WIENER-HOPF ANALYSIS OF THE ELECTROMAGNETIC WAVES RADIATION FROM A CIRCULAR WAVEGUIDE CAVITY WITH AN IMPEDANCE TERMINATION: VECTOR DIFFRACTION PROBLEM

Kazuya Kobayashi, Project Leader, Chuo University

Dozyslav B. Kuryliak, Project Member, Karpenko Physico-Mechanical Institute, National Academy of Sciences of Ukraine

Shoichi Koshikawa, Project Member, Laboratory, Antenna Giken Co., Ltd.

Zinoviy T. Nazarchuk, Project Member, Karpenko Physico-Mechanical Institute, National Academy of Sciences of Ukraine

Susumu Okada, Project Member, Chuo University

Ken'ichi Isobe, Project Member, Chuo University

1 Abstract.

An exact solution is presented for the problem of radiation of an electromagnetic wave from an open end of a circular waveguide cavity formed by a perfectly conducting cylinder and an internal plate termination with non zero impedance. The cavity is excited non-symmetrically by the TE dominant mode. This problem has been formulated as a vector diffraction problem for two scalar potentials. The key result is the correct representation of the unknown potentials in the Fourier transform domain, which shows the TE and TM waves interaction at the open end and takes into account the impedance boundary conditions at the termination.

2 Introduction.

The analysis of electromagnetic scattering by metallic waveguide cavities is an important subject in radar cross section (RCS) reduction and target identification studies. In the previous papers, we have considered several two-dimensional (2-D) cavities formed by a finite parallel-plate waveguide with a planar termination at the open end, and solved the plane wave diffraction rigorously using the Wiener-Hopf technique [1]. It has been shown by numerical computation that our results are valid over a broad frequency range. We also developed the Wiener-Hopf technique for rigorous analysis three-dimensional (3-D) perfectly conducting circular waveguide cavities [2] as a more realistic model for the RCS studies.

In our previous report we have considered the value boundary diffraction problem for vector diffraction by the cavity formed by a semi-infinite circular waveguide and an interior planar termination with non zero impedance in the case of the TM dominant mode excitation, and analysed the non-symmetric electromagnetic wave diffraction by means of the Wiener-Hopf technique [3-5]. For our theory more practical application via development of the Generalised Scattering Matrix technique is necessary to obtain the solution of this problem for both TM and TE non-symmetrical dominant modes excitations. In this report we continue to develop the Wiener-Hopf technique for the solution of the vector diffraction problems and consider the electromagnetic excitation of the circular waveguide cavity with the impedance termination by the TE non-symmetrical dominant mode.

3 Statement of the problem.

The mixed boundary value problem for mentioned above wave diffraction by cylindrical waveguide cavity involves the unknown TM and TE scalar potential that satisfy the Helmholtz equation

$$\frac{\partial^2 u_l}{\partial \rho^2}+\frac{1}{\rho}\frac{\partial u_l}{\partial \rho}+\frac{\partial^2 u_l}{\partial z^2}+\left(k^2-\frac{m^2}{\rho^2}\right)u_l=0,\quad l=\overline{1,2},\quad (1)$$

and boundary conditions:

Cylindrical faces $\{\ -L < z < L$ with $\rho = b-0\ \}$ and $\{\ -\infty < z < L$ with $\rho = b+0\ \}$

$$\left.\begin{array}{c}\frac{\partial^2 u_1}{\partial z^2}+k^2 u_1=0,\\ -\frac{m}{i\omega\varepsilon\rho}\frac{\partial u_1}{\partial z}+\frac{\partial u_2}{\partial \rho}=0;\end{array}\right\}$$

Impedance plate termination: $\{0 < \rho < b \text{ with } z = -L\}$

$$\left.\begin{aligned} \frac{\partial}{\partial\rho}\left(Z_1 u_1^t - \frac{1}{i\omega\varepsilon}\frac{\partial u_1^t}{\partial z}\right) + \frac{m}{\rho}\left(u_2^t - \frac{Z_1}{i\omega\mu}\frac{\partial u_2^t}{\partial z}\right) = 0, \\ \frac{m}{\rho}\left(Z_1 u_1^t - \frac{1}{i\omega\varepsilon}\frac{\partial u_1^t}{\partial z}\right) + \frac{\partial}{\partial\rho}\left(u_2^t - \frac{Z_1}{i\omega\mu}\frac{\partial u_2^t}{\partial z}\right) = 0. \end{aligned}\right\} \quad (2)$$

