Electromagnetic Wave Scattering by Open-Ended Cylindrical and Conical Waveguide Cavities: Wiener-Hopf Analysis
開口端円筒状・錐体状導波管キャビティによる電磁波の散乱：ウィーナー・ホッフ法による解析

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Introduction. The analysis of the scattering and diffraction by open-ended metallic waveguide cavities has been of great interest recently in connection with the prediction and reduction of the radar cross section (RCS) of a target. This problem serves as a simple model of duct structures such as jet engine intakes of aircrafts and cracks occurring on surfaces of general complicated bodies. Therefore the investigation of a scattering mechanism in case of the existence of open cavities is an important subject in the field of the RCS prediction and reduction. Some of the cavity diffraction problems have been analyzed thus far using a variety of different analytical and numerical methods. If the cavity dimensions are small in comparison to the incident wavelength, numerical techniques such as the method of moments (Senior, 1976) and the finite element method (Jeng, 1990) can be applied efficiently. For large cavities with uniform cross sections, the results based on the waveguide modal approach by the use of the reciprocity relationship and the Kirchhoff approximation have been reported (Altintas, Pathak, and Liang, 1988; Ling, Lee, and Chou, 1989). In order to describe systematically the scattering mechanism as related to a fairly general class of large cavities with reasonable accuracy, the three ray-based approaches, namely, the method of shooting and bouncing rays, the Gaussian beam method, and the generalized ray expansion method have been developed (Ling, Lee , and Chou, 1989; Pathak and Burkholder, 1991). Furthermore, hybrid techniques such as the asymptotic/modal approach and the boundary integral/modal approach (Ling, 1990) have also been established. These hybrid approaches take advantage of the efficiency of the modal analysis as well as the flexibility of asymptotic or numerical techniques. Most of these analysis methods incorporate the scattering from the interior of the cavity including the rim diffraction at the open end, but they do not rigorously take into account the scattering effect arising from the entire exterior surface of the cavity. Therefore, final solutions due to these approaches are valid only for the restricted range of incidence and observation angles. In addition, these solutions may not be uniformly valid for arbitrary dimensions of the cavity.

The Wiener-Hopf technique is known as a powerful tool for analyzing electromagnetic wave problems associated with canonical geometries, which is mathematically rigorous in the sense that the edge condition is explicitly incorporated into the analysis. Kobayashi (1993) considered a finite parallel-plate waveguide with a planar termination at the open end as an example of simple two-dimensional (2-D) cavity structures, and solved the plane wave diffraction problem rigorously using the Wiener-Hopf technique. As a result, an efficient approximate solution has been obtained, which is valid for the cavity depth greater than the incident wavelength. Kobayashi and Koshikawa (1993, 1994, 1996) have further considered 2-D material-loaded cavities formed by finite and semi-infinite parallel-plate waveguides, and carried out a rigorous RCS analysis by means of the Wiener-Hopf technique. It has been shown by numerical computation that the results are valid over a broad frequency range and can be used as a reference solution for validating more general-purpose computer codes based on approximate methods.

In this paper, we shall consider two value boundary diffraction problems. In the first part we shall consider the vector diffraction by a cavity formed by a semi-infinite circular waveguide and an interior planar termination with non zero impedance as a generalisation of our previous problems, and analyse the non-symmetric electromagnetic wave diffraction by means of the Wiener-Hopf technique. The method of solution is similar to that we have developed for the scalar diffraction analysis of parallel-plate and circular waveguide cavities, but is more complicated because the TM and TE wave interaction at the circular edge and an impedance termination are also involved. In the second part we consider the axial symmetric wave diffraction by the semi-infinite truncated cone and develop the Wiener-Hopf technique for it solution. The exact solution of these problems is a great impotence for horn antennas theory, near field study, radiated elements for phased antennas grating designing and non-destructive testing. The time factor is assumed to be $e^{-i\omega t}$ and suppressed throughout this paper.

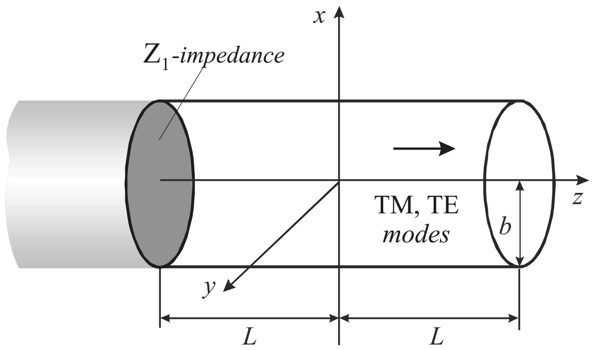


Fig.1 Geometry of the problem.

