\partial}{\partial q_2}\frac{\partial\, \Pi^1(q_1,q_2)}{\partial q_1} = \frac{\partial}{\partial q_2}((a - bq_2 - c) - 2b_1q_1) = -b < 0,$$

which means that herdsman 1's marginal payoff declines with an increase in the rival's number q_2. We also see that herdsman 1's payoff declines with an increase in his own quantity q_1.

(25) $$\Pi^1_{11} \equiv \frac{\partial}{\partial q_1}\frac{\partial\, \Pi^1(q_1,q_2)}{\partial q_1} = -2b < 0.$$

The properties (24) and (25) are related to the slope of the best response function. To see how, consider $\Pi^1_1 \equiv \dfrac{\partial\, \Pi^1(q_1,q_2)}{\partial q_1}$:

(26) $$\Pi^1_1(q_1,q_2) \equiv \frac{\partial\, \Pi^1(q_1,q_2)}{\partial q_1} = a - bq_2 - c - 2bq_1.$$

If the best response function $q_1^*(q_2)$ is substituted in, (26) vanishes identically.

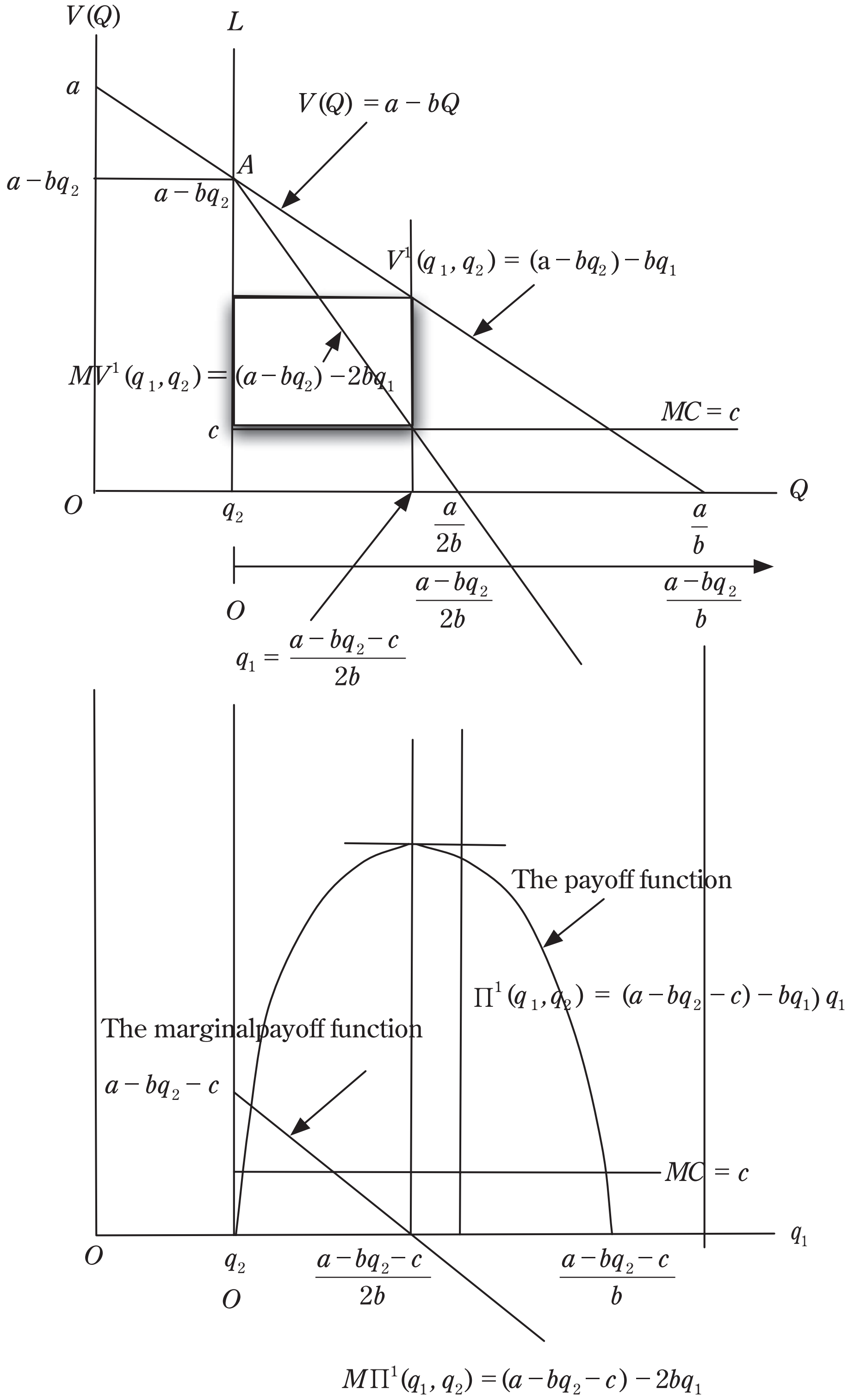
(27) $$\Pi^1_1 \equiv \frac{\partial\, \Pi^1(q_1^*(q_2),q_2)}{\partial q_1} = a - bq_2 - c - 2bq_1^*(q_2) \equiv 0.$$

Differentiating this identity function with respect to q_2 gives

(28) $$\frac{dq_1^*(q_2)}{dq_2} = -\frac{\Pi^1_{12}(q_1,q_1^*(q_2))}{\Pi^1_{11}(q_1,q_1^*(q_2))} = -\frac{-b}{-2b} = -\frac{1}{2} < 0.$$

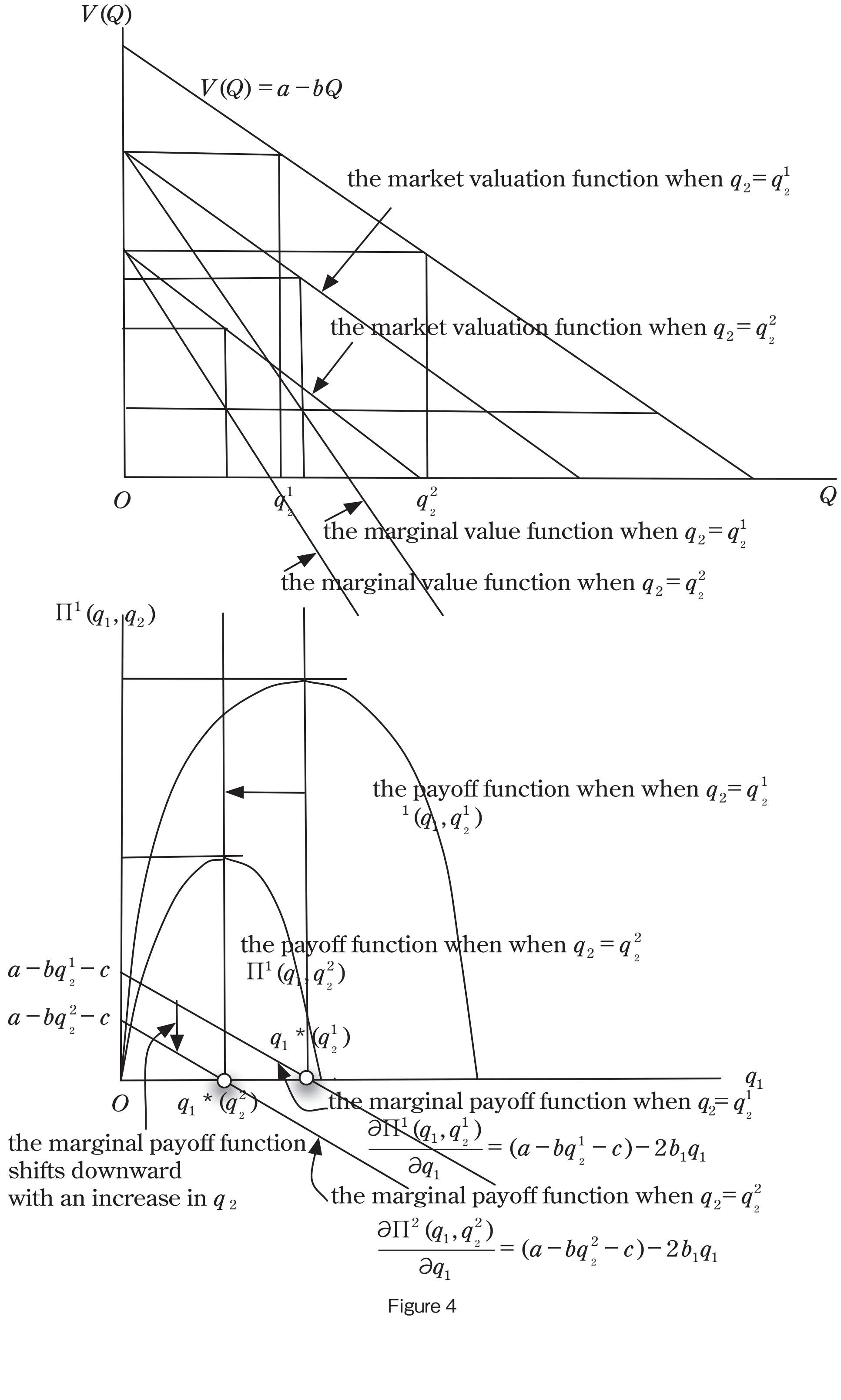
Thus, the slope of the best response function is related to Π^1_{12} and Π^1_{11}.

