# Development of Rigorous Methods for the Radar Cross Section Analysis of Complicated Structures

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Analysis of the scattering from open-ended metallic waveguide cavities has received much attention recently in connection with the prediction and reduction of the radar cross section (RCS) of a target. A number of two- and three-dimensional (2-D and 3-D) cavity diffraction problems have been analyzed thus far by means of high-frequency ray techniques and numerical methods, but it appears that the solutions obtained by these approaches are not uniformly valid for arbitrary cavity dimensions.

In this research, we have considered several 2-D and 3-D waveguide structures that can form cavities, and have analyzed diffraction problems by means of the Wiener-Hopf technique, which is known as a powerful tool for analyzing wave scattering problems involving canonical structures. In the following, we shall report some of the important results obtained in this research. The time factor is assumed to be $e^{-i\omega t}$ and suppressed throughout this report.

We consider a 2-D finite parallel-plate waveguide with four-layer material loading, being illuminated by an *E*-polarized plane wave, as shown in Fig. 1, where $-L < D_1 < D_2 < D_3 < D_4 < D_5 < L$ and *E* polarization implies that the incident electric field is parallel to the *y*-axis. The waveguide plates are perfectly conducting and of zero thickness, and the four material layers I $(D_1 < z < D_2)$, II $(D_2 < z < D_3)$, III $(D_3 < z < D_4)$, V $(D_4 < z < D_5)$ are characterized by the relative permittivity/permeability $(\varepsilon_{rm}, \mu_{rm})$ for $m = 1, 2, 3, 4$, respectively.

Define the total electric field $\phi^t(x,z)[\equiv E_y^t(x,z)]$ by

$$\phi^t(x,z) = \phi^i(x,z) + \phi(x,z), \tag{1}$$

where $\phi^i(x,z)$ is the incident field given by

$$\phi^i(x,z) = e^{-ik(x\sin\theta_0 + z\cos\theta_0)}, \quad 0 < \theta_0 < \pi/2 \tag{2}$$

with $k\left[k = \omega(\varepsilon_0\mu_0)^{1/2}\right]$ being the free-space wavenumber. We assume that the medium is slightly lossy as in $k = k_1 + ik_2$ with $0 < k_2 \ll k_1$. The solution for real $k$ is obtained by letting $k_2 \to +0$ at the end of analysis. We now define the Fourier transform of the scattered field as

$$\Phi(x,\alpha) = (2\pi)^{-1/2}\int_{-\infty}^{\infty}\phi(x,z)e^{i\alpha z}dz, \qquad \alpha = \mathrm{Re}\,\alpha + i\,\mathrm{Im}\,\alpha(\equiv \sigma + i\tau). \tag{3}$$

In view of the radiation condition, it is found that $\Phi(x,\alpha)$ is regular in the strip $|\tau| < k_2\cos\theta_0$ of the complex $\alpha$-plane. Taking the Fourier transform of the 2-D Helmholtz equation and solving the resulting equations, we derive the scattered field representation in the Fourier transform domain as in

$$\begin{aligned}\Phi(x,\alpha) &= \Psi(\pm b,\alpha)e^{\mp\gamma(x\mp b)} \text{ for } x \gtrless \pm b,\\ &= \Psi(b,\alpha)\frac{\sinh\gamma(x+b)}{\sinh 2\gamma b} - \Psi(-b,\alpha)\frac{\sinh\gamma(x-b)}{\sinh 2\gamma b}\\ &\quad -\frac{1}{b}\sum_{n=1}^{\infty}\frac{e^{-i\alpha D_5}c_{5n}^{+}(\alpha) - e^{-i\alpha D_1}c_{1n}^{-}(\alpha)}{\alpha^2+\gamma_n^2}\sin\frac{n\pi}{2b}(x+b)\\ &\quad -\frac{1}{b}\sum_{m=1}^{4}\sum_{n=1}^{\infty}\frac{e^{i\alpha D_m}c_{mn}^{+}(\alpha) - e^{i\alpha D_{m+1}}c_{m+1,n}^{-}(\alpha)}{\alpha^2+\Gamma_{mn}^2}\sin\frac{n\pi}{2b}(x+b)\\ &\qquad \text{for } (|x|<b),\end{aligned} \tag{4}$$