1 Development of the Wiener-Hopf Technique for Wave Diffraction by a Circular Waveguide Cavity

We shall generalise the technique, previously developed for a rigorous analysis of the 2-D diffraction by parallel-plate waveguide cavities, to the analysis of the three-dimensional (3-D) vector diffraction by open-ended cavity structures. Let us consider a semi-infinite circular waveguide with an interior planar termination as shown in Fig. 1., where (ρ, φ, z) are cylindrical coordinates. The mixed boundary value problem for wave diffraction by a cylindrical waveguide cavity mentioned above involves the unknown TM and TE scalar potentials. Let the total field $u^t_{1(2)}(\rho, z)$ be given by

$$u^t_1(\rho,z) = \begin{cases} u^i_1(\rho,z) + u_1(\rho,z), \\ u_1(\rho,z), \end{cases}$$

$$u^t_2(\rho,z) = \begin{cases} u_2(\rho,z), & 0<\rho<b \quad -L \le z < \infty, \\ u_2(\rho,z), & \rho > b \quad -\infty < z < \infty, \end{cases} \quad (1.1)$$

where $u^i_1(\rho,z) = c^i_{mj} J_m(\xi_j \rho/b) e^{-\gamma_j z}$ is the incident field that consists with TM- dominant mode for perfectly conducting infinite cylinder, with the complex amplitude c^i_{mj}; ξ_j for $j = 1, 2, 3, \dots$ denote the zeros of Bessel function $J_m(\cdot)$, $\gamma_j = [(\xi_j/b)^2 - k^2]^{1/2}$ (Re $\gamma_j > 0$). Then the mathematical formulation of this diffraction problem is looks as follows:

$$\begin{pmatrix} \Delta + k^2 & 0 \\ 0 & \Delta + k^2 \end{pmatrix} \begin{pmatrix} u_1(\rho,z) \\ u_2(\rho,z) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad (1.2)$$

The boundary condition at the cylindrical surface: $z \in (-\infty, L)$ with $\rho = b+0$ and $z \in (-L, L)$ with $\rho = b-0$

$$\begin{pmatrix} \vartheta[\partial^2/\partial z^2 + k^2] & 0 \\ \vartheta m\rho^{-1}\partial/\partial z & \partial/\partial\rho \end{pmatrix} \begin{pmatrix} u^t_1 \\ u^t_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

The boundary condition at the absorbing plate termination: $\rho \in (0,b), z = -L$

$$\begin{pmatrix} [\partial/\partial\rho(Z_1 - \vartheta\partial/\partial z)] & [m\rho^{-1}(1 - \eta Z_1\partial/\partial z)] \\ [m\rho^{-1}(Z_1 - \vartheta\partial/\partial z)] & [\partial/\partial\rho(1 - \eta Z_1 \partial/\partial z)] \end{pmatrix} \begin{pmatrix} u^t_1 \\ u^t_2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{1.3}$$

Here Z_1 is the plate termination impedance, $\vartheta = i(\omega\varepsilon)^{-1}$, $\eta = i(\omega\mu)^{-1}$ and m is the number of the azimuth mode.

Taking the Fourier transform of (1.2) and (1.3) appropriately, we derive the transformed wave equations with unknown inhomogeneous terms comprising the field potentials and their normal derivatives on the surface of the interior planar termination, with the result that

$$\begin{pmatrix} \hat{T} & 0 \\ 0 & \hat{T} \end{pmatrix} \begin{pmatrix} U_1(\rho,\alpha) \\ U_2(\rho,\alpha) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ in } \rho > b \quad \text{for } |\tau| < k_2, \tag{1.4}$$

$$\begin{pmatrix} \hat{T} & 0 \\ 0 & \hat{T} \end{pmatrix} \begin{pmatrix} \Phi_1(\rho,\alpha) + e^{i\alpha L}\Psi_1^+(\rho,\alpha) \\ \Phi_2(\rho,\alpha) + e^{i\alpha L}\Psi_2^+(\rho,\alpha) \end{pmatrix} = e^{-i\alpha L} \times \begin{pmatrix} \tilde{g}_1(\rho) - i\alpha\tilde{f}_1(\rho) \\ \tilde{g}_2(\rho) - i\alpha\tilde{f}_2(\rho) \end{pmatrix} \text{ in } 0 < \rho < b \text{ for } \tau > -k_2, \tag{1.5}$$