Cooper and John (1988) identified the condition $\Pi^1_{12}(q_1,q_1^*(q_2)) > 0$ as the strategic complementarity and the condition $\Pi^1_{12}(q_1,q_1^*(q_2)) < 0$ as the strategic substitutability. Following this identification, we see that as long as $\Pi^1_{11}(q_1,q_1^*(q_2))$ is negative, the strategic complementarity is identical to the condition that the best response function is positively sloped, and the strategic substitutability to the condition that this function is negatively sloped. Whether we have strategic complementarity or strategic substitutability is critical to the question of the multiplicity of equilibria in a game under analysis. If a game is symmetric in that all players share the same payoff function, the Nash equilibrium, if it exists, will be represented as a sym-



V(Q)
L
a
V(Q) = a − bQ
A
a − bq2
a − bq2
V1(q1, q2) = (a − bq2) − bq1
MV1(q1, q2) = (a − bq2) − 2bq1
MC = c
c
O
q2
a/2b
a/b
Q
O
(a − bq2)/2b
(a − bq2)/b
q1 = (a − bq2 − c)/2b
The payoff function
Π1(q1, q2) = (a − bq2 − c) − bq1) q1
The marginalpayoff function
a − bq2 − c
MC = c
q1
O
q2
O
(a − bq2 − c)/2b
(a − bq2 − c)/b
MΠ1(q1, q2) = (a − bq2 − c) − 2bq1

Figure 3



$V(Q)$
$V(Q) = a - bQ$
the market valuation function when $q_2 = q_2^1$
the market valuation function when $q_2 = q_2^2$
O
q_2^1
q_2^2
Q
the marginal value function when $q_2 = q_2^1$
the marginal value function when $q_2 = q_2^2$
$\Pi^1(q_1, q_2)$
the payoff function when when $q_2 = q_2^1$
$\Pi^1(q_1, q_2^1)$
the payoff function when when $q_2 = q_2^2$
$\Pi^1(q_1, q_2^2)$
$a - bq_2^1 - c$
$a - bq_2^2 - c$
$q_1^*(q_2^1)$
q_1
O
$q_1^*(q_2^2)$
the marginal payoff function when $q_2 = q_2^1$
$\frac{\partial \Pi^1(q_1, q_2^1)}{\partial q_1} = (a - bq_2^1 - c) - 2b_1 q_1$
the marginal payoff function shifts downward with an increase in q_2
the marginal payoff function when $q_2 = q_2^2$
$\frac{\partial \Pi^2(q_1, q_2^2)}{\partial q_1} = (a - bq_2^2 - c) - 2b_1 q_1$

Figure 4

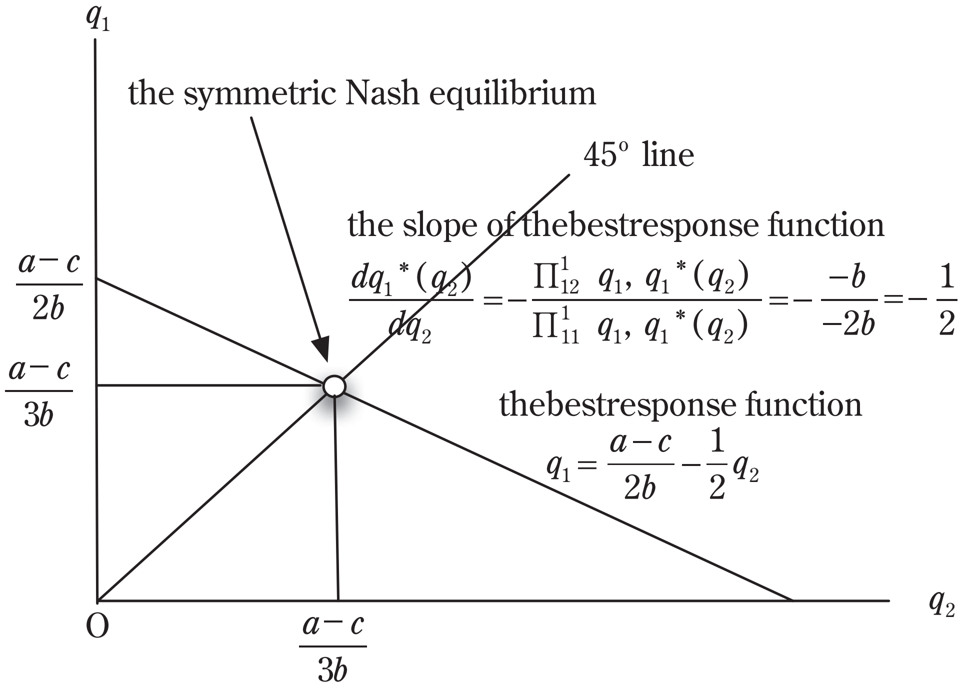


Figure 5

metric equilibrium in which all strategies will be identical across the players. The question then is whether a game has multiple symmetric equilibria or at most one equilibrium. Cooper and John pointed out that the strategic complementarity, or, in more general terms, the condition that the best response function is positively sloped, is a necessary condition for the multiplicity of the symmetric Nash equilibrium. The condition is necessary but not sufficient since it is conceivable to have cases in which the best response function, even if positively sloped, may intersect the 45° line only once. Here, in our analysis, the two herdsmen are symmetric as they share the same payoff function. The best response function is negatively sloped as the payoff function satisfies the strategic substitutability : $\Pi^1_{12}(q_1, q_1^*(q_2)) < 0$. Thus, there is only one symmetric Nash equilibrium in our case.

Let us view the best response function from the standpoint of the strategic substitutability as in Fig. 5 and to identify the unique symmetric Nash equilibrium along the 45° line by intersecting the best response function with this line. This way of finding the unique Nash symmetric equilibrium can be applied to all cases in which an arbitrary number of herdsmen are playing a simultaneous move game, as will be shown.

The best response functions, as written in (22) and (23), were obtained with this generality in mind. Intersecting these best response function with the 45° line (i.e., $q_2 = q_1$) gives

(29) $q_1 = \frac{a-c}{2b} - \frac{1}{2}q_1,$

from which the symmetric Nash equilibrium is obtained : $q_1 = q_2 = q = \frac{a-c}{3b}$. Or, alternatively, combining the best response functions (22) and (23) gives

(30) $q_1 + q_2 = \frac{a-c}{b} - \frac{1}{2}(q_1 + q_2)$.

Applying the notion of symmetric Nash equilibrium, i.e., $q_1 = q_2 = q$, we obtain the same solution : $q = \frac{a-c}{3b}$.