Let the total field $u_2^i(\rho, z)$ be given by

$$u_1^t(\rho,z) = \begin{cases} u_1(\rho,z), \\ u_1(\rho,z), \end{cases}$$

$$u_2^t(\rho,z) = \begin{cases} u_2^i(\rho,z) + u_2(\rho,z), & 0<\rho<b \quad -L \le z < \infty, \\ u_2(\rho,z), & \rho > b \quad -\infty < z < \infty, \end{cases} \quad (3)$$

where $u_2^i(\rho, z)$ is the incident field that consists with TE - mode for perfectly conducting infinite cylinder, being define as $u_2^i(\rho,z) = \tilde{c}_{mj}^i J_m(\eta_j \rho/b) e^{-\tilde{\gamma}_j z}$ with the complex amplitude $\tilde{c}_{mj}^i$; η_j for $j = 1, 2, 3, \ldots$ denote the zeros of function $J_m'(\cdot)$, $\tilde{\gamma}_j = [(\eta_j/b)^2 - k^2]^{1/2}$ ($\mathrm{Re}\,\tilde{\gamma}_j > 0$).

Next we take the Fourier transform of the Helmholtz equation and use the radiation condition. Applying the method established in our previous papers [1-5], we derive the transformed wave equations as in

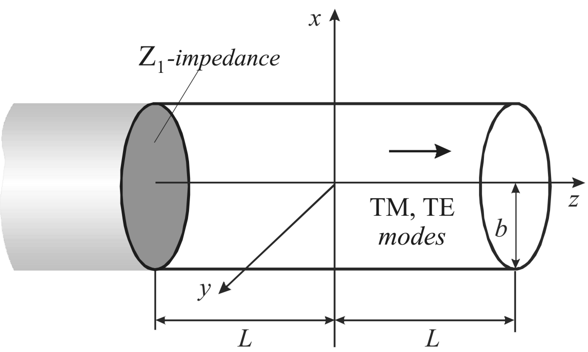

Fig.1 Geometry of the problem.

$$\widehat{T} U_l(\rho, \alpha) = 0, \quad (4)$$

in $\rho > b$ for $|\tau| < k_2$,

$$\widehat{T}\left[\Phi_l(\rho,\alpha) + e^{i\alpha L}\Psi_l^+(\rho,\alpha)\right] = e^{-i\alpha L}\left[\tilde{g}_l(\rho) - i\alpha \tilde{f}_l(\rho)\right] \quad (5)$$

in $0 < \rho < b$ for $\tau < -k_2$, for $l = 1$ and 2, where $\widehat{T} = \lfloor d^2/d\rho^2 + \rho^{-1} d/d\rho - (\gamma^2 + m^2/\rho^2)\rfloor$; $\gamma = (\alpha^2 - k^2)^{1/2}$ with $\mathrm{Re}\,\gamma > 0$. In (5), $\tilde{f}_l(\rho)$ and $\tilde{g}_l(\rho)$ are the unknown inhomogeneous terms defined by $\tilde{f}_l(\rho) = (2\pi)^{-1/2} u_l^t(\rho, -L)$, $\tilde{g}_l(\rho) = (2\pi)^{-1/2} \partial u_l^t(\rho, z)/\partial z|_{z=-L}$.