where $\alpha = \text{Re } \alpha + i\text{Im}\alpha(\equiv \sigma + i\tau)$ with $l = 1, 2$, $\hat{T} = \lfloor d^2/d\rho^2 + \rho^{-1}d/d\rho - (\gamma^2 + m^2/\rho^2)\rfloor$, $\gamma = (\alpha^2 - k^2)^{1/2}$ with Re $\gamma > 0$, and $\tilde{f}_l(\rho)$, $\tilde{g}_l(\rho)$ are the unknown inhomogeneous terms defined by

$$\tilde{f}_l(\rho) = (2\pi)^{-1/2}u_l^t(\rho, -L),$$
$$\tilde{g}_l(\rho) = (2\pi)^{-1/2}\partial u_l^t(\rho, z)/\partial z|_{z=-L}. \tag{1.6}$$

The terms on the left-hand sides of (1.4) and (1.5) are the Fourier transforms of the unknown functions in (1.2) and (1.3), being defined by

$$U_l(\rho,\alpha) = (2\pi)^{-1/2}\int_{-\infty}^{+\infty} u_l(\rho,z)e^{i\alpha z}dz, \text{ for } \rho > b;$$
$$U_l(\rho,\alpha) = (2\pi)^{-1/2}\int_{-L}^{+\infty} u_l(\rho,z)e^{i\alpha z}dz, \text{ for } \rho < b \tag{1.7a}$$

$$U_l^+(\rho,\alpha) = \frac{1}{\sqrt{2\pi}}\int_{+L}^{+\infty} u_l(\rho,z)e^{i\alpha(z-L)}dz,$$
$$\Phi_l(\rho,\alpha) = \frac{1}{\sqrt{2\pi}}\int_{-L}^{+L} u_l^t(\rho,z)e^{i\alpha z}dz. \tag{1.7b}$$

It is found that $U_l^+(\rho,\alpha)$ are regular in the half-plane $\tau > -k_2$ and $\Phi_l(\rho,\alpha)$ with $l = 1, 2$ are entire functions. Using the notation as given by (1.7), we may express $U_l(\rho,\alpha)$ as

$$U_l(\rho,\alpha) = \Phi_l(\rho,\alpha) + e^{i\alpha L}\Psi_l^+(\rho,\alpha) - U_l^i(\rho,\alpha) \tag{1.8}$$

for $0 < \rho < b$, where

$$\Psi_1^+(\rho,\alpha) = U_1^+(\rho,\alpha) + Q_1^+(\rho,\alpha),$$
$$\Psi_2^+(\rho,\alpha) = U_2^+(\rho,\alpha), \tag{1.9}$$

$$Q_1^+(\rho,\alpha) = -\frac{c_{mj}^i e^{-\gamma_j L}}{i\sqrt{2\pi}(\alpha + i\gamma_j)}J_m(\xi_j\rho/b),$$
$$U_1^i(\rho,\alpha) = -\frac{c_{mj}^i e^{-i(\alpha+i\gamma_j)L}}{i\sqrt{2\pi}(\alpha + i\gamma_j)}J_m(\xi_j\rho/b). \tag{1.10}$$

The main idea is to derive the expressions of the functions in (1.6) in terms of the Fourier-Bessel and Dini series as well as the static terms with common unknown coefficients due to the correct separation of the variables for (1.2) and (1.3) and account the interaction of TM and TE waves. This allows finding the field image in Fourier transform domain. Since the scattered field for the region $\rho > b$ must vanish as $\rho \to \infty$ according to the radiation condition, we find by taking into account the boundary conditions at the termination the solutions of (1.4) and (1.5). This leads to a scattered field representation in the Fourier transform domain. Using the boundary conditions for the field components $e_z(\rho, z)$, $e_\varphi(\rho, z)$ at the cylindrical surfers with $\rho = b$ and the conditions of continuity for the field components $h_z(\rho, z)$, $h_\varphi(\rho, z)$ with $\rho = b$ and $L < z < \infty$ in the Fourier transform domain, we derive the Wiener-Hopf equation as well as the set of linear algebraic equations of the second kind after the factorization and decomposition procedure, which leads to a rigorous solution for arbitrary physical parameters. An approximate solution is further derived for the case where the dominant propagating TE and TM modes consecutively appear in the circular cavity of large depth.