When the symmetric Nash equilibrium is reached, the total number of cattle equals $\frac{2(a-c)}{3b}$. At this quantity, the additional benefit of increasing the cattle stock by one must be balanced with the additional cost (the marginal cost c and the devaluation cost on the stock of each herdsman). To see that such balance holds at the Nash equilibrium, compute the market value per head at this equilibrium.

(31) $V\left(\frac{2(a-c)}{3b}\right) = a - b\frac{2(a-c)}{3b} = \frac{a+2c}{3}$.

Increasing the cattle stock by one assures each herdsman the additional benefit (value) of $\frac{a+2c}{3}$. But, doing so incurs the additional cost comprised of the marginal cost c and the induced devaluation of each herdsman's cattle stock amounting to $b\left(\frac{2(a-c)}{3b}\right)$. These costs add up to $c + b\left(\frac{2(a-c)}{3b}\right)$, which equals $\frac{a+2c}{3}$, hence matches the market value (the additional benefit to be received). Thus, the additional benefit is balanced with the additional cost for each herdsman at the Nash equilibrium.

(32) $\frac{a+2c}{3} = c + b\left(\frac{a-c}{3b}\right)$.

This equilibrium is identical to *the adjustment equilibrium*, which is reached as the incumbent reduced and the entrant increased the stock of their possession. If, in this calculation, each herdsman took into account the devaluation that his action induces on the entire stock (of both herdsmen), the following condition would hold.

(33) $\frac{a+2c}{3} < c + b\left(\frac{a-c}{3b}\right) + b\left(\frac{a-c}{3b}\right)$.

Faced with this condition, each herdsman would reduce his stock to the level given by $\frac{a-c}{4b}$ so that the total stock settles at $\frac{a-c}{2b}$, and the market value per head of each herdsman's cattle would be $\frac{1}{2}(a+c)$; at this equilibrium, for each herdsman, it holds that $\frac{1}{2}(a+c) = c + b\frac{a-c}{4b} + b\frac{a-c}{4b}$. This is exactly the equilibrium of the monopolist case. This point should be emphasized. If each herdsman's calculation of devaluation cost were extended to the rival's stock, equilibrium would be the same as the monopolist case. Of course, such equilibrium would never be reached, for the egocentric cost–benefit calculation of each herdsman ignores the devaluation effect of their own actions on the value of the rival's stock.

These conditions are shown in Fig. 6, where point A represents the market value of the cattle at the symmetric Nash equilibrium, where the total number equals $\frac{2(a-c)}{3b}$. The market value at this total is given by the distance AE. If each herdsman adds one more to his stock, it incurs the marginal cost c (the

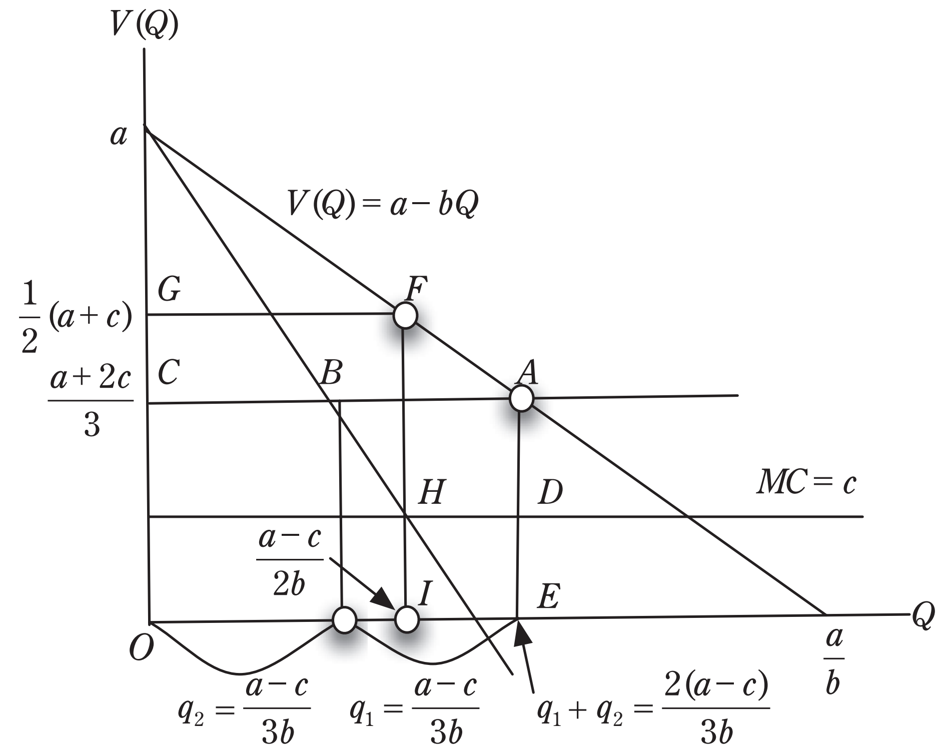


Figure 6

distance DE) plus the induced devaluation on his stock, which is the distance BA times the slope of the market valuation curve in absolute value (b). Hence, the following equality holds

(34) $\text{DistanceAE} = \text{DistanceDE} + |b|\,(\text{DistanceBA})$,

which is what condition (32) says. If all of the devaluation on the entire stock of cattle were taken into account by each herdsman, it would hold :

(35) $\text{DistanceAE} < \text{DistanceDE} + |b|\,(\text{DistanceBA} + \text{DistanceCB})$,

which is a graphical meaning of condition (33) above. Conditions (32) and (34), and condition (33) and (35), make it clear why the tragedy of the commons is caused by each individual's egocentric seeking of self–interest that ignores the externalities of his action on the conditions faced by others.

Compare this case of duopoly with that of monopolist analyzed above. The monopolist maximizes his value by choosing $Q = \frac{a-c}{2b}$, which is valued in the market at $V\left(\frac{a-c}{2b}\right) = \frac{1}{2}(a+c)$; therefore, his equilibrium is conditioned by the equality between the additional benefit given by the market value $\frac{1}{2}(a+c)$ and the additional cost, which is the sum of the marginal cost c and the induced devaluation cost of his stock $b\left(\frac{a-c}{2b}\right)$. It was this condition that was represented by (10) above.

(10) $\frac{1}{2}(a+c)=c+b\left(\frac{a-c}{2b}\right)$

In Fig. 6, the additional benefit is represented by the distance FI and the additional cost by the sum of the two distances: the distance HI and the distance GF times b. The monopolist takes into account the entire additional cost, for he has to bear all of the devaluation cost on the entire stock already in the field. Since the monopolist's equilibrium is at $Q=\frac{a-c}{2b}$, the tragedy of the commons does not arise through his action. His additional benefit is balanced with the sum of the marginal cost and the devaluation cost.

But, as we saw earlier, it is the prospect of getting a share of the profit extracted by the incumbents that invites the entry of other herdsmen into the commons. At the point of entry, the second herdsman knows that his cost of raising his first cow is given by c whereas this cow is valued in the market at $\frac{1}{2}(a+c)$ which is greater than c; he does not have any stock yet to be concerned about the induced devaluation cost. As he increases his stock, he will weigh the falling market value with the rising devaluation cost plus the constant marginal cost. The stock will be increased as long as the former is greater than the latter, and equilibrium is reached when the two are balanced. But, as a new entrant increases his stock to q_2, the market value of the cattle falls to $\frac{1}{2}(a+c)-bq_2$, hence the incumbent herdsman, faced with the condition:

(36) $\frac{1}{2}(a+c)-bq_2<c+b\left(\frac{a-c}{2b}\right)$,

will be forced to change his stock by Δq_1 to $q_1=\frac{a-c}{2b}+\Delta q_1$ in order to restore the balance:

(37) $\frac{1}{2}(a+c)-b\ (\Delta q_1+q_2)=c+b\left(\frac{a-c}{2b}-\Delta q_1\right)$.