The terms on the left-hand sides of (5) are the Fourier transforms of the functions appearing in (3), and are defined by

$$U_l(\rho,\alpha)|_{\rho>b} = (2\pi)^{-1/2} \int_{-\infty}^{+\infty} u_l(\rho,z) e^{i\alpha z} dz,$$

$$U_l(\rho,\alpha)|_{\rho<b} = (2\pi)^{-1/2} \int_{-L}^{+\infty} u_l(\rho,z) e^{i\alpha z} dz$$

where $\alpha = \mathrm{Re}\,\alpha + i\,\mathrm{Im}\,\alpha (\equiv \sigma + i\tau)$ and

$$\begin{aligned} U_1(\rho,\alpha) &= \Phi_1(\rho,\alpha) + e^{i\alpha L}\Psi_1^+(\rho,\alpha), \\ U_2(\rho,\alpha) &= \Phi_2(\rho,\alpha) + e^{i\alpha L}\Psi_2^+(\rho,\alpha) - U_2^i(\rho,\alpha), \end{aligned} \quad (6)$$

for $0 < \rho < b$, where

$$\begin{aligned} \Psi_1^+(\rho,\alpha) &= U_1^+(\rho,\alpha), \\ \Psi_2^+(\rho,\alpha) &= U_2^+(\rho,\alpha) + Q_2^+(\rho,\alpha), \end{aligned} \quad (7)$$

$$\begin{aligned} U_l^+(\rho,\alpha) &= \frac{1}{\sqrt{2\pi}} \int_{+L}^{+\infty} u_l(\rho,\alpha) e^{i\alpha(z-L)} dz, \\ \Phi_l(\rho,\alpha) &= \frac{1}{\sqrt{2\pi}} \int_{-L}^{+L} u_l^t(\rho,\alpha) e^{i\alpha z} dz. \end{aligned} \quad (8)$$

Here $U_2^i(\rho,\alpha)$ and $Q_2^+(\rho,\alpha)$ are known functions. In (6)-(8), the subscripts '$\pm$' imply that the functions are regular in the half-planes $\tau \gtrless \mp k_2$; $\Phi_l(\rho,\alpha)$ is an entire function.

The unknown inhomogeneous terms are expanded as in

$$\begin{aligned} \tilde{f}_1(\rho) &= f_{10}(\rho/b)^m + \sum_{n=1}^{\infty} f_{1n} I_m(i\xi_n \rho/b), \\ \tilde{g}_2(\rho) &= g_{20}(\rho/b)^m + \sum_{n=1}^{\infty} g_{2n} I_m(i\eta_n \rho/b), \end{aligned} \quad (9)$$

$$\begin{aligned} \tilde{f}_2(\rho) &= -c_1\left(\frac{\rho}{b}\right)^m + \frac{Z_1}{i\omega\mu}\tilde{g}_2(\rho), \\ \tilde{g}_1(\rho) &= -i\omega\varepsilon c_1\left(\frac{\rho}{b}\right)^m + i\omega\varepsilon Z_1 \tilde{f}_1(\rho). \end{aligned} \quad (10)$$

In (9)-(10), f_{1n} and g_{2n} for $n = 0, 1, 2, 3, \ldots$ and c_1 are the unknown coefficients; ξ_j for $j = 1, 2, 3, \ldots$ denote the zeros of Bessel function $J_m(\cdot)$, $I_m(\cdot)$ is the modified Bessel function of the first kind. Next by following a procedure similar to that developed in [2], we arrive at the solution of (4) and (5) that leads to the scattered field representation in the Fourier transform domain (6) as follows

$$U_1(\rho,\alpha) = \begin{cases} i\omega\varepsilon E_1^+(b,\alpha) e^{i\alpha L} \frac{K_m(\gamma\rho)}{\gamma^2 K_m(\gamma b)} \text{ for } \rho > b, \\ i\omega\varepsilon E_1^+(b,\alpha) e^{i\alpha L} \frac{I_m(\gamma\rho)}{\gamma^2 I_m(\gamma b)} + \\ + ikZ^{-1}\gamma^{-2}[c_1 - (Z_1 - Zk^{-1}\alpha) f_{10}] e^{-i\alpha L}\left(\frac{\rho}{b}\right)^m + \\ + ikZ^{-1}(Zk^{-1}\alpha - Z_1)\sum_{n=1}^{\infty} \frac{f_{1n} e^{-i\alpha L}}{\alpha^2 + \gamma_n^2} \\ I_m(i\xi_n \rho/b) \text{ for } 0 < \rho < b. \end{cases}$$