2 Development of the Wiener-Hopf Technique for Wave Diffraction by a Semi-Infinite Truncated Cone. Axially symmetric case.

Conical surfaces are important for the study of wide -band antennas, target identification, waveguide junctions, and material defects. The problem of wave scattering by cones has been studied by means of different approaches such as asymptotic, numerical, and numerical-analytical methods. Accounting for the importance of such structures in the wide range of problems arising in technical physics as well as for the complexity of the analysis of the corresponding diffraction processes, this paper proposes a new rigorous method of solution for diffraction problems involving canonical, conical structures.

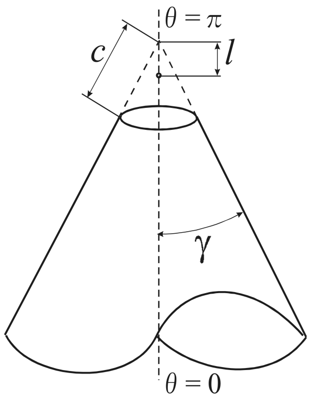


Fig.2 Geometry of the problem.

Let us consider the axial symmetric excitation of a truncated, perfectly conducting semi-infinite cone $C = \{c < r < \infty, -\pi < \varphi < \pi, \theta = \gamma\}$ by the radial electric dipole located at the conical axis, where (r, θ, φ) are spherical coordinates (see Fig. 2). The problem is to determine the unknown scalar TM potential $u(r, \theta)$ satisfying the Helmholtz equation

$$\Delta u + k^2 u = 0, \tag{2.1}$$

where k is free-space wavenumber and the boundary condition in a cone C

$$E_r^t(r,\theta)\left[\equiv \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}u^{(t)}\right)\right] = 0$$
$$\text{for } \theta=\gamma \text{ and } c<r<\infty. \tag{2.2}$$

Here $u = u(r, \theta)$ is the diffracted field and $u^{(t)} = u^{(t)}$ (r, θ) is the total field; the symbol Δ denotes the Laplacian in spherical coordinates (r, θ, φ) for axial symmetric case.

Let the total field $u^{(t)}(r, \theta)$ be given by

$$u^{(t)}(r,\theta) = \begin{cases} u^{(i)}(r,\theta)+u(r,\theta) \text{ for } 0<\theta<\gamma, \\ u(r,\theta) \text{ for } \gamma<\theta<\pi, \end{cases} \tag{2.3}$$

where $u^{(i)}(r, \theta)$ is the incident field excited in semi infinite perfectly conducting circular cone due to the radial electric dipole, which takes the form

$$u^{(i)}(r,\theta) = \frac{A_0}{2i\sqrt{srsl}}\int_\Gamma \frac{\nu[P_{\nu-1/2}(\cos\gamma)P_{\nu-1/2}(-\cos\theta)-P_{\nu-1/2}(-\cos\gamma)P_{\nu-1/2}(\cos\theta)]}{\cos\pi\nu\, P_{\nu-1/2}(\cos\gamma)} K_\nu(sl)I_\nu(sr)d\nu \tag{2.4}$$

with $\Gamma \in \Pi : \{|\text{Re } \nu| < 1/2\}$, $s = -ik$ and $\theta < 2\gamma$; $P_{\nu-1/2}(\cdot)$ is the Legendre function of the first kind, $I_\nu(\cdot)$, $K_\nu(\cdot)$ are the modified Bessel function of the first and second kinds respectively.

Taking the Kantorovich-Lebedev transform of (2.1), (2.2) and apply the condition of tangensial electric field component continuety $(E_r^t(r, \gamma + 0) = E_r^t(r, \gamma - 0))$ we derive the Kantorovich-Lebedev transform of the diffracted field as follows

$$\Phi(\nu,\theta) = \begin{cases} \frac{E_1(\nu,\gamma)}{\nu^2-1/4}\frac{P_{\nu-1/2}(\cos\theta)}{P_{\nu-1/2}(\cos\gamma)} \text{ for } 0<\theta<\gamma, \\ \frac{E_1(\nu,\gamma)}{\nu^2-1/4}\frac{P_{\nu-1/2}(-\cos\theta)}{P_{\nu-1/2}(-\cos\gamma)} \text{ for } \gamma<\theta<\pi. \end{cases} \tag{2.5}$$