But, the new entrant keeps adding to his stock as long as it holds

(38) $\frac{1}{2}(a+c)-b\ (q_1+q_2)>c+bq_2$.

This again forces the incumbent to reduce his stock even further. The process of the entrant adding to his stock and the incumbent reducing his stock continues until the market value is matched with the sum of the marginal cost and the devaluation cost on each herdsman's stock, i.e., until the condition: $\frac{a+2c}{3}=c+b\frac{a-c}{3b}$ is attained, which shows that the market value is equal to the marginal cost plus the devaluation cost on the stock of each herdsman. This is identical to the symmetric Nash equilibrium with $q_1=q_2=\frac{a-c}{3b}$. Since each herdsman considers the devaluation effect of his action only on his own stock (ignoring the effect on the rival's stock), the total stock ends up exceeding the monopolist's herd size. At the symmetric Nash equilibrium, each herdsman's profit equals $\frac{(a-c)^2}{9b}$.

(39) $\Pi^1(q_1,q_2)=V^1(q_1,q_2)\,q_1-C\,(q_1)=\frac{a-c}{3b}\left(\frac{a+2c}{3}-c\right)=\frac{(a-c)^2}{9b}=\Pi^2(q_1,q_2)$.

Adding the two, the total profit amounts to

(40) $\Pi^1(q_1,q_2)+\Pi^2(q_1,q_2)=\frac{2(a-c)^2}{9b}$,

which is less than the monopolist's profit : $\dfrac{(a-c)^2}{4b}$.

(41) $\dfrac{(a-c)^2}{4b} > \dfrac{2(a-c)^2}{9b}$.

To summarize, the equilibrium of the monopolist is disrupted by the entry of a new herdsman. The new entrant increases his number, and the incumbent reduces his number, and the equilibrium that is reached in the limit is the same as the symmetric Nash equilibrium of two herdsmen playing a simultaneous move game. But, the logic is entirely different.

Summary :

Each herdsman's number of cattle at the Nash equilibrium : $\dfrac{a-c}{3b}$

The market value at the Nash equilibrium : $V\left(\dfrac{2(a-c)}{3b}\right) = \dfrac{a+2c}{3}$

The profit of each herdsman : $\Pi^1(q_1, q_2) = \Pi^2(q_1, q_2) = \dfrac{(a-c)^2}{9b}$

The total profit of the two herdsmen combined : $\dfrac{2(a-c)^2}{9b}$

The total profit of the two herdsmen combined is less than the total profit of the monopolist.

$\dfrac{2(a-c)^2}{9b} < \dfrac{(a-c)^2}{4b}$

4. The case of three herdsmen

With the two incumbent herdsmen getting a positive profit from the use of the commons, another herdsman inevitably enters the commons. His first number brings the marginal benefit (the market value) of $\dfrac{a+2c}{3}$ while the marginal cost is given by c ; at the time of entry, he does not have any stock to be affected by a fall in the average value of cattle. Thus, with no stock, it holds that

(42) $\dfrac{a+2c}{3} > c$.

As the entrant adds his first number to his stock, the market value of cattle possessed by the two incumbents $\dfrac{2(a-c)}{3b}$ falls by b. As a result, the equilibrium condition (32) : $\dfrac{a+2c}{3} = c + b\left(\dfrac{a-c}{3b}\right)$ will be disturbed to

(43) $\dfrac{a+2c}{3} - b < c + b\left(\dfrac{a-c}{3b}\right)$.

In restoring the balance, the incumbents have to reduce their stocks. As long as the additional benefit exceeds the additional cost for the entrant, his stock keeps increasing and the incumbents' stocks are adjusted downward. As the market price falls, the devaluation cost rises. This process continues until the new equilibrium is reached, which can be identified by the symmetric Nash equilibrium with three herdsmen.

To identify *adjustment equilibrium* by way of this identity, write the truncated market valuation function of the first herdsman when the second and third herdsmen's stocks are q_2 and q_3.

(44) $V^1(q_1,q_2,q_3) = (a - b\ (q_2 + q_3)) - bq_1.$

His payoff function is given by

(45) $\Pi^1(q_1,q_2,q_3) = V^1(q_1,q_2,q_3)\,q_1 - C\,(q_1) = (a - b\ (q_2 + q_3) - c)\,q_1 - bq_1^2.$

And, maximization of this function with respect to q_1 gives

(46) $\dfrac{\partial\,\Pi^1(q_1,q_2)}{\partial q_1} = a - b\ (q_2 + q_3) - c - 2bq_1 = 0,$

from which is obtained the best response function :

(47) $q_1 = \dfrac{a-c}{2b} - \dfrac{1}{2}(q_2 + q_3).$

At the symmetric Nash equilibrium, it holds : $q = q_1 = q_2 = q_3$. Hence, from (47) the Nash equilibrium (as the symmetric Nash equilibrium) is obtained as

(48) $q = \dfrac{a-c}{4b}.$

In Fig. 7, the best response function (47) is drawn as a function of a symmetric action by herdsmen 2 and 3, represented by q_{-1} ; by a symmetric action by herdsmen 2 and 3 is meant that they are taking the same action. As in the case of duopoly, the strategic substitutability holds : As in (28), combining $\Pi^1_{11}(q_1,q_1^*(q_{-1})) = -b < 0$ and $\Pi^1_{11}(q_1,q_1^*(q_{-1})) = -2b$, we see that the slope of the best response function is negative with slope $-\frac{1}{2}$, which means that as $q_1 + q_2$ increases, q_1 decreases at the rate of $\frac{1}{2}$. If the best response function is drawn as a function of a symmetric action as in Figure 7, the Nash equilibrium is ob-

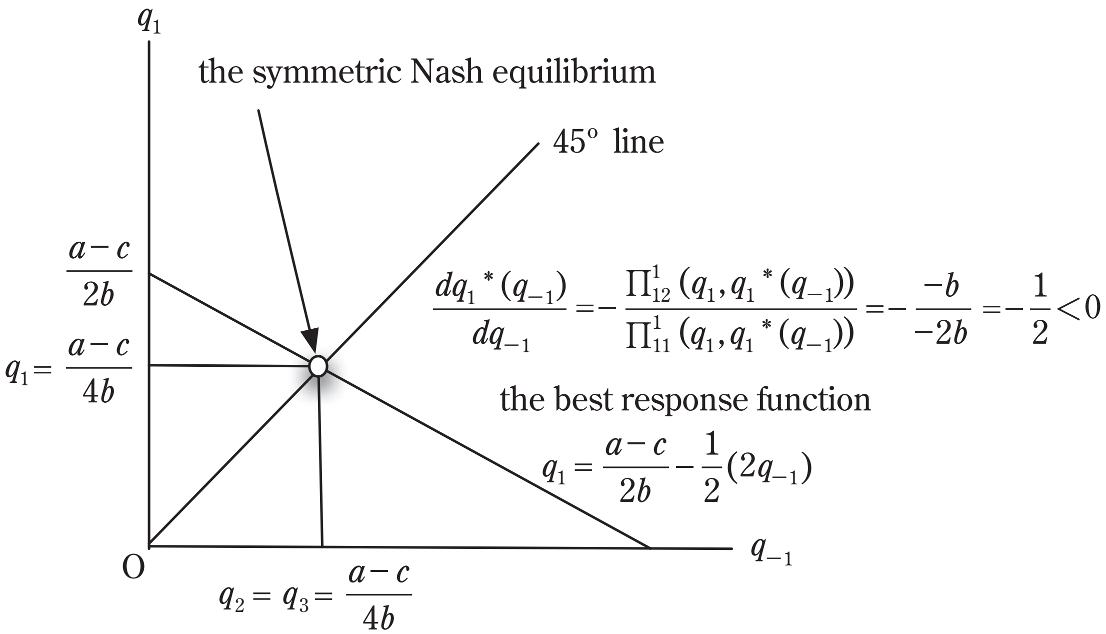


Figure 7

tained by intersecting it with the 45° line.