(11a)

$$
U_2(\rho,\alpha)=\begin{cases} \tilde{V}_2^+(\alpha)\frac{K_m(\gamma\rho)}{\gamma(\alpha-k)K_m'(\gamma b)} \text{ for } \rho>b, \\ \tilde{V}_2^+(\alpha)\frac{I_m(\gamma\rho)}{\gamma(\alpha-k)I_m'(\gamma b)}+ \\ +\gamma^{-2}\left[\left(\alpha\frac{Z_1}{\omega\mu}-1\right)g_{20}-i\alpha c_1\right]e^{-i\alpha L}\left(\frac{\rho}{b}\right)^m+ \\ +\left(\alpha\frac{Z_1}{\omega\mu}-1\right)\sum_{n=1}^{\infty}\frac{g_{2n}e^{-i\alpha L}}{\alpha^2+\tilde{\gamma}_n^2}I_m(i\eta_n\rho/b)- \\ -U_2^i(\rho,\alpha) \text{ for } 0<\rho<b. \end{cases} \tag{11b}
$$

Here $E_1^+(b,\alpha)$ $\tilde{V}_2^+(\alpha)$ are the unknown functions regular in the upper half-plane; $\{g_{2n}\}$ and $\{f_{1n}\}$ for $n=1,2,3,\dots$ are two sets of unknown coefficients which can be expressed by means of the unknown functions in the discrete points as follows

$$
\begin{aligned}
\tilde{V}_1^+(k)&=-\frac{1}{2iZ}[c_1-(Z_1-Z)f_{10}]e^{-ikL},\\
\tilde{V}_2^+(k)&=-\frac{m}{2kb}[(ZZ_1-1)g_{20}-ikc_1]e^{-ikL},\\
\tilde{V}_1^+(i\gamma_n)&=\frac{kb^2}{2Z\xi_n}(Z_1-iZk^{-1}\gamma_n)(i\gamma_n-k)I_m'(i\xi_n)e^{\gamma_n L}f_{1n},\\
\tilde{V}_2^+(i\tilde{\gamma}_n)&=\frac{b}{2}\left(i\tilde{\gamma}_n\frac{Z_1}{\omega\mu}-1\right)(i\tilde{\gamma}_n-k)I_m''(i\eta_n)e^{\tilde{\gamma}_n L}g_{2n},
\end{aligned}
$$

where $\tilde{V}_1^+(\alpha) = i\omega\varepsilon(\alpha+k)^{-1}E_1^+(b,\alpha)e^{i\alpha L}$; $\tilde{\gamma}_n = [(\eta_n/b)^2-k^2]^{1/2}$. Equations (11) hold in the strip $|\tau|<k_2$ and is non standard because the terms with static multiplier $(\rho/b)^m$ is involved. In order to ensure the non dependence of the field components with respect to the static terms is found, that $g_{20}=i\omega\mu f_{10}$.

4 Exact solution of the boundary problem.

Using (11), we find the Fourier transform of the magnetic components $h_\varphi(\rho=b\pm0,z)$, $h_z(\rho=b\pm0,z)$ and derive, that

$$
\begin{aligned}
&\frac{i\omega\varepsilon E_1^+(b,\alpha)}{\gamma^2 b M_1(\alpha)}-\frac{m}{\omega\mu b^2}\frac{\alpha\tilde{V}_2^+(\alpha)e^{-i\alpha L}}{(\alpha-k)M_2(\alpha)}-\\
&\quad-\frac{k}{bZ}(Zk^{-1}\alpha-Z_1)\sum_{n=1}^{\infty}\frac{f_{1n}\xi_n e^{-2i\alpha L}}{\alpha^2-\gamma^2}I_m'(i\xi_n)+\\
&\quad+\frac{m\alpha}{\omega\mu b}\left(\alpha\frac{Z_1}{\omega\mu}-1\right)\sum_{n=1}^{\infty}\frac{g_{2n}e^{-2i\alpha L}}{\alpha^2-\tilde{\gamma}^2}I_m(i\eta_n)+\\
&\quad+\frac{Z_1}{i\omega\mu}\sum_{n=1}^{\infty}g_{2n}e^{-2i\alpha L}I_m(i\eta_n)+\\
&\quad+\frac{\tilde{a}_{mj}\tilde{\gamma}_j e^{\tilde{\gamma}_j L}}{\omega\mu b\eta_j^2\sqrt{2\pi}(\alpha+i\tilde{\gamma}_j)}\left[e^{-2\tilde{\gamma}_j L}-e^{-2i\alpha L}\right]\\
&=J_2^-(b,\alpha),
\end{aligned} \tag{12a}
$$