Here

$$E_1(\nu,\gamma) = \int_0^c rE_r(r,\gamma)K_\nu(sr)\frac{dr}{\sqrt{r}}. \tag{2.6}$$

It follows from the boundary condition for tangential magnetic fields that

$$H_\varphi^{(t)}(r,\gamma+0) - H_\varphi^{(t)}(r,\gamma-0) = \begin{cases} 0 \text{ for } 0<r<c, \\ j_1(r) \text{ for } c<r<\infty. \end{cases} \tag{2.7}$$

Taking into account that $H_\varphi(r, \theta) = i\omega\varepsilon\partial/\partial\theta\{u\}$ and using the equations (2.5), (2.7) we arrive at the Wiener-Hopf equation that takes the form

$$E_1(\nu,\gamma)M(\nu,\gamma) - \frac{A_0}{2s\sqrt{l}}\frac{K_\nu(sl)}{P_{\nu-1/2}(\cos\gamma)} = -\frac{\pi\sin\gamma}{2i\omega\varepsilon}J_1(\nu,\gamma). \tag{2.8}$$

with $\nu \in \Pi$ $(\Pi(\equiv: \{|\text{Re } \nu| < 1/2\}))$, where $M(\nu, \gamma)$ is the kernel function and

$$J_1(\nu,\gamma) == \int_c^\infty [H_\varphi^{(t)}(r,\gamma-0)-H_\varphi^{(t)}(r,\gamma-0)]K_\nu(sr)\frac{dr}{\sqrt{r}}. \tag{2.9}$$

Let us represent the unknown function $E_1(\nu, \gamma)$ as follows

$$E_1(\nu,\gamma) = \frac{1}{2}E_1^+(\nu,\gamma)\left(\frac{sc}{2}\right)^\nu\Gamma(-\nu) + \frac{1}{2}E_1^-(\nu,\gamma)\left(\frac{sc}{2}\right)^{-\nu}\Gamma(\nu), \tag{2.10}$$

where

$$E_1^\pm(\nu,\gamma) = \Gamma(1\pm\nu)\left(\frac{sc}{2}\right)^{\mp\nu} \int_0^c rE_r(r,\gamma)I_{\pm\nu}(sr)\frac{dr}{\sqrt{r}}. \tag{2.11}$$

$E_1^\pm(\nu, \gamma)$ are an analytic functions at the half planes Re $\nu^{>-1/2}_{<1/2}$; $E_1^\pm(\nu, \gamma) = O(\nu^{-1/2})$ for $\nu \to \infty$ at the half planes of regularity. The kernel function is factorized as $M(\nu, \gamma) = M_+(\nu, \gamma)M_-(\nu, \gamma)$, where $M_+(\nu, \gamma) = M_-(-\nu, \gamma)$ and split functions $M_\pm(\nu, \gamma)$ are regular ad

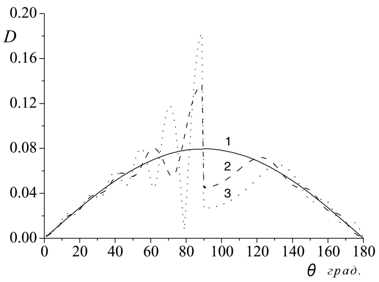


Fig.3 *Far field pattern for semi-infinite truncated cone:* $kc = 24$, $\gamma = 89°$: 1 – $kl = 0.01$; 2 – $kl = 5$; 3 – $kl = 10$

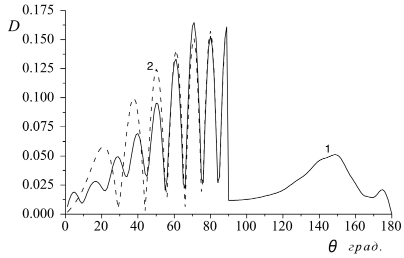
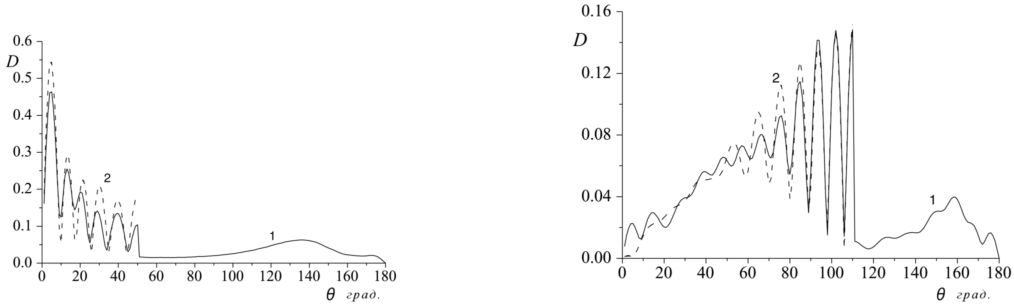