Summary : The case of three herdsmen

The Nash equilibrium herd size of each herdsman : $q = q_1 = q_2 = q_3 = \dfrac{a-c}{4b}$

The combined total stock on the commons : $q_1 + q_2 + q_3 = \dfrac{3(a-c)}{4b}$

The average market value of the cattle : $\dfrac{a+3c}{4}$

The profit of each herdsman : $\dfrac{(a-c)^2}{16b}$

The combined total profit of all herdsmen : $\dfrac{3(a-c)^2}{16b}$, which is less than the total profit in the duopoly case, $\dfrac{2(a-c)^2}{9b}$, which, in turn, is less than the profit in the monopolist case, $\dfrac{(a-c)^2}{4b}$.

5. The case of n herdsmen

At the Nash equilibrium of the three herdsmen, another entrant enters the commons, for the benefit from his first cow, which equals the market value $\dfrac{a+3c}{4}$, is greater than the marginal cost c. As the herd size of the entrant increases, the incumbent herdsmen are forced to adjust their stocks downward. The entrant keeps adding to his stock as long as

(49) $\dfrac{a+3c}{4} - b\,(\varDelta q_1 + \varDelta q_2 + \varDelta q_3 + q_4) < c + bq_4.$

At the same time, the incumbent herdsmen keep reducing their stocks, which will mitigate the fall in the market value. This process of inreasing and reducing finally ends at the next equilibrium, which is obtained as the symmetric Nash equilibrium of a simultaneous move game of the four herdsmen. The market valuation function that herdsman 1 faces is now given by

(50) $V^1(q_1, q_2, q_3, q_4) = (a - b\,(q_2 + q_3 + q_4)) - bq_1.$

The profit maximization condition of herdsman 1 gives the following best response function.

(51) $q_1 = \dfrac{a-c}{2b} - \dfrac{1}{2}(q_2 + q_3 + q_4)$ or

At the symmetric Nash equilibrium, it holds that $q_1 = q_2 = q_3 = q_4$; hence, this equilibrium is obtained at $q_1 = q_2 = q_3 = q_4 = \dfrac{a-c}{(n+1)\,b}$ where $n = 4$. The market value per head at the equilibrium equals

(52) $V\left(\dfrac{4(a-c)}{5b}\right) = a - b\,\dfrac{4(a-c)}{5b} = \dfrac{a+4c}{5}.$

And the profit of each herdsman amounts to

(53) $\left(\dfrac{a+4c}{5} - c\right)\dfrac{a-c}{5b} = \dfrac{(a-c)^2}{25b}.$

This process of a new entrant entering the commons and disrupting the equilibrium of the incumbent herdsmen continues, with an increasing number of herdsmen, until all values that can be obtained from the commons are exhausted. To see this, let the best response function of herdsman 1 be written as in (54), where there are $n-1$ incumbent herdsmen.

(54) $$q_1 = \frac{a-c}{2b} - \frac{1}{2}(q_2 + q_3 + q_4 + \cdots q_{n-1})).$$

At the symmetric Nash equilibrium, the stocks of all herdsmen take the same value. Hence, the stock of each herdsman equals

(55) $$q = \frac{a-c}{(n+1)b},$$

and the market value per head at the equilibrium equals

(56) $$V\left(\frac{n(a-c)}{(n+1)b}\right) = \frac{a+nc}{n+1},$$

The profit that each herdsman gets amounts to

(57) $$\Pi^i(q_i, q_{-i}) = \frac{(a-c)^2}{b(n+1)^2},$$

and the total profit of all herdsmen combined to

(58) $$\sum_{i=1}^{n} \Pi^i(q_i, q_{-i}) = \frac{n(a-c)^2}{b(n+1)^2},$$

which is less than the total profit of the preceding case where there are n herdsmen. That is,

(59) $$\sum_{i=1}^{n} \Pi^i(q_i, q_{-i}) = \frac{n(a-c)^2}{b(n+1)^2} < \sum_{i=1}^{n} \Pi^i(q_i, q_{-i}) = \frac{(n-1)(a-c)^2}{bn^2}.$$

At the equilibrium with n herdsmen, the market value per head of cattle is higher than the marginal cost c.

(60) $$\frac{a-nc}{n+1} > c.$$

Faced with this condition, a new herdsman enters the commons and starts raising his cattle. In the limit as the number of herdsmen increases without limit, each herdsman's stock converges to zero, and the combined total stock to $\frac{a-c}{b}$:

(61) $$\lim_{n\to\infty} \frac{a-c}{(n+1)b} = 0,$$

(62) $$\lim_{n\to\infty} \frac{n(a-c)}{(n+1)b} = \lim_{n\to\infty} \frac{a-c}{\left(1+\frac{1}{n}\right)b} = \frac{a-c}{b}.$$

The market value per head of cattle converges to the level of the marginal cost c.

(63) $$\lim_{n\to\infty} \frac{a+nc}{n+1} = \lim_{n\to\infty} \frac{\frac{a}{n}+c}{1+\frac{1}{n}} = c.$$

The profit that each herdsman gets and the profit of all herdsmen combined will converge to zero:

(64) $\lim_{n\to\infty}\Pi^i = \lim_{n\to\infty}\frac{(a-c)^2}{b\,(n+1)^2} = 0,$

(65) $\lim_{n\to\infty}\sum_{i=1}^{n}\Pi^i\,(q_i, q_{-i}) = \lim_{n\to\infty}\frac{n\,(a-c)^2}{b\,(n+1)^2} = \lim_{n\to\infty}\frac{(a-c)^2}{b\left(n+2+\frac{1}{n}\right)} = 0.$

Figure 8 shows how the Nash equilibrium shifts with an increase in the population of herdsmen. As long as the population size remains fixed, the equilibrium can be represented by the Nash equilibrium with a positive value obtained by all herdsmen. But, the tragedy of the commons is not the tragedy caused by a fixed number of herdsmen, but rather it is an inevitable outcome resulting from an indefinite increase in the population. In this regard, we recall Hardin's point that it is the unchecked population growth that causes the abuse of the commons. If the number of the herdsmen is fixed, regardless of the externalities that one herdsman's decision causes on the value of the stocks owned by the others, the commons reaches an equilibrium that can be formally identified by the Nash equilibrium. This equilibrium shifts as the number of herdsmen increases, with a consequent fall in the market value per head of cattle, but, as demonstrated, at any finite equilibrium the market value exceeds the marginal cost. It is this fact that entices a