$$
\begin{aligned}
&\frac{\tilde{V}_2^+(\alpha)e^{i\alpha L}(\alpha+k)}{M_2(\alpha)}\\
&\quad+\frac{1}{b}\left(\alpha\frac{Z_1}{\omega\mu}-1\right)\sum_{n=1}^{\infty}\frac{e^{-2i\alpha L}g_{2n}\eta_n^2}{\alpha^2+\tilde{\gamma}_n^2}I_m(i\eta_n)+\\
&\quad+\frac{\tilde{a}_{mj}ie^{\tilde{\gamma}_j L}}{\sqrt{2\pi}b(\alpha+i\tilde{\gamma}_j)}\left[e^{-2\tilde{\gamma}_j L}-e^{-2i\alpha L}\right]\\
&=i\omega\mu b J_1^-(b,\alpha),
\end{aligned} \tag{12b}
$$

where $\tilde{a}_{mj}=\tilde{c}_{mj}^i\eta_j^2 J_m(\eta_j)$. In (12), $M_1(\alpha)=I_m(\gamma b)$ $K_m(\gamma b)$, $M_2(\alpha)=\gamma^2 I_m'(\gamma b)K_m'(\gamma b)$ are the kernel functions. Equations (12) are the desired coupled Wiener-Hopf equations hold in the strip $|\tau|<k_2$. Next, we apply the factorisation and decomposition procedure for solution of the Wiener-Hopf equations (12). This leads to the infinite linear algebraic system, that can be solve for arbitrary geometrical parameters and frequency with pre-specify accuracy. We also derive the field representation and analyse the particular cases.

Field representation: (i) *Mode field structure for the cavity region* $0<\rho<b; -L<z<L$. The field representation for the cavity region is as follows:

$$
\begin{aligned}
u_1^t(\rho,z)&=\frac{i\omega\varepsilon}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{E_1^+(\alpha)}{\gamma^2}\frac{I_m(\gamma\rho)}{I_m(\gamma b)}e^{-i\alpha(z-L)}d\alpha+\\
&+\frac{i\gamma\varepsilon}{\sqrt{2\pi}}\left(\frac{\rho}{b}\right)^m\int_{-\infty}^{\infty}\frac{[c_1-(Z_1-Zk^{-1}\alpha)f_{10}]}{\gamma^2}e^{-i\alpha(z+L)}d\alpha+\\
&+\sum_{n=1}^{\infty}f_{1n}I_m(i\xi_n\rho/b)\frac{i\omega\varepsilon}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{(Zk^{-1}\alpha-Z_1)}{\alpha^2+\gamma_n^2}e^{-i\alpha(z+L)}d\alpha,
\end{aligned} \tag{13a}
$$

$$
\begin{aligned}
u_2^t(\rho,z)&=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{\tilde{V}_2^+(\alpha)e^{-i\alpha L}I_m(\gamma\rho)}{\gamma(\alpha-k)I_m'(\gamma b)}e^{-i\alpha(z-L)}d\alpha+\\
&+\frac{1}{\sqrt{2\pi}}\left(\frac{\rho}{b}\right)^m\int_{-\infty}^{\infty}\frac{\left(\alpha\frac{Z_1}{\omega\mu}-1\right)g_{20}-i\alpha c_1}{\gamma^2}e^{-i\alpha(z+L)}d\alpha+\\
&+\sum_{n=1}^{\infty}g_{2n}I_m(i\eta_n\rho/b)\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{\left(\alpha\frac{Z_1}{\omega\mu}-1\right)}{\alpha^2+\tilde{\gamma}_n^2}e^{-i\alpha(z+L)}d\alpha.
\end{aligned} \tag{13b}
$$