Fig.4 *Far field pattern:* $kc = 24$, $kl = 20$, $\gamma = 89°$: 1 – *semi-infinite truncated cone*; 2 – *semi-infinite cone.*



A) б)

Fig.5 *Far field pattern:* 1 – *semi-infinite truncated cone*, 2 – *semi-infinite cone* $kc = 24$, $kl = 23.5$; *a)* $\gamma = 50°$; *б)* $\gamma = 110°$

nonzero in Re $\nu^{>-1/2}_{<1/2}$, and show the asymptotic behaviour $M_\pm(\nu,\gamma) = O(\nu^{-1/2})$ as $\nu \to \infty$ with Re $\nu^{>-1/2}_{<1/2}$. Having used the edge condition, we find that the unknown functions in (2.9) and (2.11) behave like $E_1^+(\nu,\gamma) = O(\nu^{-1/2})$ for $\nu > -1/2$, $E_1^-(\nu,\gamma) = O(\nu^{-1/2})$ and $J_1(\nu,\gamma)(sc/2)^{-\nu}\Gamma^{-1}(-\nu) = O(\nu^{-3/2})$ for $\nu < 1/2$ as $\nu \to \infty$ and $J_1(\nu,\gamma)$ is an entire function. Multiplying both sides of (2.8) by $2(sc/2)^{-\nu}M_-^{-1}(\nu,\gamma)\Gamma^{-1}(-\nu)$ and applying the decomposition procedure we derive that

$$\begin{aligned} &E_1^+(\alpha,\gamma)M_+(\alpha,\gamma) \\ &+\left[E_1^-(\alpha,\gamma)(sc/2)^{-2\alpha}M_+(\alpha,\gamma)\Gamma(\alpha)/\Gamma(-\alpha)\right]^+ \\ =&\frac{A_0\pi}{s\sqrt{l}}\left[\frac{(sc/2)^{-\alpha}K_\alpha(sl)}{M_-(\alpha,\gamma)P_{\alpha-1/2}(\cos\gamma)\Gamma(-\alpha)}\right]^+. \end{aligned} \tag{2.12}$$

Here

$$[\cdots]^+ = -\frac{1}{2\pi i}\int_\Gamma [\cdots]\frac{d\nu}{\nu-\alpha}$$

with Re $\alpha >$ Re ν and $\nu \in \Pi$ defines the regular function in the right half plane Re $\alpha > -1/2$. This leads to the exact solution, which involves a numerical solution of the set of linear algebraic equations of the second kind. The results are valid for arbitrary physical parameters. It is to be noted that for the static case ($k \to 0$), we obtain the analytical solution explicitly. Based on the above results, we have carried out numerical computation for various physical parameters. Illustrative numerical examples presented for the radiation patterns of amplitudes of the field components H_φ that show in the Fig. 3-5 with $D(\theta) = \lim_{r\to\infty}|rH_\varphi(r,\theta)e^{-ikr}|$.

Conclusions. An exact solutions of two new problems we have presented here. The electromagnetic wave radiation from an open end of a circular waveguide cavity formed by a perfectly conducting cylinder and an internal plate termination with non zero impedance that is excited non symmetrically by TM dominant mode is the first problem that has been solved. This solution has wide range of application in particular can be used for design a new approaches as well as a benchmark for the comparison of approximate technique applied to more practical problems. This problem has formulated as a vector diffraction problem for two scalar potentials. The method of the solution presented here is a generalisation of the approach we have established previously for the analysis of the perfectly conducting parallel-plate and circular waveguide cavities with a planar termination. The key result is the correct analytical representation of the unknown scalar potentials in the Fourier transform domain which shows the TM and TE waves interaction at the open end and takes into account the impedance boundary conditions at the termination. These allow to recast the vector diffraction problem as a coupled Wiener-Hopf equations with respect to unknown analytical functions in an overlap complex half-planes. Finally, the problem is reduced to infinite system of linear algebraic equations due to the factorization and decomposition procedure.