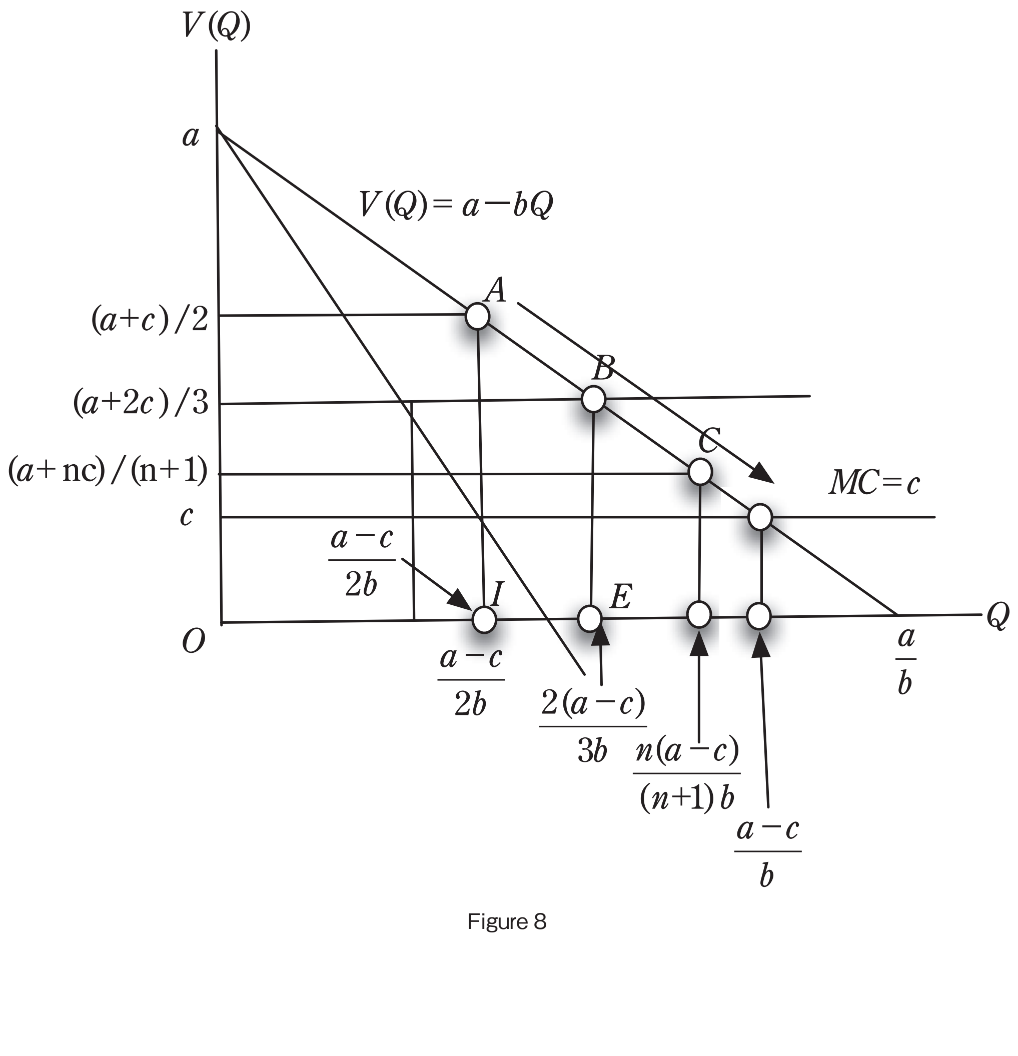


Figure 8

new herdsman to enter the commons, and it is the population growth that perpetuates the entry of a new herdsman to already crowded commons. With this entry, the market value of cattle keeps falling. Eventually, each herdsman's share in the total stock of cattle falls to zero while this total stock converges to the maximum number that can be sustained under the marginal cost, which is given by $(a-c)/b$. The profit that each herdsman can extract from the commons falls to zero, and so does the profit of all herdsmen combined.

In Fig. 9, the adjustment process is shown schematically, with the number of the herdsmen increasing one by one over time. The graph shows that at any stage the entrant increases his number and the incumbents reduce their stocks, both toward the same equilibrium number. The total stock approaches its maximum size $(a-c)/b$ while the stock of each herdsman approaches its limit, zero. Note that at any stage the incumbents reduce their stocks and the entrant increases his stock, all to the same equilibrium level.

It is possible that many new herdsmen may enter the commons at the same time at any stage of temporary equilibrium (adjustment equilibrium), or that new herdsmen keep entering before this equilibrium is reached. Such cases can be analyzed without any problem, and the process of exploitation proceeds with a much faster pace. The path of adjustment over time in Fig. 9, for both entrants and incumbents, becomes

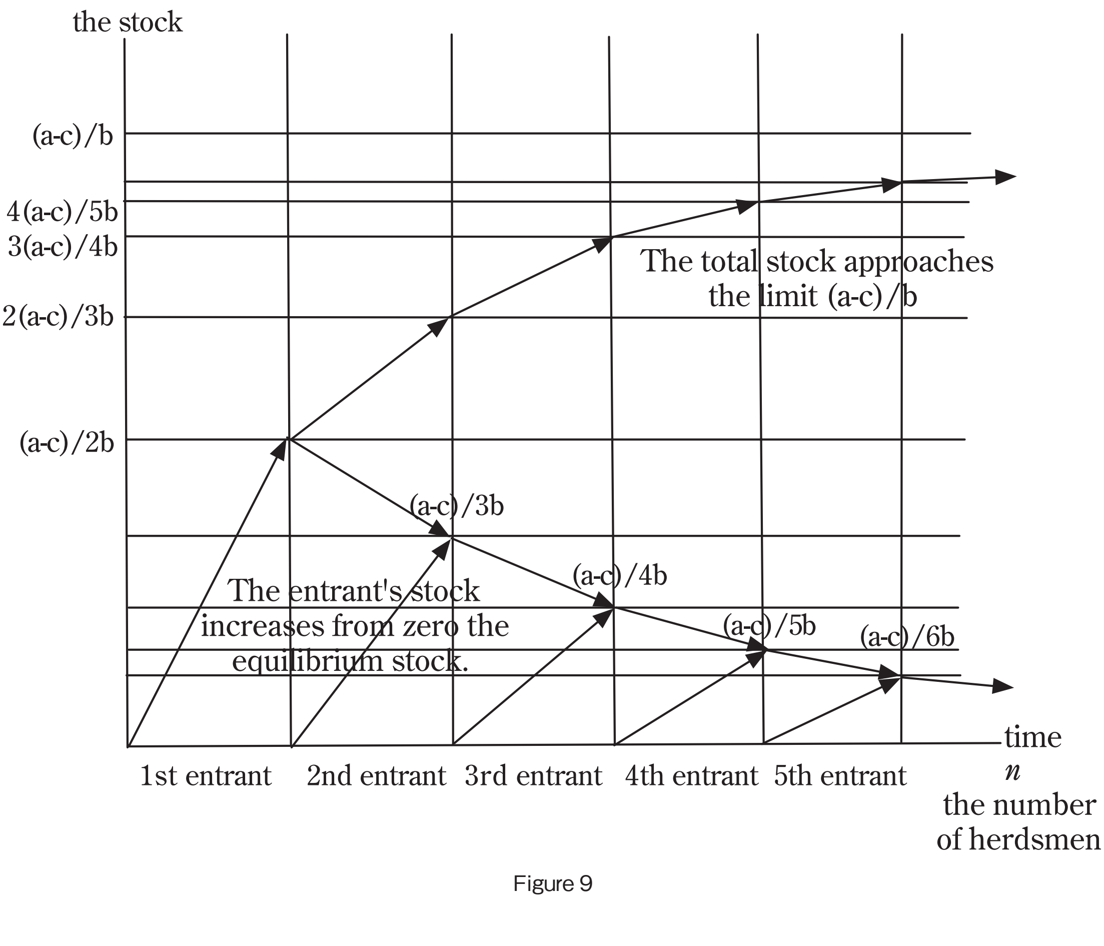


Figure 9

more drastic in early stages and asymptotically approaches the limit faster : $(a-c)/b$ for the total stock and zero for the stock of each individual herdsman. This analysis shows that Hardin's logic that a herdsman gets the full benefit from an added animal, i.e., the positive utility of +1, while he suffers only a fraction of the negative utility of −1 from overglazing needs to be modified. At any stage, as long as the number of herdsmen stays constant, equilibrium is reached, although it is still true that the positive utility outweighs the negative utility. At this equilibrium, each herdsman gets the full benefit of the average value of the cattle in the market but this benefit is balanced with the cost of adding another animal comprised of the private marginal cost and the devaluation that his action induces on his own stock. The fact that a herdsman can share the negative utility from overglazing with other herdsmen does not by itself account for the arrival of the tragedy. The tragedy of the commons, represented game theoretically, stresses the fact that each herdsman does not pay for the negative externalities that his decision exerts on the value of the stocks owned by other herdsmen. This account is equally incomplete, for it does not address the fundamental problem created by the entry of a new herdsman, who finds it profitable to enter at any equilibrium reached by a finite number of herdsmen. We have demonstrated here that it is the logic of entry and an accompanying adjustment process that is ultimately response for the arrival of the day of reckoning.