Since the region inside the cavity is identified by $0<\rho<b$ and $|z|<L$, the first integrals involved in (13a) and (13b) can be evaluated by deforming the contours into the upper half-plane. It is found that the singularities for these integrals in such a process of deformation are only simple poles at $\alpha=k$, and $\alpha=i\gamma_n$ with $n=1,2,3,\dots$ for (13a) and, $\alpha=k$, $\alpha=i\tilde{\gamma}_n$ with $n=1,2,3,\dots$ for (13b). The second and third integrals

involved in (13a) and (13b) can be evaluated by deforming the contours into lower half-plane. It can be shown that two second integrals in this process of deformation clearly have simple poles at $\alpha = -k$. The third integral in (13a) and (13b) has only simple poles at $\alpha = -i\gamma_n$, and $\alpha = -i\tilde{\gamma}_n$ respectively. Thus, the evaluation of the integrals leads to the following scattered field representation:

$$u_1^t(\rho, z) = u_{01}(\rho, z) + \\ + \sum_{n=1}^{\infty} \left\{ S_{1n}^{(1)} e^{\gamma_n(z-L)} + S_{1n}^{(2)} e^{-\gamma_n(z+L)} \right\} J_m(\xi_n \rho/b), \tag{14a}$$

$$u_2^t(\rho, z) = u_{02}(\rho, z) + \\ + \sum_{n=1}^{\infty} \left\{ S_{2n}^{(1)} e^{\tilde{\gamma}_n(z-L)} + S_{2n}^{(2)} e^{-\tilde{\gamma}_n(z+L)} \right\} J_m(\eta_n \rho/b), \tag{14b}$$

Here the terms $u_{01}(\rho, z)$, $u_{02}(\rho, z)$ correspond to the residues of the integrands (6.1) in the simple poles at $\alpha = k$ and $\alpha = -k$, and we arrival at

$$u_{01}(\rho, z) = -\omega\varepsilon k^{-1}\sqrt{\pi/2}(\rho/b)^m e^{ikL} \times \\ \times \left\{ E_1^+(k) e^{-ikz} + [c_1 - (Z_1 + Z) f_{10}] e^{ikz} \right\}, \tag{15a}$$

$$u_{02}(\rho, z) = \sqrt{\pi/2}(\rho/b)^m e^{ikL} \times \\ \times \left\{ -iE_1^+(k) e^{-ikz} + [ikc_1 - (1 + Z^{-1} Z_1) g_{20}] k^{-1} e^{ikz} \right\}, \tag{15b}$$

The terms $S_{1n}^{(1)} e^{\gamma_n(z-L)} J_m(\xi_n\rho/b)$, $S_{1n}^{(2)} e^{-\gamma_n(z+L)} J_m(\xi_n\rho/b)$ correspond to the residues of the integrands (13a) in the poles at $\alpha = i\gamma_n$ and $\alpha = -i\gamma_n$, and $S_{2n}^{(1)} e^{\tilde{\gamma}_n(z-L)} J_m(\eta_n\rho/b)$, $S_{2n}^{(2)} e^{-\tilde{\gamma}_n(z+L)} J_m(\eta_n\rho/b)$ correspond to the residues of integrands (13b) in the poles at $\alpha = i\tilde{\gamma}_n$ and $\alpha = -i\tilde{\gamma}_n$ respectively. These terms may be expressed as follows

$$S_{1n}^{(1)} = -\frac{i\sqrt{2\pi}\omega\varepsilon E_1^+(i\gamma_n)}{\xi_n \gamma_n J_m'(\xi_n)}, \\ S_{1n}^{(2)} = \sqrt{2\pi}\omega\varepsilon e^{im\pi/2} f_{1n} [\gamma_n^{-1} Z_1 + ik^{-1} Z], \tag{16a}$$

$$S_{2n}^{(1)} = -\frac{\sqrt{2\pi} V_2^+(i\tilde{\gamma}_n)}{b\tilde{\gamma}_n (i\tilde{\gamma}_n - k) J_m''(\eta_n)}, \\ S_{2n}^{(2)} = -\frac{\sqrt{\pi/2}e^{im\pi/2}}{b\tilde{\gamma}_n} g_{2n} \left(1 + i\tilde{\gamma}_n \frac{Z_1}{\omega\mu} \right). \tag{16b}$$