where $\gamma = (\alpha^2 - k^2)^{1/2}$ with $\mathrm{Re}\,\gamma > 0$, and

$$\begin{aligned}\Psi(\pm b,\alpha) = (1/2)\{&e^{-i\alpha L}[U_-(\alpha) \pm V_-(\alpha)]\\ &+e^{i\alpha L}[U_{(+)}(\alpha) \pm V_{(+)}(\alpha)]\}.\end{aligned} \tag{5}$$

The definition of the other quantities appearing in (4) is omitted here due to space limitations. It is found that $U_-(\alpha), U_+(\alpha), V_-(\alpha)$, and $V_{(+)}(\alpha)$ in (5) satisfy the Wiener-Hopf equations

$$\begin{aligned}J_1^d(\alpha) = &-\frac{e^{-i\alpha L}U_-(\alpha) + e^{i\alpha L}U_{(+)}(\alpha)}{M(\alpha)}\\ &-\sum_{n=1,\mathrm{odd}}^{\infty}\frac{n\pi}{b^2}\left[\frac{e^{i\alpha D_5}c_{5n}^{+}(\alpha) - e^{i\alpha D_1}c_{1n}^{-}(\alpha)}{\alpha^2+\Gamma_n^2} + \sum_{m=1}^{4}\frac{e^{i\alpha D_m}c_{mn}^{+}(\alpha) - e^{i\alpha D_{m+1}}c_{(m+1)n}^{-}(\alpha)}{\alpha^2+\Gamma_{mn}^2}\right],\end{aligned} \tag{6}$$

$$\begin{aligned}J_1^s(\alpha) = &-\frac{e^{-i\alpha L}V_-(\alpha) + e^{i\alpha L}V_{(+)}(\alpha)}{N(\alpha)}\\ &-\sum_{n=2,\mathrm{even}}^{\infty}\frac{n\pi}{b^2}\left[\frac{e^{i\alpha D_5}c_{5n}^{+}(\alpha) - e^{i\alpha D_1}c_{1n}^{-}(\alpha)}{\alpha^2+\Gamma_n^2} + \sum_{m=1}^{4}\frac{e^{i\alpha D_m}c_{mn}^{+}(\alpha) - e^{i\alpha D_{m+1}}c_{m+1,n}^{-}(\alpha)}{\alpha^2+\Gamma_{mn}^2}\right]\end{aligned} \tag{7}$$

with

$$M(\alpha) = \frac{e^{-\gamma b}\cosh\gamma b}{\gamma}, \quad N(\alpha) = \frac{e^{-\gamma b}\sinh\gamma b}{\gamma}, \tag{8}$$

where $J_1^{d,s}(\alpha)$ are unknown entire functions denoting the surface currents induced on the waveguide plates.

Applying the factorization and decomposition procedure, the Wiener-Hopf equations (6) and (7) are solved leading to the exact solution. However, the solution thus obtained is formal since it contains infinite series with unknown coefficients as well as infinite branch-cut integrals with unknown integrands. Employing a rigorous asymptotics with the aid of the edge condition, an approximate solution convenient for numerical computation is obtained.

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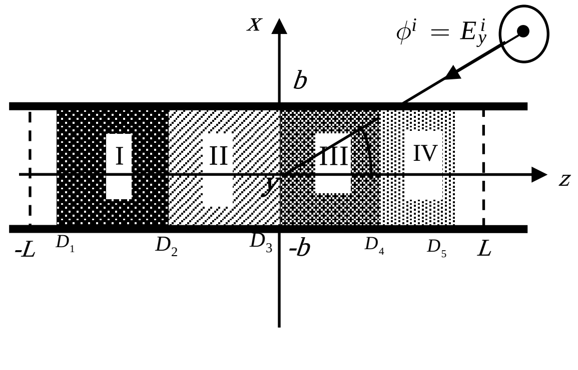


Fig. 1. Geometry of the problem.