6. The difficulty of colluding

At any state in which there is a finite number of herdsmen, the state of the commons reaches an equilibrium that can formally be identified by the Nash equilibrium. If the number of herdsmen remains fixed at n, a collusion, based on trigger strategies, is possible if the discount rate for future profits is small enough (or the discount factor applied to future profits are close enough to one). If such a collusion is formed, the value that the monopolist would extract from the commons would be distributed equally among the members with each restraining his stock to a fraction $1/n$ of the monopolist's stock. A collusion of this kind will, however, be fated to dissolve as soon a new herdsman (under the unchecked population growth) enters the commons by the logic that he pays only the marginal cost (no devaluation cost) while getting the full market value for his first number, and nothing can stop him from doing so, as the commons is open to anybody who desires to use it. If collusion is ill−fated, the commons may be protected by way of social norms, rules, and conventions. Dasgupta (1986, 1996, 1997) discussed many cases in which local commons, as opposed to global commons, have been preserved in this way. But, whether commons are local or global, as long as the population keeps increasing, it will become increasingly difficult to rely on social norms and conventions as a control device as norms themselves will be forced to change under the pressure from overpopulation. It is, therefore, difficult to negate Hardin's point that it is the unchecked population growth that underlies the problem of the tragedy of the commons.

7. Competition in the product market and the use of the commons

Perfect competition often serves as a reference model, by which to evaluate the performance of a given market structure for its efficiency. Our analysis has shown that at each equilibrium with a finite number of herdsmen the market value per cattle head exceeds the marginal cost. If another herdsman enters the commons and supplies his product to the market, the market price falls and comes closer to the marginal cost, which will be considered beneficial to the consumers. For this reason, we have been receptive to the entry of new herdsmen as a desirable movement toward higher market efficiency. With such permissiveness, however, we leave the use of the commons unchecked without imposing the social costs on the users. The entry of new herdsmen is inevitable since the market value of cattle is always greater than the marginal cost at any equilibrium temporarily reached by a finite number of herdsmen. As an entrant increases his stock, the incumbents will be forced to cut back their stocks, and this adjustment process leads to the next equilibrium, which will be disturbed again by another entrant. The process continues with a constant flow of new comers, until the commons is exhausted to its limit stopped only by the private marginal cost. If the efficiency of the goods market is closely related to an excessive use of the commons, our incessant support of competition in the goods market has to be balanced with an equally conscientious effort to control the entry of the users into the commons, which, in the long−run, is probably possible only if the population growth is placed under human control.

8. Concluding remarks

Rather than viewing the problem of the tragedy of the commons as a simultaneous move game of n players, this paper examined the problem from the standpoint of an adjustment process that accommodates an increasing number of the users. Because the users of the commons are not playing a simultaneous move game at any stage, a Nash equilibrium characterization of the tragedy of the commons as resulting from a simultaneous move game is misrepresenting the problem. What has to be demonstrated is the logic of entry that drives the dynamic process that is responsible for the arrival of the tragedy. If the number of herdsmen stays fixed, the commons reaches an equilibrium when, for each individual herdsman, the market value per head (which defines the additional benefit from an extra head added) is balanced with the additional cost of adding to his stock, which is comprised of the private marginal cost and the devaluation cost that his decision induces on his own stock. This equilibrium is broken as soon as a new user enters the market. For this user, the marginal benefit from adding an animal to his stock, which is given by the average value of the entire cattle raised on the commons, is always greater than the cost of adding, for the devaluation cost induced by his decision on his stock is non−present at the point of entry, although this entry imposes the devaluation cost on the stocks owned by the incumbent users. It is precisely this condition that invites the entry of a new herdsman. The entry is inevitable. As the number of herdsmen increases from n to $n+1$, the entrant increases his stock while the incumbent herdsmen are forced to cut back their stocks because of the devaluation cost imposed by the entrant. This process continues until new equilibrium is reached, in which, for each herdsman again, the marginal benefit is equalized with the additional cost comprised of the private marginal cost and the induced devaluation cost on his own stock. The new equilibrium is identical to the Nash equilibrium of a simultaneous move game of the $n+1$ users, but this equilibrium is not reached as a result of a simultaneous move game reset with $n+1$ herdsmen. The total stock on the commons increases from one temporary equilibrium to another (each approximated by the Nash equilibrium), approaching asymptotically to the maximum stock that is bounded by the private marginal cost alone, at which no more entry can occur. In this limit, the profit that each herdsman obtains from the commons falls to zero, and so does the combined total profit received by all herdsmen. The commons is exploited to its full, with no more extractable profit left. In Hardin's words, this stationary state is the long−desired goal of social stability.

We have thus confirmed Hardin's insight that it is the unchecked population growth that underlies the tragedy of the commons as its fundamental cause, although his logic itself is misrepresented. It is certainly true that the total stock of many users exceeds what the social planner prescribes as the optimal stock, and the total profit they get as a whole always falls short of the profit obtainable from the social optimum. One may ask why the incumbents stop at their equilibrium stocks, when they know that new entrants will come and take away some of their equilibrium shares. The answer to this question lies in the fact that raising extra cattle is profitable only for a new entrant, who does not own any stock waiting to be devalued by his own action. This is not true for the incumbents who already possess their equilibrium stocks. If they increase their stocks, such decisions induce the devaluation cost on their own stocks, thereby tipping the balance between the additional benefit and the additional cost. If the market valuation curve (analogous to the market demand curve) shifts outward with the growth of the economy, the stock owned by each herdsman at any finite equilibrium will increase, for an increase in the market value justifies adding to his stock as long as this value exceeds the sum of the marginal cost and the devaluation cost. With such shifting, the limit on the use of the commons is pushed even further, and the seriousness of the tragedy will be so much greater. Many of the commons are thus fated to be exploited beyond recovery.

Hardin's logic of the tragedy of the commons was based on the computation of the positive utility of adding one more cow relative to the negative utility due to overgrazing. He argued that since the latter is shared by all herdsmen, hence is less than the former, every herdsman is compelled to increase his herd without limit, ultimately to the ruin of the commons. This logic, based on the pursuit of ego-centric interest at the neglect of the induced impact of each herdsman's decision on the utilities of the rest of the group, led him to deplore the fate of the commons as long as the commons is left to the invisible hand. We have demonstrated that this logic is misleading, the reason being that it is not to the benefit of the herdsmen to increase their stocks without limit as long as the number of herdsmen is finite. With a finite number, there will always be an equilibrium with a finite herd size, since each herdsman finds the profit maximizing stock by equating the additional benefit with the additional cost arising from the private marginal cost and the induced devaluation cost. How a new equilibrium is reached through the process in which a new entrant disturbs the status-quo and in which the incumbents are forced to cut back their stocks is very different from the way an entrant and the incumbents, resetting the initial conditions, play a simultaneous move game. This is why one needs to look at this process to examine what sort of adjustment actually takes place from one temporary equilibrium to another. This paper has demonstrated that a new herdsman's decision to enter the commons puts the benefit-cost calculus of the incumbent herdsmen out of balance through the devaluation cost induced by the entrant's decision to have his own stock. The process ends with the maximum utilization of the commons under the given cost structure, with each herdsman getting an infinitely small portion of the total profit that is nearly zero, therefore extracting practically no profit from the commons. This is the doomsday picture of the ultimate stability in which every herdsman is forced to live with the minimum subsistence while the commons, so precious to the society as a whole, is ruined beyond hope of recovery.