Next we express f_{1n} and g_{2n} through the $E_1^+(i\gamma_n)$ and $V_2^+(i\tilde{\gamma}_n)$, respectively and derive the field components representations. It can be shown that the terms (15a) and (15b) do not make any contribution in the field components expressions because are reduced.

(ii) *Far field pattern.* The scattered field for $\rho > b$ is found to be

$$u_1(\rho, z) = \frac{i\omega\varepsilon}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{E_1^+(\alpha) K_m(\gamma\rho)}{\gamma^2 K_m(\gamma b)} e^{-i\alpha(z-L)} d\alpha, \\ u_2(\rho, z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{\tilde{V}_2^+(\alpha) e^{-i\alpha L} K_m(\gamma\rho)}{\gamma(\alpha - k) K_m'(\gamma b)} e^{-i\alpha(z-L)} d\alpha. \tag{17}$$

Far field can be evaluated in the usual way by deforming the contour onto its stationary phase path. We used the polar coordinate $z = R\cos\theta$, $\rho = R\sin\theta$ for $0 < \theta < \pi$ to derive the far field asymptotic expression. Omitting the all details of this lengthy but straightforward calculation, the final result is as follows

$$\begin{aligned} e_z &= -\sin(m\varphi + \varphi_0) S_1(\theta) \frac{e^{ikR}}{R}, \\ e_\rho &= \sin(m\varphi + \varphi_0) S_1(\theta) \frac{\cos\theta}{\sin\theta} \frac{e^{ikR}}{R}, \\ h_\varphi &= \sin(m\varphi + \varphi_0) \frac{S_1(\theta)}{Z\sin\theta} \frac{e^{ikR}}{R}; \\ h_z &= \cos(m\varphi + \varphi_0) S_2(\theta) \frac{\sin\theta}{kZ} \frac{e^{ikR}}{R}, \\ h_\rho &= \cos(m\varphi + \varphi_0) \frac{S_2(\theta)\cos\theta}{kZ\sin\theta} \frac{e^{ikR}}{R}, \\ e_\varphi &= \cos(m\varphi + \varphi_0) k^{-1} S_2(\theta) \frac{e^{ikR}}{R} \end{aligned} \tag{18}$$

for $\rho > b$ as $|k\rho| \to \infty$, were $S_1(\theta)$ and $S_2(\theta)$ can be expressed as

$$S_1(\theta) = \frac{(\pi/2)^{1/2} e^{-ik\cos\theta} E_1^+(-k\cos\theta)}{K_m(-kb\sin\theta)}, \\ S_2(\theta) = \frac{(\pi/2)^{1/2} \tilde{V}_2^+(-k\cos\theta)}{(1+\cos\theta) K_m'(-kb\sin\theta)}. \tag{19}$$

Conclusions. An exact solution of the new problem for electromagnetic wave radiation from an open end of a circular waveguide cavity is presented here. The key result of this paper is the correct analytical representation of the unknown scalar potentials in the Fourier transform domain (11) which shows the TM and TE waves' interaction at the open end and takes into account the impedance boundary conditions at the termination. This leads to the coupled Wiener-Hopf equations (12). Finally, the problem is reduced to the infinite system of linear algebraic equations due to the factorisation and decomposition procedure. The method provides a straightforward formulation of the solution and is valid for arbitrary geometrical and frequency parameters. The modes field representation for the cavity region and far field pattern using the saddle point technique for integration is also obtained. This results (the cavity is exited by the dominant TE mode) together with our previous results of the opposite case (the cavity is exited by the dominant TM mode) allow to receive the elements of the generalized scattering matrix (S-matrix) and apply the results for study of electromagnetic wave radiation from more complicated cavities.

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