We also considered the mixed value boundary problem for the Helmholtz equation in a spherical coordinate system for the conical region which is bounded by the circular perfectly conducting truncated cone. It is the second problem that was exactly solved here. For this purpose a new approach is proposed. The scheme of the solution includes applying of the Kontorovich-Lebedev transformation, derivation of the Wiener-Hopf equation and its reduction to the set of linear algebraic equation of the second kind. We analyse the Wiener Hopf equation for the case of an axial symmetric excitation of the semi infinite truncated cone by the radial electric dipole (E polarization wave diffraction problem) and the representative numerical results for far field pattern is presented. An exact analysis of this problem by an alternative semi-inversion technique was presented recently by Kuryliak, D., *Radiophysics and Radio Astronomy*, vol.4, no.2, pp.121–128, 1999; vol.5, no.3, pp.284–290, 2000.

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References

[1] Kuryliak, D. B., S. Koshikawa, K. Kobayashi, and Z. T. Nazarchuk, “Rigorous analysis of the vector diffraction problem for a cylindrical waveguide cavity”, *Reports of National Academy of Sciences of Ukraine* - 2001. - no.3, pp.85–94.

[2] Kuryliak D. B., S. Koshikawa, K. Kobayashi, and Z. T. Nazarchuk, “Wiener-Hopf technique for the diffraction by circular waveguide cavity”, *Proc. Progress in Electromagnetic Research Symposium (PIERS 2001)*, July 18–22, 2001, Osaka, Japan, p.327.

[3] Kuryliak D. B., S. Koshikawa, K. Kobayashi, and Z. T. Nazarchuk, “Wiener-Hopf technique for the vector diffraction by circular waveguide cavity”, *Digest Asia-Pacific Radio Science Conference (AP-RASC’01)*, August 1–4, 2001, Tokyo, Japan, p.295.

[4] Kuryliak D. B., K. Kobayashi, S. Koshikawa, and Z. T. Nazarchuk, “Wiener-Hopf analysis of the axial symmetric wave diffraction problem for a circular waveguide cavity”, *International Workshop on Advanced Electromagnetics (IWAE’01)*, July 30–31, 2001, Tokyo, Japan, p.32.

[5] Kuryliak D. B., K. Kobayashi, S. Koshikawa, and Z. T. Nazarchuk, “Axial symmetric wave diffraction by a circular waveguide cavity”, *Proc. 2002 International Conference on Mathematical Methods in Electromagnetic Theory* (MMET*02), September 10–13, 2002, Kiev, Ukraine, pp.370–372.

[6] Kuryliak D. B., K. Kobayashi, S. Koshikawa, Z. T. Nazarchuk, “Axial symmetric wave diffraction by a circular waveguide cavity”, *Proc. 2002 Asia-Pacific Microwave Conference (APMC 2002)*, November 19–22, 2002, Kyoto, Japan, *to be presented.*

[7] K. Kobayashi, D. B. Kuryliak, S. Koshikawa, Z. T. Nazarchuk, E. I. Veliev, T. Owaki, T. Sanai, K. Yamagami “Development of New Rigorous Methods for Wave Scattering Problems and Their Applications to Radar Cross Section Analysis and Non-Destructive Testing”, Jour. Inst. Sci. and Eng., Chuo Univ. no.9, 32–37, 2002.

[8] Kuryliak D. B., K. Kobayashi, S. Koshikawa, Z. T. Nazarchuk, “Diffraction by a truncated, semi-infinite cone: comparison of the Wiener-Hopf and semi - inversion methods” *Proc. Progress in Electromagnetic Research Symposium (PIERS 2003)*, October 13–16, 2003, Hawaii, Honolulu, USA, *to be presented.*

[9] Kuryliak D. B., K. Kobayashi, S. Koshikawa, Z. T. Nazarchuk, “Wiener-Hopf analysis of the diffraction by a circular waveguide cavity: axial symmetric case” *Proc. Progress in Electromagnetic Research Symposium (PIERS 2003)*, October 13–16, 2003, Hawaii, Honolulu, USA, *to be presented.*