How to keep the tragedy of the commons from emerging is a political (moral) issue. To control the unchecked population growth, Hardin (1968) suggested that some sort of mutual coercion (a system of coercion that is mutually agreed upon) will be necessary. But, in order for such coercion to be tenable, the society must share a common set of values leading to a certain criterion of judgment that can be used to compare what is inherently incommensurable with some agreed–upon weights as well as an administrative system that can be used to enforce the needed coercion. While Hardin speaks for this coercion, Crowe (1969) alerts us to the difficulty of attaining it by pointing out that the common value system, the monopoly of coercive force, and the fair administering of the coercive power are increasingly eroded in our times. In an extension to his own paper, Hardin (1998) questions his long–held Adam Smith's insight that the decisions of individuals who seek self–interest will lead to the best state of affairs (in terms of allocation of scarce resources), in favor of Lloyd's picture (1833) of what actually happened to the commons when greedy herdsmen made free use of it. As shown in this analysis, herdsmen seeking self–interest are well aware that the additional benefit and the additional cost of their actions must be balanced, which implies that as long as the number of herdsmen is fixed, there will be a definite equilibrium, which is reached when the total herd size has reached a size large enough so that, together with an accompanying market value, the marginal cost and the devaluation cost add up precisely to this market value. We should keep this point separate from the ultimate cause of the tragedy of the commons itself. Hardin's point is well taken that that it is the unregulated population growth that is ultimately responsible for the arrival of the tragedy of the commons.

In an article that addresses Hardin's logic and the related moral issues from a more general standpoint, Elliott (1997) proposes that it is best to understand Hardin's thesis as a thought experiment analogous to Einstein's theory of relativity *vis–à–vis* Newtonian physics. It is a thought experiment in the sense that Hardin laid bare the logic of how the tragedy becomes a reality, without any factual statement. Elliott points out that the assumptions of individualism and the free market system are not essential to Hardin's argument, because human behavior, whether grounded in self–serving economic interest or other–serving altruism, has, in his words, a built–in feedback mechanism that feeds economic and population growth. Elliott argues that the tragedy of the commons is caused by any activity that increasingly exploit the finite ecosystem, regardless of the motivations that drive it, whether in industrialized economies or developing countries (part II). It is worth heeding what he says on the four general premises that cause the tragedy of the commons: (1) The Earth is finite. (2) Larger natural commons are a limited biosystem, which cannot be sustained by an increasing number of any organism. (3) The human beings, for the first time, are exceeding the capacity of the ecosystem. (4) The exploitation of the Earth's resources is caused not only by the pursuit of self–interest of individuals but also by the self–sacrificing altruists who work for the elimination of human miseries and injustice (part III). Faced with this destructive situation, the human civilization is at a point of juncture to examine how our moral values and principles can be modified so that they may become more consistent with the sustenance of the precious biosystem of the Earth, thereby assuring the continuation of our freedoms. Any moral principles that cannot be sustained under the restraint of the ecosystem of the Earth are no longer viable.

As a final word, it should be mentioned that Hardin advocates interdisciplinary approaches to solving the problems of over–utilization of common resources before it becomes too late. This is not an easy task since different disciplines have their own ways of theorizing, which are often opposed to each other. But, as Hardin says, we should not forget to focus on "the nature of things" when different hands are joined across diverse disciplines, social and natural.

Footnotes

1. The observation that the humans are inclined to care about things that are owned and to care little about public or common properties has been made by a number of prominent figures throughout the history. In ancient Greece, Thucydides wrote in *The History of the Peloponnesian War* :

They devote a very small fraction of the time to the consideration of any public object, most of it to the prosecution of their own objects. Meanwhile, each fancies that no harm will come to his neglect, that it is the business of somebody else to look after this or that for him ; and so, by the same notion being entertained by all separately, the common cause imperceptibly decays.

Thucydides, *The History of the Peloponnesian War*, Book I, Sec. 141.

Likewise, Aristotle also observed :

That all persons call the same thing mine in the sense in which each does so mny be a fine thing, but it is impracticable ; or if the words are taken in the other sense, such a unity in no way conduces to harmony. And there is another objection to the proposal. For that which is common to the greatest number has the least care bestowed upon it. Every one thinks chiefly of his own, hardly at all of the common interest ; and only when he is himself concerned as an individual. For besides other considerations, everybody is more inclined to neglect the duty which he expects another to fulfill ; as in families many attendants are often less useful than a few.

Aristotle, *Politics* II, Chapter III, 1261 b.

In the 13th century, St. Thomas Aquinas, in *Summa Theologica*, addresing the question of theft and robbery, wrote, in Article 1 : Whether it is natural for man to possess external things?

I answer that, External things can be considered in two ways. First, as regards their nature, and this is not subject to the power of man, but only to the power of God Whose mere will all things obey. Secondly, as regards their use, and in this way, man has a natural dominion over external things, because, by his reason and will, he is able to use them for his own profit, as they were made on his account : for the imperfect is always for the sake of the perfect, as stated above (Question 64, Article 1). It is by this argument that the Philosopher proves (Polit. i, 3) that the possession of external things is natural to man. Moreover, this natural dominion of man over other creatures, which is competent to man in respect of his reason wherein God's image resides, is shown forth in man's creation (Genesis 1 : 26) by the words : "Let us make man to our image and likeness : and let him have dominion over the fishes of the sea," etc.

This answer is followed by Article 2 : Whether it is natural for man to possess external things?, in which he writes :

I answer that, Two things are competent to man in respect of exterior things. One is the power to procure and dispense them, and in this regard it is lawful for man to possess property. Moreover this is necessary to human life for three reasons. First because every man is more careful to procure what is for himself alone than that which is common to many or to all: since each one would shirk the labor and leave to another that which concerns the community, as happens where there is a great number of servants. Secondly, because human affairs are conducted in more orderly fashion if each man is charged with taking care of some particular thing himself, whereas there would be confusion if everyone had to look after any one thing indeterminately. Thirdly, because a more peaceful state is ensured to man if each one is contented with his own. Hence it is to be observed that quarrels arise more frequently where there is no division of the things possessed.

The second thing that is competent to man with regard to external things is their use. On this respect man ought to possess external things, not as his own, but as common, so that, to wit, he is ready to communicate them to others in their need. Hence the Apostle says (1 Timothy 6: 17–18): "Charge the rich of this world ... to give easily, to communicate to others," etc.

Summa Theologica, Question 66: Theft and Robbery

In *Human Action*, von Mises writes:

...... If land is not owned by anybody, although legal formalism may call it public property, it is utilized without any regard to the disadvantages resulting. Those who are in a position to appropriate to themselves the returns—lumber and game of the forests, fish of the water areas, and mineral deposits of the subsoil—do not bother about the later effects of their mode of exploitation. For them the erosion of the soil, the depletion of the exhaustible resources and other impairments of the future utilization are external costs not entering into their calculation of input and output. They cut down the trees without any regard for fresh shoots or reforestation. In hunting and fishing they do not shrink from methods preventing the repopulation of the hunting and fishing grounds. In the early days of human civilization, when soil of a quality not inferior to that of the utilized pieces was still abundant, people did not find any fault with such predatory methods. When their effects appeared in a decrease in the net returns, the ploughman abandoned his farm and moved to another place. It was only when a country was more densely settled and unoccupied first class land was no longer available for appropriation, that people began to consider such predatory methods wasteful. At that time they consolidated the institution of private property in land. They started with arable land and then, step by step, included pastures, forests, and fisheries. The newly settled [p. 657] colonial countries overseas, especially the vast spaces of the United States, whose marvelous agricultural potentialities were almost untouched when the first colonists from Europe arrived, passed through the same stages. Until the last decades of the nineteenth century there was always a geographic zone open to newcomers—the frontier. Neither the existence of the frontier nor its passing was peculiar to America. What characterizes American conditions is the fact that at the time the frontier disappeared ideological and institutional factors impeded the adjustment of the methods of land utilization to the change in the data.

In the central and western areas of continental Europe, where the institution of private property has been rigidly established for many centuries, things were different. There was no question of oil erosion of formerly cultivated land. There was no problem of forest devastation in spite of the fact that the do-

mestic forests had been for ages the only source of lumber for construction and mining and of fuel for heating and for the foundries and furnaces, potteries and glass factories. The owners of the forests were impelled to conservation by their own selfish interests. In the most densely inhabited and industrialized areas up to a few years ago between a fifty and a third of the surface was still covered by first–class forests managed according to the methods of scientific forestry.

Human Action, XXIII : The Data of the Market,
Sec. 6, The Limits of Property Rights and
Problems of External Costs and External
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